Tribhuvan University

Institute of Science and Technology
Course of Study for Four Year Mathematics

Course Title: Linear Algebra
Course No. : MAT 201
Level : B.Sc.
Nature of Course: Theory

Full Marks: 75
Pass Mark: 26.25
Year: II
Lecture: 150 Hrs.

Course Description

This course is designed for second year of Four years B.Sc. program. The main aim of this course is to provide knowledge of linear algebra.

Course Objective

The main objectives of this course structure is to enable the students

(i) To develop in-depth knowledge and good theoretical background in linear algebra;
(ii) To take up higher studies;
(iii) To sustain interest in and promote enjoyment of linear algebra and its applications in various branches of mathematics and physical and social sciences;
(iv) To get associated with teaching in the field related to linear algebra.
(v) To compare with graduates from various other universities in the field of Linear algebra.

Course Contents

Unit 1 System of Linear Equations: Linear equations, System of linear equations, General systems of linear equations, Gaussian elimination, Elementary replacement and scale operations, Row- equivalent pairs of matrices, Elementary row operations, Reduced row echelon form, Row echelon form, Intuitive interpretation, Application: Feeding Bacteria. [15 Lectures]

Unit 2 Vectors and Matrices: Vectors, Linear combinations of vectors, Matrix- vector products, The span of a set of vectors, Interpreting linear systems, Row- equivalent systems, Consisting and inconsistent systems, Kernel or null space of a matrix, Homogeneous equations, Uniqueness of the reduced row echelon form, Rank of a matrix, General solution of a system, Matrix- matrix product, Indexed sets of vectors: Linear dependence and independence, Using the row- reduction process, Determining linear dependence or independence, Application: Linear ordinary differential equations. [15 Lectures]

Unit 3 Vector Spaces: n- tuples and vectors, Vector addition and multiplication by scalar, Properties of $\mathbb{R}^n$ as a vector space, Linear combinations, Span of a set of vectors, Geometric interpretation of vectors, Line passing through origin, Lines in $\mathbb{R}^2$, lines in $\mathbb{R}^3$, Planes in $\mathbb{R}^3$, Lines and planes in $\mathbb{R}^n$, General solution of a system of equations , Applications: Elementary mechanics, network problems, traffic flow. [15 Lectures]

Unit 4 Linear Transformation: Functions, Mappings, and transformations, Domain, Co- domain, and range, various examples, Injective and surjective mappings, Linear transformations, Using matrices to define linear maps, Injective and surjective linear transformations, Effects of linear transformations, Effects of transformations on geometrical figures, Composition of two linear mapping, Vector spaces, Theorems on Vector spaces, Various examples, Linearly dependent sets, Linear mapping, Application: Models in economic theory. [15 Lectures]

Unit 5 Matrix Operations: Matrix addition and scalar multiplication, Matrix- matrix multiplication, Pre-multiplication and post- multiplication, Dot product, Special matrices, Matrix transpose, Symmetric matrices, Skew-symmetric matrices, Non- commutativity of matrix multiplication, Associativity law for matrix multiplication, Linear transformations, Elementary matrices, More on the matrix- matrix product, Vector- matrix product, Solving systems with a left inverse, Solving systems with right inverse, Analysis, Square matrices, Invertible matrices, Elementary matrices and LU factorization, Computing an inverse, More on left and right Inverse of Non- square matrices, Invertible matrix, Application: Diet problems theorem. [15 Lectures]

Unit 6 Determinants: Properties of determinants, An algorithm for computing determinants, Algorithm without scaling, Zero determinant, Calculating areas and volumes, Minors and cofactors, Direct methods for computing determinants, Properties of determinants, Cramer's rule, Planes in $\mathbb{R}^n$, Computing inverses using determinants. [15Lectures]

Unit 7 Vector Subspaces: Introduction, Linear transformations, Revisiting kernels and null spaces, The row space and column space of a matrix, Basis for a vector spaces, Coordinate vector, Isomorphism and equivalence relations, Finite- dimensional and infinite- dimensional vector spaces, Linear transformation of a set, Dimensions of various subspaces, Coordinate vectors, Changing coordinates, Linear transformations, Mapping a vector space into itself, Similar matrices, More on equivalence relations. [15 Lectures]

Unit 8 Eigen Systems: Introduction, Eigenvectors and eigenvalues, Using determinants in finding eigenvalues, Linear transformations, Distinct eigenvalues, Bases of eigenvectors, Characteristic equation and Characteristic polynomial, Diagonalization involving complex numbers, Application: Powers of a matrix. [15 Lectures]

Unit 9 Inner- Product Vector Spaces: Inner product spaces and their properties, The norm in an inner- product space, Distance function, Mutually orthogonal vectors, Orthogonal projection, Angle between vectors, Orthogonal compliments, Orthonormal bases, Subspaces in inner- product spaces, The Gram- Schmidt algorithm, Modified Gram- Schmidt process, Linear least- square solution. [15 Lectures]

