Numericals: Remaining.

P.S.: Marks are flexible

(Theory only).

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References: Communication systems, Dr. Sanjay Sharma.
Handwritten note of communication System - I
Old question solution.

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## 1. Introduction (3 Hours / 5 marks)

Block diagram of digital communication system : 73sh[5]

The following figure shows the model of a digital communication system. The overall purpose of the system is to transmit the message or sequences of symbols coming out of a source to a destination point at as high a rate and accuracy as possible.
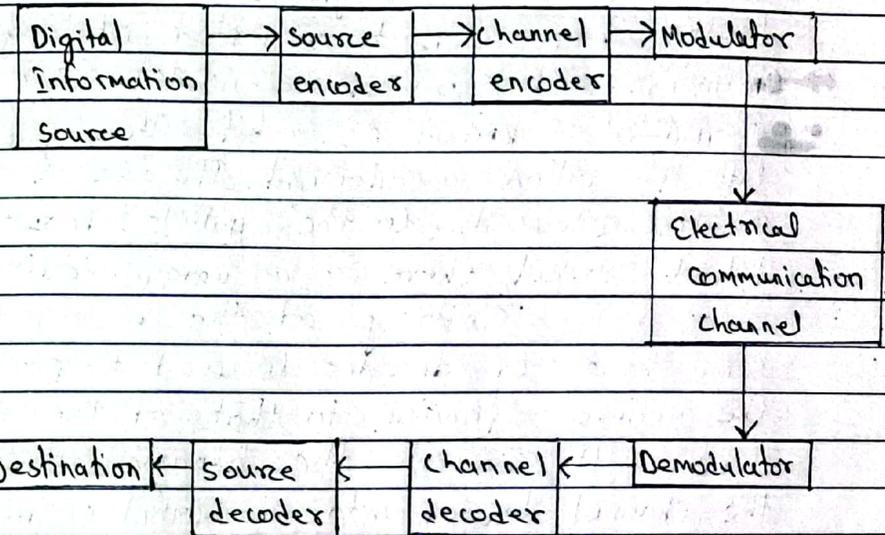


① Digital information source: In case of DIS, the information source produces a message signal which is not continuously varying with time. Rather

the message signal is discontinued with time.
The output of discrete information sources
consists of a sequence of discrete
symbols or letters. An analog information source
may be transformed into discrete information
source through the process of sampling &
Quantizing.

(ii) Source Encoder and Decoder: The symbols produced
by the information source are given to the
source encoder. These symbols cannot be transmitted
directly. They are first converted into digital
form (0's & 1's) by source encoder. Each binary
'1' and '0' is known as a bit. The group of
bits is called a codeword. The source encoder
assigns codewords to the symbols. For each
distinct symbol, there is a unique codeword.
        Similarly, at the receiver end,
some sort of decoder is used to perform
the reverse operation to that of the source
encoder. It converts the binary output of
the channel decoder into a symbol sequence.
Some decoders also use memory to store codewords.
The decoders and encoders are can be synchronous
or asynchronous.

(iii) Channel encoder and decoder: After signal conversion
via source encoder, the signal is transmitted
through the channel. The communication channel adds
noise and interference to the signal being
transmitted. Hence errors are introduced in the
binary sequence received at the receiver end.
Thus channel coding is done to avoid these
kind of errors. Channel encoder adds some
redundant binary bits to the input sequence.
These redundant bits are always added with
some properly defined logic.
Similarly channel decoder at receiving end
decodes these redundant bits and checks
for any errors thus reconstructing error free
accurate bit sequence reducing the effects of
channel noise and distortion.

(iv) Digital Modulators and Demodulators: Modulation
is the process of superimposing low frequency
message signal on high frequency carrier signal
for efficient transmission of signal. Similarly
Demodulation at receiver gets the original message
signal back from modulated signal.
Here in digital communication system, digital
modulation techniques are used. The carrier signal
used by digital modulators is always continuous

sinusoidal wave of high frequency. Amplitude shift keying (ASK), Phase shift keying (PSK), frequency shift keying (FSK), differential phase shift keying (DPSK) and minimum shift keying (MSK) are the examples of various digital modulators. However, since these modulators use a continuous carrier wave, therefore they are known as digital CW modulators. At the receiving end, the digital demodulator converts the input modulated signal into the sequence of binary bits.

ⓒ Communication channel: The connection between transmitter and receiver is established through a communication channel. The communi--cation can take place through wireline, wireless or fiber optic channels. The other media such as optical disks, Magnetic tapes & disks, etc may also be called as a communica--tion channel since they can also carry data through them.

Advantages of Digital communication system: [9ch [2]

(i) The digital communication systems are simpler and cheaper compared to analog communication systems because of advances made in the IC technology.

(ii) In digital communication, the speech, video and other data may be merged and transmitted over a common channel using multiplexing.

(iii) Using data encryption, only permitted receivers may be allowed to detect the transmitted data.

(iv) Since the transmitted signal is digital in nature, therefore a large amount of noise interference may be tolerated.

(v) Since in digital communication, channel coding is used, therefore errors may be detected and corrected in receivers.

(vi) Digital communication is adaptive to other advanced branches of data processing such as digital signal processing, image processing & data compression, etc.

76 ch Q) Define any three types of noises: 76ch [3]
= Noise in a communication system is basically
undesired or unwanted signals that get randomly
added to the actual information carrying signal.
They are subcategorized into:

(i) External noise: Natural & Man-made noise
(ii) Internal noise: Generated by electronic equipment
involved in the system itself.

Three types of noise:

① Thermal noise: Electrical signal is transmitted when
electrons move randomly. The random motion of
the electrons is the reason for the thermal
energy received by the conductor. These free electrons
are non-uniformly distributed within conductor.
Due to this a possibility exists that at one end
the number of free electrons will be comparatively
higher than at the other end. This non-uniform
distribution of electrons provides the average
voltage to be zero, however the average power is
not zero in this case. So, this non-zero power
is nothing but noise. And as it is the action of
thermal action, Hence also known as thermal noise
power. It is also known as Johnson or white noise,

(ii) Shot noise: It is the result of random variation
in the appearance of electrons and holes at the
output side of the device. These random movemen
-ts are the result of discontinuities in the
device which is used by the system. The shot
noise generates sound like several lead shots
are striking over a metal plate or tube.
Non-linearity or discontinuity in the system
generates shot noise.

(iii) Partition Noise: Here the name itself is indicating
the cause for generation of this type of no
As it gets generated when the system is
composed of multiple paths, & during the flow
the current gets divided in those paths. These
are nothing but the result of random variation
in the divisions. Due to this reason some devices
offer low partition noise while some offer,
high.

Source coding: 70 Asd [1]

A conversion of the output of a discrete memoryless source (DMS) into a sequence of binary symbols (ie, binary code word) is called source coding. The device that performs this conversion is called the source encoder. Figure below shows a source encoder.



| Discrete Memoryless Source (DMS) | $x_i$ | Source encoder | Binary sequence |
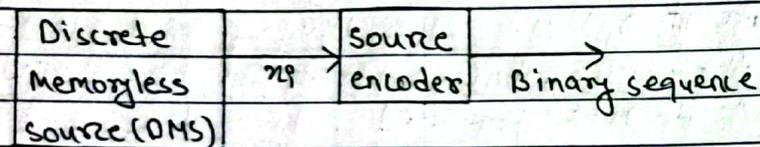
$x = \{x_1, x_2, \dots x_i, \dots x_m\}$

Fig: Block diagram for Source coding.

**Importance of source coding in Digital communication system:** 73ch, 71ch, 70ch, 69ch [6,2]

An objective of source coding is to minimize the average bit rate required for representation of the source by reducing the redundancy of the information source. By reducing the redundancy of the encoding, we can make the probability

of error approach zero.

**significance of variable length coding: w/ Example: 72ka8**

A variable length coding technique in which codeword length is not fixed. To obtain this, short codes are assigned to frequently occuring events and long codes are assigned to infrequent events. The advantage / significance of VLC is that it does not degrade the signal quality in any way. The reconstructed signal will exactly match the input signal so that if the signal is adequately described by a series of events, using VLCs to communicate them to the decoder will not change the events. A example is shown in table below

| $x_i$ | code 1 | code 2 | code 3 | code 4 |
|-------|--------|--------|--------|--------|
| $x_1$ | 0 | 0 | 0 | 1 |
| $x_2$ | 1 | 10 | 01 | 01 |
| $x_3$ | 00 | 110 | 011 | 001 |
| $x_4$ | 11 | 111 | 0111 | 0001 |

69ch[5] **Importance of channel coding:** They are used to protect the digital information from noise & interference & reduce the number of bit errors by introducing redundant bits

Source coding vs channel coding! 74 As [3]

| Source coding | Channel coding |
|---|---|
| • It is defined as the conversion of output of discrete memoryless source into a sequence of binary symbols. | It is defined as adding extra bits so that we can control error at the receiver side. |
| • It's main purpose is to reduce redundancy | It's main purpose is to reduce noise & error correction. |
| • This causes reduce in Bandwidth consumption | This causes extra Bandwidth consumption as extra bits are added. |
| • Its methods are<br>  - shannon-fano coding<br>  - Huffman coding | It's methods are<br>  - Block code<br>  - cyclic code |

Q) Explain shannon~Fano coding. 73 Sh [3]

= The steps for shannon~Fano algorithm are given below:

1) List the source symbols in the order of decreasing probability.

2) Partition the set into two sets that are as close to equiprobables as possible, & assign 0 to upper set and 1 to lower set

3) Continue this process, each time partitioning the sets with as nearly equal probabilities as possible until further partitioning is not possible.

An example is given below: In shannon fano coding the ambiguity may arise in the choice of approximately equiprobable sets.

| $x_i$ | $P(x_i)$ | step 1 | step 2 | step 3 | step 4 | code |
|---|---|---|---|---|---|---|
| $x_1$ | 0.30 | 0 | 0 | | | 00 |
| $x_2$ | 0.25 | 0 | 1 | | | 01 |
| $x_3$ | 0.20 | 1 | 0 | | | 10 |
| $x_4$ | 0.12 | 1 | 1 | 0 | | 110 |
| $x_5$ | 0.08 | 1 | 1 | 1 | 0 | 1110 |
| $x_6$ | 0.05 | 1 | 1 | 1 | 1 | 1111 |

Entropy $H(n) = -\sum\limits_{i=1}^{6} P_i \log_2 (P_i)$

$= \sum\limits_{i=1}^{6} P_i \log_2 (1/P_i)$

$= 0.3 \times \log_2\left(\dfrac{1}{0.3}\right) + 0.25 \times \log_2\left(\dfrac{1}{0.25}\right) + 0.20 \times \log_2\left(\dfrac{1}{0.20}\right)$

$+ 0.12 \log_2\left(\dfrac{1}{0.12}\right) + 0.08 \log_2\left(\dfrac{1}{0.08}\right) + 0.05 \log_2\left(\dfrac{1}{0.05}\right)$

$= 2.36$ bits/symbol.

Average length $(L) = \sum\limits_{i=1}^{6} P(x_i) n_i$

$= 2(0.3) + 2(0.25) + 2(0.2) + 3(0.12) +$
$\quad 4(0.08) + 4(0.05)$

$= 2.38$ bits/symbol.

efficiency, $\eta = \dfrac{H(n)}{L} \times 100\% = 99\%.$
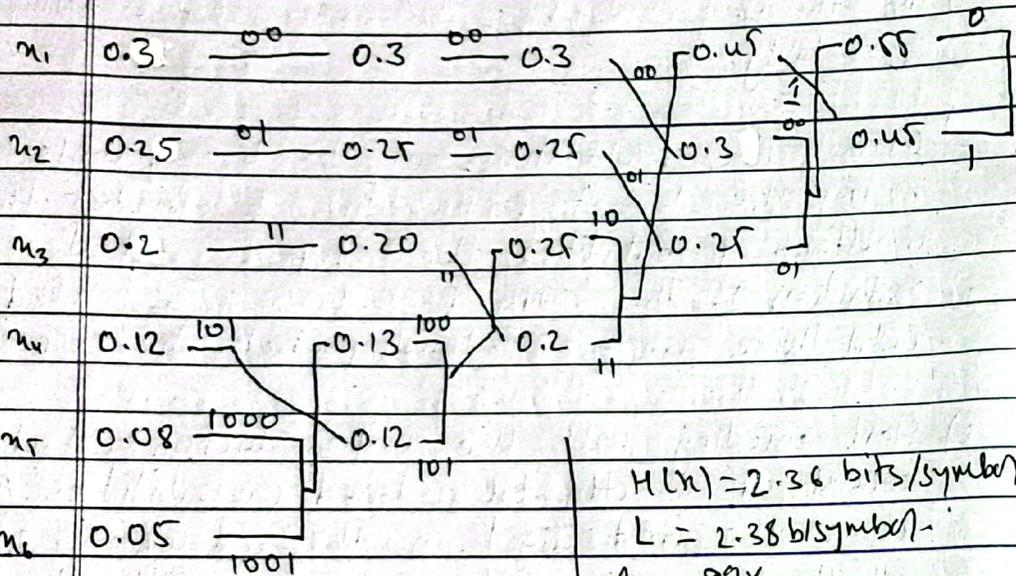
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Huffman coding: 72ch[4], 70ch[3]

Huffman encoding results in optimum code. Thus it is the code that has the highest efficiency. The huffman encoding procedure is as follows:

1) list the source symbols in order of decreasing probability.

2) Combine the probabilities of the two symbols having the lowest probabilities, & reorder the resultant probabilities, this step is called reduction 1. The same procedure is repeated until there are two ordered probabilities remaining.

3) Start encoding with the last reduction, which consist of exactly two ordered probabilities. Assign 0 as the first digit in the codewords for all the source symbols associated with the first probability; assign 1 to the second probability.

4) Now go back & assign 0 and 1 to the second digit for the two probabilities that were combined in the previous reduction step, Retaining all assignments made in step 3.

5) keep regressing this way until the first column is reached.

Lets take the same example taken above.

| $x_i$ | $P(x_i)$ | code |
|---|---|---|

$x_1$ | 0.3 $\xrightarrow{00}$ 0.3 $\xrightarrow{00}$ 0.3 $\xrightarrow[00]{}$ 0.45 $\xrightarrow[00]{}$ 0.55 $\xrightarrow{0}$

$x_2$ | 0.25 $\xrightarrow{01}$ 0.25 $\xrightarrow{01}$ 0.25 | 0.3 $\xrightarrow[00]{1}$ 0.45 $\xrightarrow{1}$

$x_3$ | 0.2 $\xrightarrow{11}$ 0.20 | 0.25 0.25 $\xrightarrow[01]{}$

$x_4$ | 0.12 $\xrightarrow{101}$ 0.13 $\xrightarrow{100}$ 0.2 $\xrightarrow[11]{}$

$x_5$ | 0.08 $\xrightarrow{1000}$ 0.12 $\xrightarrow{101}$

$x_6$ | 0.05 $\xrightarrow{1001}$

$H(x) = 2.36$ bits/symbol
$L = 2.38$ b/symbol.
$\eta = 99\%$.

$$\therefore \quad 0.3 \quad 00$$
0.25    01
0.2    11
0.12    101
0.08    1000
0.05    1001

Sampling theory: (4 Hours / 8 Marks)

* Sampling theorem: Nyquist theorem and its proof.
76ch[3] 7 3sh [5] , 78Bh[2], 72ch[2], 71ch[2], 74t6[4]

Sampling _____ is the fundamental operation in signal processing. It converts continuous time signal to discrete time signal.
The statement of sampling theorem can be given in two parts:

(i) A band-limited signal of finite energy, which has no frequency-component higher than $f_m$ Hz, is completely described by its sample values at uniform intervals less than or equal to $1/2f_m$ second apart.

(ii) A band-limited signal of finite energy, which has no frequency components higher than $f_m$ Hz, may be completely recovered from the knowledge of its samples taken at rate of $2f_m$ samples per second.

The first part represents the representation of signal in its samples & minimum sampling rate

required to repeat a continuous-time signal into its samples.

The second part of the theorem represents reconstruction of the original signal from its samples. It gives sampling rate required for satisfactory reconstruction of signal from its samples.

Combining two parts, the sampling theorem may be stated as under:

"A continuous-time signal may be completely represented in its samples & recovered back if the sampling frequency is $f_s \geq 2f_m$. Here $f_s$ is the sampling frequency & $f_m$ is the maximum frequency present in the signal."

Proof: 73 sh[5],

To prove the sampling theorem, we shall show that a signal whose spectrum is band-limited to $f_m$ Hz, can be reconstructed exactly without any error from its samples taken uniformly at a rate $f_s > 2 f_m$ Hz

Let us consider continuous time signal $x(t)$ whose spectrum is band-limited to $f_m$ Hz. This means

that the signal $x(t)$ has no frequency components beyond $f_m$ Hz. Therefore, $X(\omega)$ is zero for $|\omega| > \omega_m$, i.e,

$$X(\omega) = 0 \quad \text{for} \quad |\omega| > \omega_m$$
$$\text{where,} \quad \omega_m = 2\pi f_m.$$

figure ⓐ shows ct signal $x(t)$. Let $X(\omega)$ be its FT or frequency spectrum as shown in figure ⓑ. Sampling of $x(t)$ at a rate of $f_s$ Hz ($f_s$ samples per second may be achieved by multiplying $x(t)$ by an impulse train $\delta_{Ts}(t)$. The impulse train consists of unit impulses repeating periodically every $T_s$ seconds, where $T_s = 1/f_s$. This multiplication results in sampled signal $g(t)$. This sampled signal consists of impulses spaced every $T_s$ seconds (the sampling interval).