Unit 10 Additional Topics : Introduction, Hermitian Matrices and self-adjoint mappings, Self- adjoint mapping, Unitary and orthogonal matrices, The Cayley- Hamilton theorem, Quadratic forms, Permutation matrix, LU-factorization, QR- factorization, Partitioned matrices, solving a system having a 2×2 block matrix, Richardson iterative method, Jacobi iterative method, Gauss- Seidel method. [15 Lectures]

Text book

1. Ward Cheney & David Kincaid; *Linear Algebra Theory and Applications*, Jones and Bartlett India Pvt. Ltd.

Reference books

2. S. Lang; *Introduction to Linear Algebra*, Second Edition, Springer.
3. R.M. Shrestha & S. Bajracharya; *Linear Algebra, Groups, Rings & Theory of Equations*, Sukunda Pustak Bhavan, Kathmandu .
4. T.P. Nepal, C.R. Bhatta & Ganga Ram D.C. ; *A Text Book on Algebra*, Pradhan Book House Exhibition Road, Kathmandu.
5. H.N. Bhattarai & G.P. Dhakal; *Undergraduate Algebra*, Vidharthi Pustak Bhandar, kathmandu.
6. B.S. Vatssa; *Theory of Matrices*, Wiley Eastern Ltd.

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Course of Study for Four Year Mathematics

Course Title: Differential Equations **Full Marks:** 75
Course No. : MAT 202 **Pass Mark:** 26.25
Level : B.Sc. **Year:** II
Nature of Course: Theory **Lecture :** 150 Hrs.

Course Description

This course is designed for second year of Four years B.Sc. program. The main aim of this course is to provide knowledge of Differential Equations.

Course Objectives: The objective of this course is to acquaint students with the basic concepts of differential equation like first order linear and nonlinear differential equations, second order differential equations and higher order linear equations as well as partial differential equation. It aims at enabling students to build good knowledgebase in the subject of ordinary differential equations and partial differential equation.

Course Contents

Unit 1: Definition and classification of differential equations, Solutions of differential equations, Some mathematical models and direction fields [15 Lectures]

Unit 2: First Order Linear and Nonlinear Differential Equations: Integrating factors, Separable equations, Modeling with first order equations, Difference between the linear and nonlinear equations, Autonomous equations and population dynamics, Exact equations and integrating factors, Numerical approximations, Euler's method, Existence and uniqueness theorem, First order difference equations. [17 Lectures]

Unit 3: Second Order Linear Equations: Homogeneous equations with constant coefficients,Solutions of linear homogeneous equation, The Wronskian, Complex roots of the characteristic equation, Repeated roots, Reduction of order, Nonhomogeneous equations, Method of undetermined coefficients, Variation of parameters, Mechanical and electric vibrations, Forced vibrations. [15 Lectures]

Unit 4: Higher Order Linear Equations: General theory of nth order linear equations, Homogeneous equations with constant coefficients, Method of undetermined coefficients, Method of variation of parameters. [15 Lectures]

Unit 5: System of First Order Linear Equations: Introduction, Review of matrices, Linear algebraic equations; Linear independence, Eigenvalues, Eigenvectors, Basic theory of first order linear equations. [15 Lectures]

Unit 6: Differential Equations of the First Order but not the First Degree: Equations solvable for p, Equations solvable for y, Equations solvable for x, Equations solvable for x and y, Equations homogeneous in x and y, Clairaut's equation. [8 Lectures]

Unit 7: Partial Differential Equations of the First Order: Partial differential equations, Origin of First order partial differential equations, Cauchy problem for first order equations, Linear equations of the first order, Integral surface passing through a given curve, Surfaces orthogonal to a given system of surfaces, Charpit's method, Special types of first-order equations. [20 Lectures]

Unit 8: Partial Differential Equations of the Second Order: The origin of second order equation, Linear equations with constant coefficients, Equations with variable coefficients, Nonlinear equations of the second order (Monge's Method). [15 Lectures]

Unit 9: Partial Differential Equations and Fourier Series: Two-point boundary value problems, Fourier series, Fourier convergence theorem, Even and odd functions. [15 Lectures]

Unit 10: Separation of Variables: Heat conduction in a rod, Other heat conduction problems, Wave equation, Vibration of an elastic string, Laplace's equation. [15 Lectures]

Text books

1. Boyce, W. and DiPrima, R.; *Elementary Differential Equations and BoundaryValue Problems*, 9th Ed., Wiley India.
2. Ian Sneddon; *Elements of Partial Differential Equations*, McGraw Hill International Editions.
3. Zafar Ahsan, *Differential Equations and Their Applications*, Second Edition, Printice Hall of India, 2005.

Reference book

4. James C. Robinson; *An Introduction to Ordinary Differential Equations*, Cambridge University Press