The resulting or sampled signal may be written as
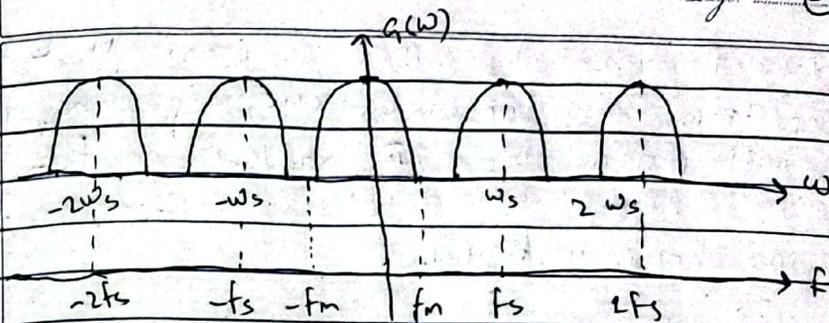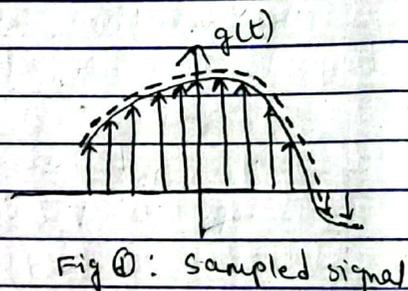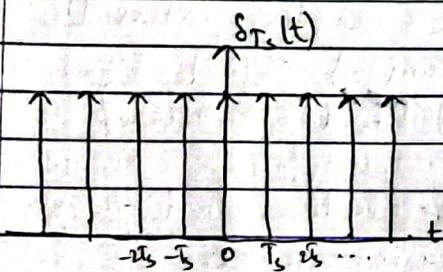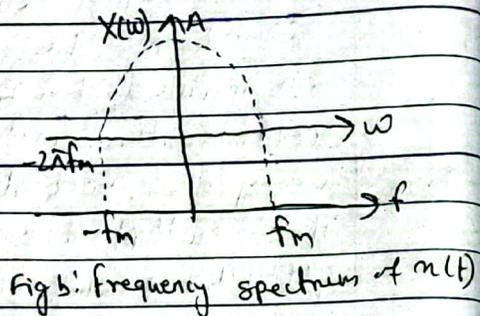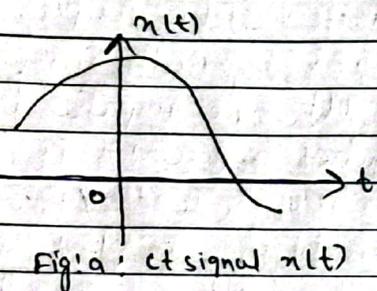
$$g(t) = x(t) \, \delta_{Ts}(t) \quad \text{——①}$$

Since, $\delta_{Ts}(t)$ is a periodic signal it can be represented as fourier series:

$$\delta_{Ts}(t) = \frac{1}{T_s} [1 + 2\cos\omega_s t + 2\cos2\omega_s t + 2\cos3\omega_s t \cdots] \quad \text{——⑪}$$

Here, $W_s = \dfrac{2\pi}{T_s} = 2\pi f_s.$

Putting the value of $\delta_{Ts}(t)$ from ① in eq^n ⓘ
the sampled signal is

$$g(t) = \frac{1}{T_s}\Big[ x(t) + 2x(t)\cos W_s t + 2x(t)\cos 2W_s t + $$
$$2x(t)\cos 3W_s t + \cdots \Big] \quad —— ⓘⓘⓘ$$



Fig a: ct signal $x(t)$



Fig b: frequency spectrum of $x(t)$



Fig c: impulse train as sampling function



Fig ⓓ: sampled signal

Fig e: spectrum of sampled signal.

Now to obtain $G(W)$ we have to take fourier transform of right hand side.

FT of $x(t) \longrightarrow X(W)$
FT of $2x(t)\cos W_s t \to [X(W-W_s) + X(W+W_s)]$
FT of $2x(t)\cos 2W_s t \to [X(W-2W_s) + X(W+2W_s)]$
and so on.

Therefore on taking FT, eq^n ③ becomes

$$G(W) = \frac{1}{T_s}\Big[ X(W) + X(W-W_s) + X(W+W_s) + X(W-2W_s)$$
$$+ X(W+2W_s) + X(W-3W_s) + X(W+3W_s)$$
$$+ \cdots \Big] \quad —— ④$$

or, $G(W) = \dfrac{1}{T_s}\displaystyle\sum_{-\lambda}^{\alpha} X(W - nW_s) \quad —— ⑤$

Thus from eq$^n$ (iv) & (v) it is clear that the spectrum $G(\omega)$ consists of $X(\omega)$ repeating periodically with period $\omega_s = \dfrac{2\pi}{T_s}$ rad/sec. or $f_s = \dfrac{1}{T_s}$ Hz

as shown in fig (e).

Now, it we have to reconstruct $x(t)$ from $g(t)$, we must be able to recover $X(\omega)$ from $G(\omega)$. This is possible if there is no overlap between successive cycles of $G(\omega)$. fig (e) shows that this requires

$$f_s > 2 f_m \quad\quad — (vi)$$

But the sampling interval $T_s = 1/f_s$

Hence, $\quad T_s < 1/2 f_m \quad\quad — (vii)$

Therefore, as long as the sampling frequency $f_s$ is greater than twice the maximum signal frequency $f_m$, $G(\omega)$ will consist of non-overlapping repetitions of $x(\omega)$. If this is true, fig (e) shows that $x(t)$ can be recovered from its samples $g(t)$ by passing the sampled signal $g(t)$ through an ideal low pass filter of bandwidth $f_m$ Hz. This provides the sampling theorem.

Factors to consider while sampling:

(i) The sampled signal waveform consists of finite amplitude & duration pulses, rather than ideal pulses

(ii) Reconstruction filters are not ideal

(iii) Input waveforms are time limited rather than band limited.

Q) If two bandlimited signals $x_1[t]$ and $x_2[t]$ have bandwidths of $\omega_1$ & $\omega_2$ Hertz respectively, estimate the maximum sampling interval required for the signal given by $y[t] = x_1[t] x_2[t]$.

= Sol$^n$

$$y[t] = x_1[t] \, x_2[t] \longrightarrow y[j\omega] = \dfrac{1}{2\pi}\{x_1(j\omega) * x_2(j\omega)\}$$

we know that,
$$x_1(j\omega) \to 0 \text{ for } |\omega| > \omega_1$$
$$x_2(j\omega) \to 0 \text{ for } |\omega| > \omega_2$$

convolution of two signal will result in a signal that is non-zero with at least one of the signal being non-zero.

Therefore, $y(j\omega) = 0$ for $|\omega| > (\omega_1 + \omega_2)$

Nyquist rate = 2 Wm

$$= 2(w_1 + w_2)$$

Max$^m$ sampling interval = $\dfrac{2\pi}{\text{min sampling frequency}}$

$$= \dfrac{2\pi}{2(w_1 + w_2)}$$

$$= \dfrac{\pi}{w_1 + w_2} \,//$$

Q) with mathematical derivations show that original bandlimited signal can be reconstructed from its samples taken at Nyquist rate. 70 Asd [5]

= The low pass filter that is used to recover original signal from its samples is known as interpolation filter. Since an ideal low pass filter is practically impossible we use a practical low-pass filter as shown in figure below.
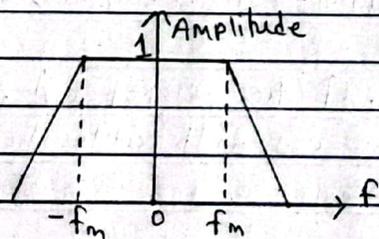


Fig: Practical low pass filter.

The process of reconstructing a continuous time signal $n(t)$ from its samples is called as interpolation As discussed earlier, a signal $n(t)$ band limited to $f_m$ Hz can be reconstructed (interpolated) completely from its samples by passing through an ideal low passed filter of cut off frequency. $f_m$ Hz.

The expression for sampled signal is written as

$$g(t) = n(t) \cdot \delta_{Ts}(t) \qquad —①$$

$$= \dfrac{1}{T_s}\left[ n(t) + 2n(t)\cos w_s t + 2n(t)\cos 2w_s t + \cdots \right] \qquad —②$$

From above eq$^n$ it can be seen that sampled signal contains a component $\dfrac{1}{T_s} \times n(t)$.

To recover $n(t)$, sampled signal must be passed through LPF of BW $f_m$ Hz and gain $T_s$. Therefore, the reconstruction or interpolating filter Transform function is expressed as;

$$H(w) = T_s \times \text{rect}\left( \dfrac{w}{4\pi f_m} \right) \qquad —③$$

The impulse response $h(t)$ of this filter is the inverse fourier transform of $H(\omega)$, i.e,

$$h(t) = F^{-1}[H(\omega)] = F^{-1}\left[ T_s \, \text{rect}\left(\frac{\omega}{4\pi f_s}\right) \right]$$

$$= 2 f_m T_s \, \text{Sinc}(2\pi f_m t) \quad —(iv)$$

Assuming sampling is done at Nyquist rate, then

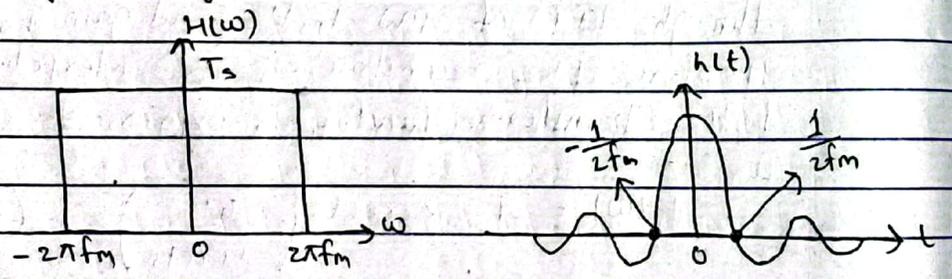$$T_s = \frac{1}{2 f_m}$$

$$2 f_m T_s = 1 \quad —(v)$$

Putting the value of $2 f_m T_s$ in eq$^n$ (iv)

$$h(t) = 1 . \, \text{Sinc}(2\pi f_m t) = \text{Sinc}(2\pi f_m t) —(vi)$$

graphically;

From figure above, it can be seen that $h(t)$ = 0 at all Nyquist sampling instants $t = \pm n/2 f_m$ except at $t = 0$.

Now when the sampled signal $g(t)$ is applied to this filter, the o/p will be $x(t)$. Each sample in $g(t)$ produces a sinc pulse of height equal to strength of sample. Addition of the sinc pulses produced by all the samples results in $x(t)$.

For instant, the $k^{th}$ sample of the input $g(t)$ is the impulse $x(kT_s) \delta(t - kT_s)$. The filter output of this impulse will be $x(kT_s) h(t - kT_s)$. Therefore the filter o/p to $g(t)$, which is $x(t)$, may be expressed as a sum,

$$x(t) = \sum_k x(kT_s) h(t - kT_s) \quad —(vii)$$

$$= \sum_k x(kT_s) \text{Sinc}[2\pi f_m (t - kT_s)] —(viii)$$

$$\therefore x(t) = \sum_k x(kT_s) \text{Sinc}(2\pi f_m - k\pi) —(ix) \begin{cases} T_s = \frac{1}{2 f_m} \end{cases}$$

eq$^n$ (ix) is the interpolation formula which provides values of $x(t)$ between samples as a weighted sum of all the sample values.

**Aliasing effect:** 7UAs, 7OAsd, 73Sh, Hsh 73Ch;

when a continuous-time band-limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then successive cycles of spectrum $G(\omega)$ of the sampled signal $g(t)$ overlap with each other as shown in figure below. Here, the signal is said to be undersampled in this case and some amount of aliasing is produced in this process. Aliasing is the phenomenon in which a high frequency component in the frequency-spectrum takes identity of a lower-frequency component in the spectrum of the sampled signals


$G(\omega)$ for $f_s < 2f_m$ — Aliasing
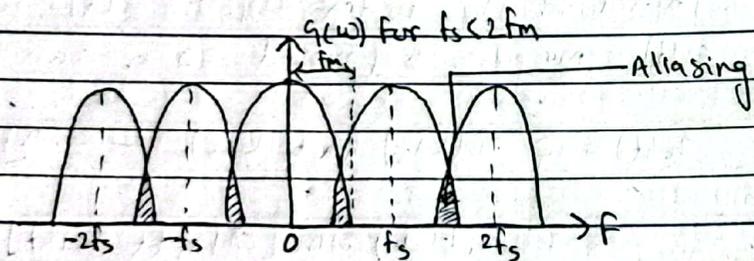
In this case shown in figure above, it is not possible to recover original signal $x(t)$ from sampled signal without distortion. To avoid aliasing we must:

(i) use pre alias filter to limit band of frequencies of the signal to $f_m$ Hz.

(ii) Sampling frequency '$f_s$' must be selected such that, $f_s > 2f_m$

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**Sub-sampling theory:** 7uch [2], 7R kar [2]

Previously, we studied sampling theorem for low-pass signals. However when the given signal is bandpass signal, then a different criteria must be used to sample the signal. The sub-sampling theory is related with sampling of bandpass signal, which is expressed as under:

'The bandpass signal $x(t)$ whose maximum bandwidth is $2f_m$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth. Here, $f_m$ is maximum frequency component present in the signal.'

Hence if the bandwidth is $2f_m$, then minimum sampling rate for bandpass signal must be $4f_m$ samples per second.

$$\therefore f_s = 4f_m$$

$$\therefore T_s = \frac{1}{f_s} = \frac{1}{4f_m}$$

## Natural sampling: 76ch[6], 78Bh[5], 71Sh[6]

Since in instantaneous sampling, $\tau$ approaches 0 which is not possible in real life. Thus this method is not suitable to use in real life. Thus we use Natural sampling which is a practical method and has a finite width equal to $\tau$.

Let us consider an analog continuous-time signal $x(t)$ to be sampled at the rate of $f_s$ Hz.

where, $f_s$ > Nyquist rate.

Let $c(t) \rightarrow$ sampling f$^n$ which is train of periodic of width $\tau$ & frequency equal to $f_s$ Hz.



Fig: functional diagram of Natural sampler.
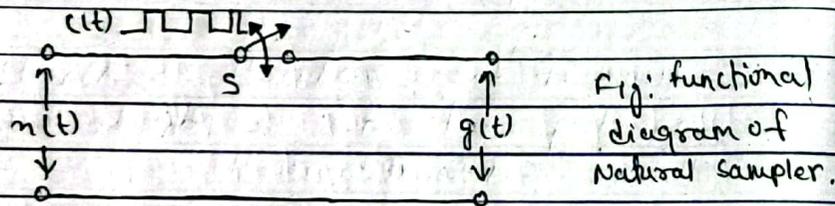
With the help of natural sampler shown above, a sampled signal $g(t)$ is obtained by multiplication of sampling function $c(t)$ & i/p signal $x(t)$

from figure, when $c(t)$ = high, switch 's' = Closed

Therefore,

$g(t) = x(t)$    when    $c(t) = A$ (Amplitude of $c(t)$)
$g(t) = 0$        when    $c(t) = 0$
       — (iv)

Mathematically,    $g(t) = c(t) \cdot x(t)$ — (iii)

we know that exponential Fourier series for any periodic waveform is expressed as

$$c(t) = \sum_{n=-\infty}^{\infty} c_n \, e^{j 2\pi n t / T_0} \quad — (v)$$

for $c(t)$,

$T_0 = T_s = 1/f_s$ = Period of $c(t)$

$$f_0 = f_s = \frac{1}{T_0} = \frac{1}{T_s} = \text{frequency of } c(t)$$

$\therefore$ from (v), we have

$$c(t) = \sum_{n=-\infty}^{\infty} c_n \, e^{j 2\pi f_s n t} \quad \left\{ \text{from } \frac{1}{T_0} = f_s \right\} \quad — (v)$$

Since $c(t)$ = rectangular pulse train,

$$c_n = \frac{\tau A}{T_0} \, \text{sinc}(f_n \cdot \tau) \quad — (vi)$$

here, $\tau$ = pulse width = $\tau$
&, $f_n$ = harmonic frequency = $n f_s$
or, $f_s = \frac{n}{T_0} = n f_0$.

Hence, $C_n = \dfrac{\tau A}{T_0} \, sinc(f_n \tau)$ —— (vii)

$\therefore$ using eq^s (v) and (vii),

$$C(t) = \sum_{n=-\alpha}^{\alpha} \dfrac{\tau A}{T_s} \, sinc(f_n \tau) \, e^{j2\pi f_s \cdot nt} \quad —(viii)$$

Now, from (iii), we have,

$$g(t) = \dfrac{\tau A}{T_s} \cdot \sum_{n=-\alpha}^{\alpha} sinc(f_n \tau) \, e^{j2\pi f_s nt} \cdot x(t) \quad —(ix)$$

This is the required time-domain representation for naturally sampled signal $g(t)$.

Now to get the frequency-domain representation of the naturally sampled signal $g(t)$, lets take its fourier transform:

$$G(f) = FT[g(t)]$$

$$G(f) = \dfrac{\tau A}{T_s} \sum_{n=-\alpha}^{\alpha} sinc(f_n \cdot \tau) \, FT[e^{j2\pi f_s nt} \cdot x(t)] \quad —(x)$$

From the frequency shifting property of FT,

$$e^{j2\pi f_s nt} \cdot x(t) \longleftrightarrow X(f - f_s n) \quad —(xi)$$

$$\therefore \quad G(f) = \dfrac{\tau A}{T_s} \sum_{n=-\alpha}^{\alpha} sinc(f_n \cdot \tau) \, X(f - f_s n) \quad —(xii)$$

since $f_n = n f_s$ = harmonic frequency

$\therefore$ eq^n (xii) becomes,

$$G(f) = \dfrac{\tau A}{T_s} \sum_{n=-\alpha}^{\alpha} sinc(n f_s \cdot \tau) \, X(f - f_s n) \quad —(xiii)$$

Hence, we write
spectrum of Naturally sampled signal :

$$G(f) = \dfrac{\tau A}{T_s} \sum_{n=-\alpha}^{\alpha} sinc(n f_s \cdot \tau) \, X(f - f_s n) \quad —(xiv)$$



Fig. ct signal x(t)

Fig. sampling function

Fig. Naturally Sampled signal waveform, g(t)
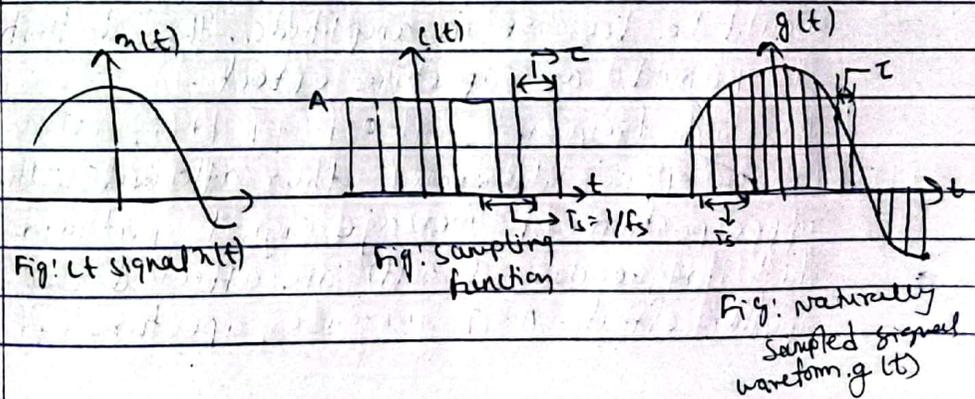
Aperture effect: 74ch[2], 73ch[7], 73sh[3], 74Ab[6]
71sh[2]  69ch[5]

Let us take one pulse of rectangular pulse
train and its spectrum as shown in figure
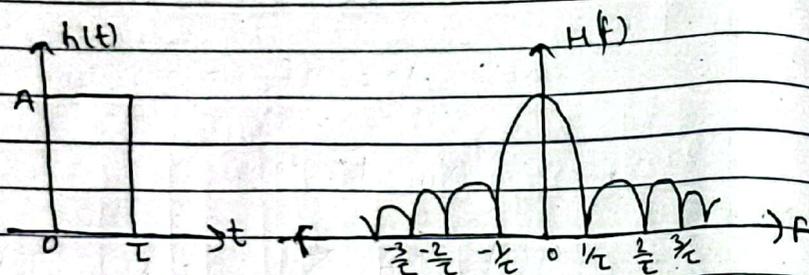below:



Fig: one pulse of
rectangular
Pulse-train

Fig: spectrum of pulse
shown in previous
figure.

During flat top sampling, to convert varying
amplitudes of pulse to flat top pulses we
use a sinc function. Because of this, there
would be decrease in amplitude. This distortion
is named as aperture effect.
The high frequency roll-off of H(f) acts like
a low-pass filter and thus attenuates the
upper portion of message signal spectrum. These
high frequencies of x(t) are affected causing
aperture effect. As τ increases, aperture effect is

more prominent.
Remedy for aperture effect:

① using the sampling pulse(τ) as narrow as possible
② By using equalizers during reconstruction, havi
transfer function of $H_{eq}(f) = \dfrac{1}{P(f)}$

③ By using good filter design by following
sampling theorem properly.

Pulse Modulation Systems: (8 Hours / 14 Marks)

70ch [1.5] Pulse Amplitude modulation may be defined as that
0.5 type of modulation in which the amplitudes of
regularly spaced rectangular pulses vary accordingly
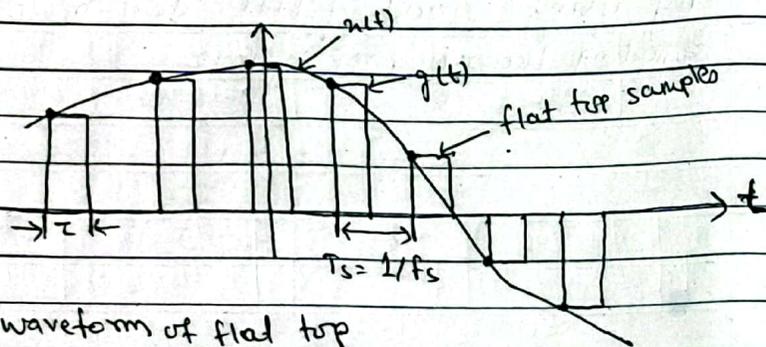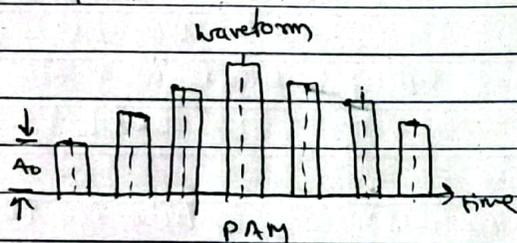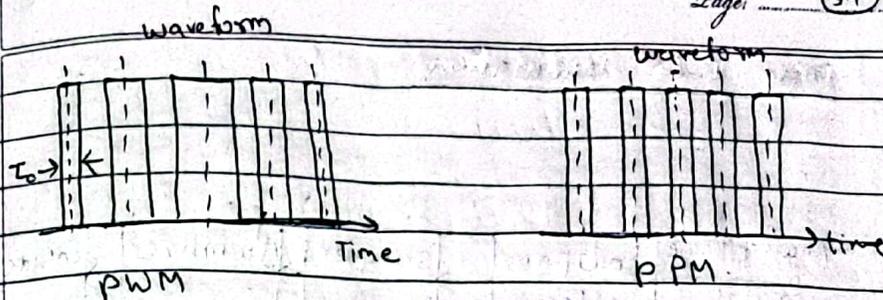to instantaneous value of the modulating or
message signal.



Fig: waveform of flat top
sampled PAM.



70ch [1.5] Pulse width modulation may be defined as the
0.5 type of modulation in which the width of
modulated pulses varies in proportion with the
amplitude of modulating signal.

waveform



PWM

Time

waveform

PPM

time

70ch In Pulse position modulation, the amplitude &
[1.5] width of the pulses are kept constant but the
0.5 position of each pulse is varied in accordance
with the amplitudes of the sampled values of
the modulating signal. The position of the pulses
is changed with respect to the position of reference
pulses.

Pulse code Modulation: In PCM, the message signal
is sampled and amplitude of each sample is
approximated (rounded off) to the nearest one of
a finite set of discrete levels. This will enable us
to represent both time and amplitude in discrete form

Figure below shows the basic elements
of a PCM system. It consists of three main parts
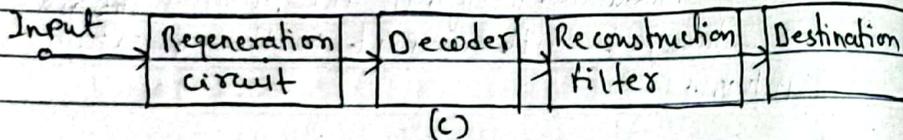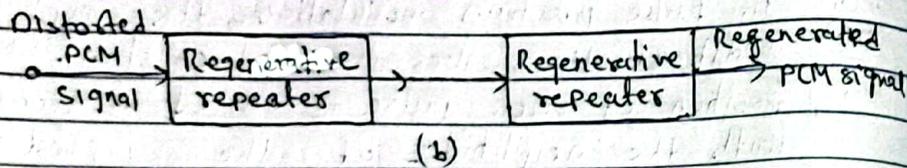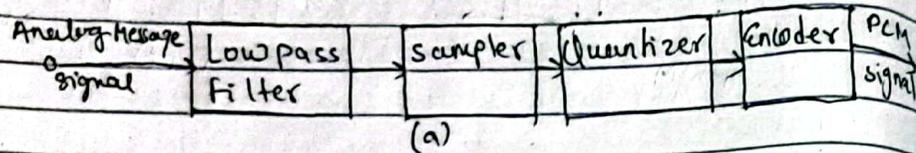ie transmitter, transmission path and receiver.

Analog Message Signal → | Low pass Filter | → | Sampler | → | Quantizer | → | Encoder | → PCM signal

(a)

Distorted PCM Signal → | Regenerative repeater | → ... → | Regenerative repeater | → Regenerated PCM signal

(b)

Input → | Regeneration circuit | → | Decoder | → | Reconstruction filter | → | Destination |

(c)

Fig: The basic elements of a PCM signal
    (a) Transmitter
    (b) Transmission path
    (c) Receiver.

The essential operations in the transmitter of a PCM system are sampling, quantizing & encoding as shown in figure above. Sampling is an operation in which analog signal is sampled according to the sampling theorem resulting in a discrete-time signal. The quantizing & encoding operation is performe

in same circuit which is known as analog-to-digital converter (ADC).

Also, essential operations in the receiver are regeneration of impaired signals, decoding & demodulation of the train of quantized samples. These operations are usually performed in the same circuit which is known as a digital-to-analog converter (DAC).

Further, at intermediate points along the transmission route, from the transmitter to the receiver, regenerative repeaters are used to reconstruct (ie. regenerate) the transmitted sequence of coded pulses in order to combat the accumulated effects of signal distortion & noise.

Quantization refers to the use of a finite set of amplitude levels and the selection of a level nearest to a particular sample value of the message signal as the representation for it. In fact, this operation combined with sampling, permits the use of coded pulses for representing the message signal. Thus, it is the combined use of quantization and coding that distinguishes Pulse code Modulation from analog modulation techniques.

70AS[8]
94AS[7]
71Ch[4]

**Signal to Quantization noise Ratio for Linear/uniform quantization:**

We know that in a pulse code Modulation system for linear quantization the signal quantization noise ratio is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

Normalized noise power is given as $\frac{\Delta^2}{12}$

Therefore $\frac{S}{N} = \frac{\text{Normalized signal power}}{(\Delta^2/12)}$ ——(i)

we know that number of bits 'v' & quantization levels are related as,

$$q = 2^v \quad \text{——(ii)}$$

Let us assume that input $x(nTs)$ to a linear quantizer has continuous amplitude in the range $-x_{max}$ to $+x_{max}$. Therefore total amplitude range becomes: $x_{max} - (-x_{max}) = 2x_{max}$.

Now, stepsize will be

$$\Delta = \frac{2x_{max}}{q} \quad \text{——(iii)}$$

$$\Delta = \frac{2x_{max}}{2^v} \quad \{\text{from (ii)}\}$$

Now substituting this value in ①

$$\frac{S}{N} = \frac{\text{normalized signal power}}{\left(\frac{2x_{max}}{2^v}\right)^2 \times \frac{1}{12}}$$

Let normalized signal power be denoted as 'P'.

Then, $\frac{S}{N} = \frac{P}{\frac{4x^2_{max}}{2^{2v}} \times \frac{1}{12}} = \frac{3P \times 2^{2v}}{x^2_{max}}$

This is the required relation for signal to quantization noise ratio for linear quantization in a PCM system.

Hence, $\boxed{SQNR: \frac{S}{N} = \frac{3P}{x^2_{max}} \cdot 2^{2v}}$ ——(iv)

This exp$^n$ shows that signal to noise power ratio of

quantizer increases exponentially with increasing bits per sample.

If $n(t) \rightarrow$ normalized.
$$x_{max} \rightarrow 1$$

$$\therefore SQNR = 3 \times 2^{2v} \times P \quad \longrightarrow \text{(V)}$$

If $P \rightarrow$ normalized
$$P \leq 1$$

$$\therefore SQNR \leq 3 \times 2^{2v} \quad \longrightarrow \text{(VI)}$$

Now as both of them are normalized we can express SQNR in decibels;

$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB \leq 10 \log_{10} [3 \times 2^{2v}] \leq (4.8 + 6v) \, dB$$

$$\therefore \boxed{\left(\frac{S}{N}\right) dB \leq (4.8 + 6v) \, dB} \quad \longrightarrow \text{(vii)}$$

## Non-Uniform quantization:

Basic Process? 72kar[3]
Need? (VVI) [2]
Uniform vs non-uniform quantization 73ch [4]

A uniform quantizer is that type of quantizer in which the 'step size' remains same throughout the input range. It is of two types: Mid thread and Mid rise. Similarly, a non-uniform quantizer is that type of quantizer in which the 'step size' varies according to the input values.

In a non-uniform quantizer, the quantizer characteristics is non-linear and step size is also not constant instead it is variable, dependent on the amplitude of the input signal. Here, the step size is reduced with the reduction in signal level. For weak signals $(P << 1)$, the step size is small, therefore the quantization noise reduces, to improve SQNR for weak signals. The step size is thus varied a/c to the signal level to keep the SQNR adequately high. This is non-uniform quantization. It is achieved through companding.

**VVI**
**2 marks**

Need of Non-Uniform Quantization:

(i) Robust Quantization: In uniform Quantization, the quantizer has linear characterstic. Thus stepsize remains same throughout the range of quantizer. Because of this, over the complete range of inputs, the maximum quantization error also remains the same. It is very prominent as sometimes quantization error can reach 30 to 50% of original voltage causing significant error. Thus in such cases, we use non-uniform quantization in which SQNR remains essentially constant over a wide range of i/p power levels. A quantizer that satisfies all these requirements is known as Robust quantizer which can be achieved by using a nonuniform quantization.

(ii) For speech and Music signals: speech and music signals are characterized by large crest factor. This means that for such signals the ratio of peak to rms value is quite high. For large crest factor of speech and music signal; P should be very very less than $1$ ($P \ll 1$) because of which SQNR is very less. Thus this type of signal should use non-uniform quantization to tackle the problem. In SQNR as P decreases,

noise increases. Noise is dependent on $\Delta$. Thus by varying stepsize ($\Delta$), we can keep SQNR at required value. This is the another need of Non-uniform quantization.

| uM | uniform quantization | non uniform quantization |
|---|---|---|
| • | It is the type of quantization in which the quantization levels are uniformly spaced. | It is the type of quantization in which quantization levels are unequal. |
| • | The step size ($\Delta$) is constant throughout i/p power levels. | The step size ($\Delta$) keeps on varying according to i/p power levels. |
| • | It is particularly divided into two categories: Mid-thread and Mid-rise quantization | It is not categorized further. However companding can be done by $\mu$-law & A-law methods. |
| • | It has significant amount of quantization error | It reduces quantization error. |
| • | SQNR is variable/changing continuously. | SQNR is essentially constant. |

## Companding:

- Explain [1.5, 2, 3]
- necessity
- Types: [4] {Block diagram & wave forms}

Non-uniform quantization is very difficult to achieve via regular methods. It can be practically achieved via a method called companding. The reason why Non-Uniform Quantization is difficult to implement is because we do not know about the changes in the signal level beforehand. Thus we use companding.

Companding is a term derived from two expression, companding = compressing + Expanding.

Compression is the process of amplifying weak signals and attenuating strong signals before applying them to a uniform quantizer. The block that provides compression is called as compressor.

Similarly at receiver exactly opposite is followed which is called expansion. The circuit used for providing expansion is called as an expander.

The compression of signal at the transmitter and expansion at receiver is combined to be called companding. The compression and expansion is obtained by passing the signal through the amplifier having non-linear transfer characteristic. The process of companding is illustrated in block diagram below:
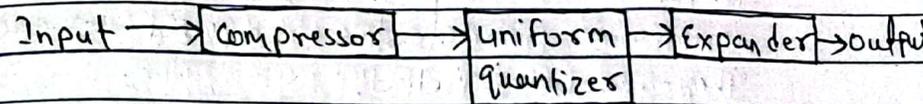
Input → Compressor → Uniform Quantizer → Expander → output
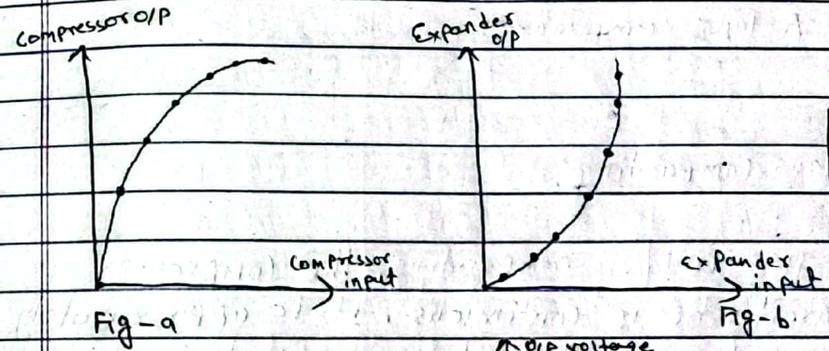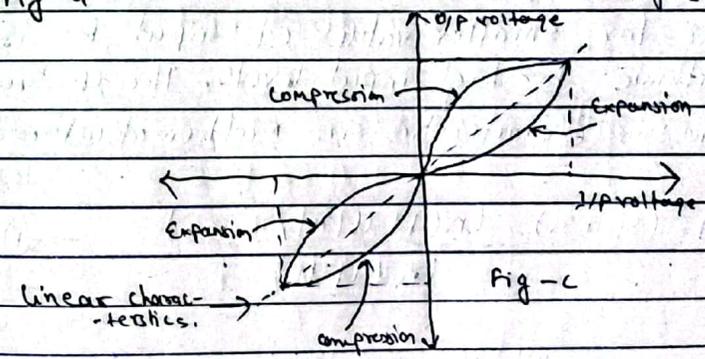
Fig: A companding model.



Fig-a

Fig-b

Fig-c

Fig a: compressor Characteristic

Fig b: Expander characteristic

Fig c: compander characteristic

# Necessity of companding: In uniform quantization, $\Delta$ is fixed. Thus quantization noise is fixed. But, signal power is not fixed which might cause poor SQNR. This might affect the quality of signal. Thus we use companding to avoid this.

# Types of companding: There are two types of companding techniques. They have different compressor characteristics compared to each other. They are listed below:

(i) $\mu$-law companding
(ii) A-law companding.

* $\mu$-law companding:

In the $\mu$-law companding, the compressor characteristic is continuous. It is approximately linear for smaller values of input levels and logarithmic for high input levels. The $\mu$-law compressor characteristics is mathematically expressed as

$$z(x) = (sgn\, x)\, \frac{\ln(1 + \mu |x| / x_{max})}{\ln(1 + \mu)} \quad —①$$

where, $0 \le |x| / x_{max} \le 1$

Here,

$z(x) \rightarrow$ output of compressor

$x \rightarrow$ input of compressor

$|x| / x_{max} \rightarrow$ normalized i/p wrt maximum value $x_{max}$

$sgn\, x \rightarrow \pm 1$ (+ve & -ve values of i/p & o/p)

The $\mu$-law compressor characteristic is shown below for different values of $\mu$. Practical value of $\mu$ is 255. $\mu \rightarrow 0$ means uniform quantization.
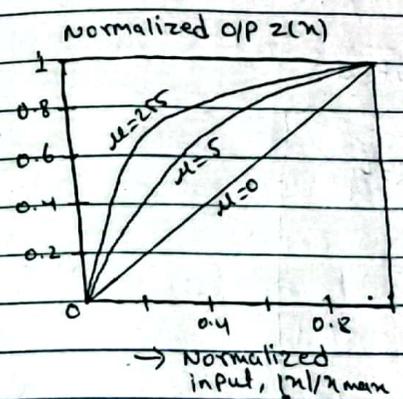


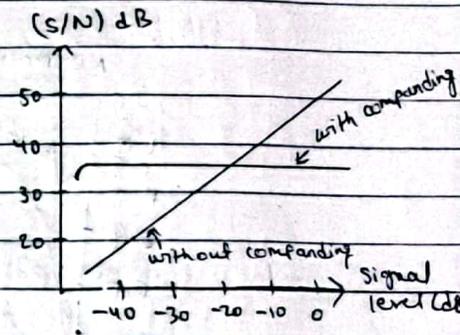Fig: compressor characteristics of a $\mu$-law compressor



Fig: PCM performance with $\mu$-law companding. SNR is almost constant while using companding

uses:
(i) It is used for speech & music signals.
(ii) It is used for PCM telephone systems in US, Canada & Japan.

* ## A law companding:

In the A law companding the compressor characteristic is piecewise, Made up of a linear segments for low level inputs and a logarithmic segments for high level inputs.
Figure below shows A-law compressor characteristics for different values of A.

$A \to 1 \to$ linear characteristic / uniform quantization
$A \to 87.56 \to$ practically used value.



Normalized output

Normalized input
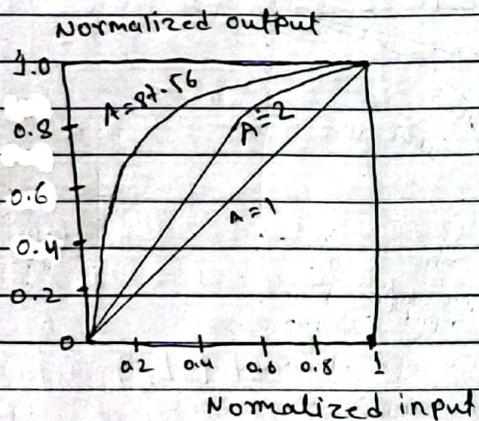
Mathematically,

$$\frac{z(x)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\log_e A} & \text{for } 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\[3mm] \dfrac{1+\log_e[A|x|/x_{max}]}{1+\log_e A} & \text{for } \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

— (ii)

uses:
• PCM telephone systems in Europe.

\# SQNR for DM: 7.6 ch [5], 7.3 sh [6], 7.0 ch [6]

we know that the condition to avoid the slope overload distortion is expressed as

$$A < \frac{\Delta}{W_{nits}} = \frac{\Delta}{2\pi} \frac{f_s}{f_m} \qquad —(i)$$

Therefore, the maximum value of the output signal power is expressed as,

$$P_{max} = \left(\frac{A}{\sqrt{2}}\right)^2 \qquad \begin{cases} \text{since P is proportional} \\ \text{to square of RMS value} \end{cases}$$

Hence, we have $P_{max} = \dfrac{A^2}{2} = \dfrac{\Delta^2 f_s^2}{8\pi^2 f_m^2}$ —(ii)

Now, we require to obtain the expression for quantization noise power.
The quantization error in DM lies in range $(-\Delta, \Delta)$

∴ maximum quantization error, $\epsilon_{man} = \pm \Delta$

The error shown above is uniformly distributed as shown in figure.

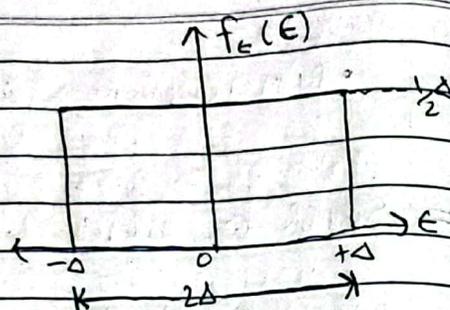Thus, the PDF is an uniform distribution function which is defined as under:

$$f_\epsilon(\epsilon) = \begin{cases} \dfrac{1}{2\Delta} & \text{for } -\Delta \le f_\epsilon(\epsilon) \le +\Delta \\ \\ 0 & \text{otherwise} \end{cases}$$



Fig: PDF of quantization

Mean square value or the variance of the quantization noise is given by,

$$\overline{\epsilon^2} = \int_{-\Delta}^{\Delta} \epsilon^2 f_\epsilon(\epsilon) \, d\epsilon$$

$$= \int_{-\Delta}^{\Delta} \epsilon^2 \cdot \frac{1}{2\Delta} \, d\epsilon = \frac{1}{2\Delta} \left[ \frac{\epsilon^3}{3} \right]_{-\Delta}^{\Delta} = \frac{1}{2\Delta} \left[ \frac{\Delta^3}{3} + \frac{\Delta^3}{3} \right]$$

$$\overline{\epsilon^2} = \frac{1}{2\Delta} \times \frac{2\Delta^3}{3}$$

$$\therefore \quad \overline{\epsilon^2} = \frac{\Delta^2}{3} \quad \text{"} \quad - (iii)$$

Normalized quantization noise power $N_q = \dfrac{\overline{\epsilon^2}}{1} = \dfrac{\Delta^2}{3}$ — (iv)

The delta modulated signal is passed through a reconstruction low pass filter (LPF) at the output of a DM receiver. The BW of this LPF is $f_M$ such that
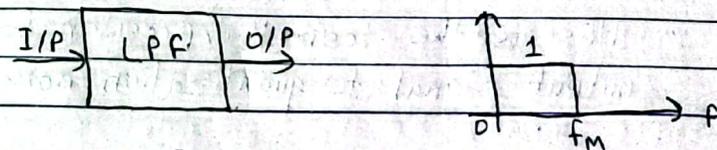
$$f_M \ge f_m \quad \text{and} \quad f_M \ll f_s$$



Fig: LPF at delta modulation.

Now assuming $N_q$ is distributed uniformly over frequency band upto $f_s$, the output quantization noise power within the bandwidth $f_M$ is given by,

Normalized noise power at filter O/P, $N_q' = \dfrac{\Delta^2}{3} \times \dfrac{f_M}{f_s}$

— (v)

Substituting, ① & ⑤ in one place, we get,

$$\boxed{\frac{S}{N_q}\Big|_0} = \frac{P_{max}}{N_q'} = \frac{\Delta^2 f_s^2}{8\pi^2 f_M^2} \times \frac{3 f_s}{\Delta^2 f_M}$$

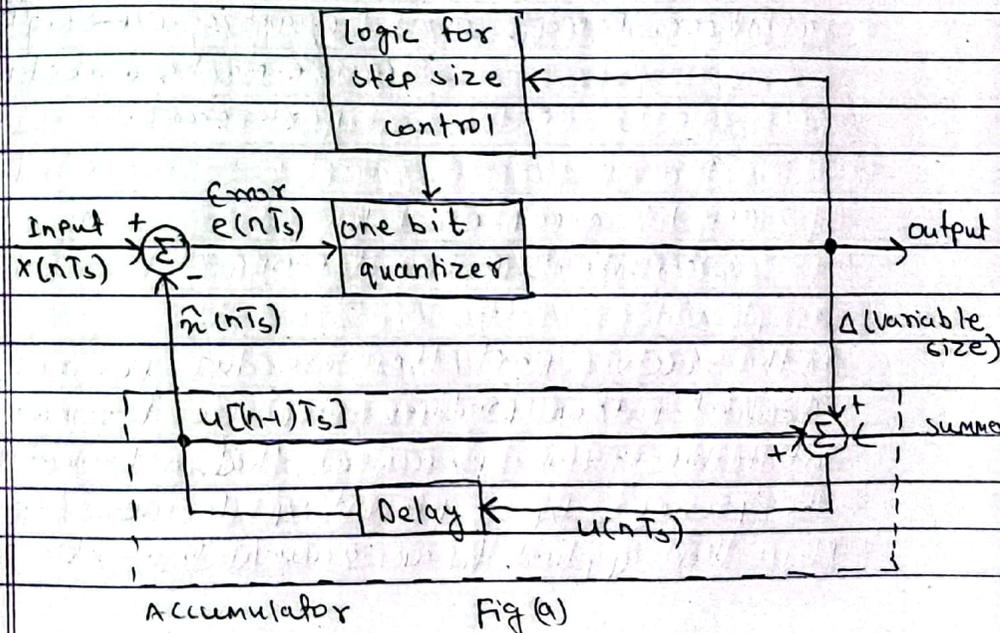$$\boxed{\frac{S}{N_q}\Big|_0} = \frac{3 f_s^3}{8\pi^2 f_M^2 f_M} \qquad (vi)$$

since $f_s = \dfrac{1}{T_s}$

$$\therefore \boxed{\frac{S}{N_q}\Big|_0 = \frac{3}{8\pi^2 f_M^2 f_M T_s^3}} \qquad (vii)$$

This is the desired expression for the output signal to quantization noise ratio.

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# Ⅱ Adaptive delta Modulation: short notes: 72kar[4]

To overcome quantization errors due to slope overload and granular noise, the stepsize ($\Delta$) is made adaptive to variations in the input signal $x(t)$. Particularly in the steep segment of the signal $x(t)$, the step size is increased. Also, if the input is varying slowly, the step size is reduced. Then, this method is known as Adaptive Delta Modulation (ADM). The adaptive delta modulators can take continuous changes in step size or discrete changes in step size.
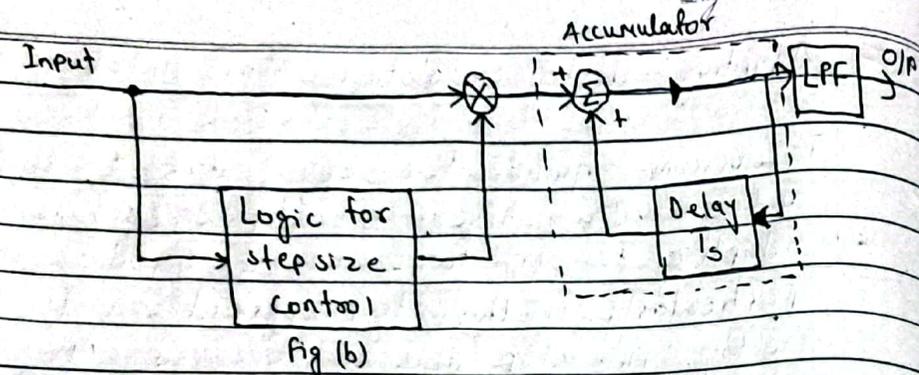


Accumulator          Fig (a)

Input

Accumulator



LPF O/P

Logic for
step size
control

Delay
1s

Fig (b)

Fig : a : Transmitter of ADM
b : Receiver of ADM.

The transmitter of ADM is shown in figure (a).
The logic for step size control is added in
the diagram. It depends on o/p of one bit
quantizer. For eg: one bit quantizer o/p→high
step size may be doubled for
next sample.

one bit quantizer → low
step size → Reduce by one step.

In the receiver part, there are two portions
The first portion produces step size for incoming
bit following same process as that in transmitter.
The previous i/p & present i/p decide step size.
It is then applied to an accumulator which builds

up staircase waveform. The lowpass filter then
smoothens out the staircase waveform to
reconstruct the original signal.

\* Advantages:

① DM use only one bit per sample thus transmitter
& receiver's design are simple.
② ADM's SNR becomes better reducing slope overload
distortion & idle noise.
③ Because of variable stepsize, dynamic range of
ADM is wider than DM.
④ utilization of bw is better than delta Modulation.

# Differential Pulse Code Modulation (DPCM) : 70ch[4]

• why DPCM is better than PCM? 71ch[2]
• Working principle with figure and equations : 71ch[5]

We may see that the samples of signal are
highly correlated with each other because any
signal does not change fast. This means its value
from present sample to next sample does not differ
by large amount. The adjacent samples of the

signal carry the same information with a little difference. When these samples are encoded by a standard PCM system, the resulting encoded signal contains some redundant infor- -mation.

If this redundancy is reduced, then overall bit rate will decrease & number of bits required to transmit one sample will also be reduced. This results in saving bandwidth as well. This type of digital pulse modulation scheme is known as Differential Pulse Code Modulation (DPCM) and because of above mentioned reasons, it is superior over PCM.

* Working Principle

DPCM works on the principle of prediction. The value of present samples is predicted from past samples. The prediction may not be exact but it is very close to the actual sample value.

Sampled signal $\longrightarrow x(nTs)$

Predicted signal $\longrightarrow \hat{x}(nTs)$ {produced by prediction filter}

comparator finds difference between these two

signals which is known as prediction error $e(nTs)$
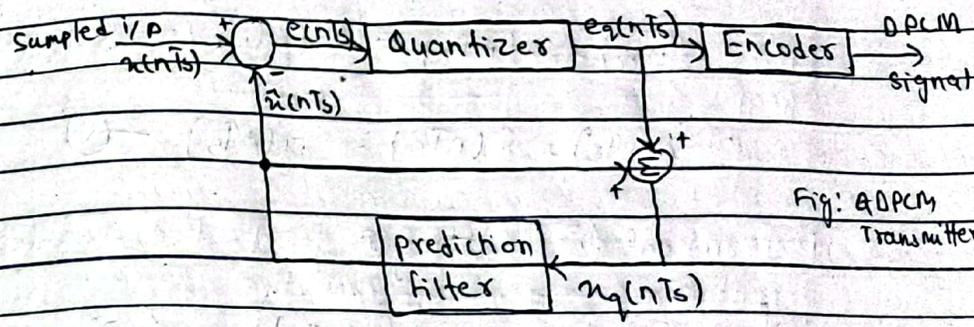
$$e(nTs) = x(nTs) - \hat{x}(nTs) \quad —①$$



Fig: ADPCM Transmitter

The prediction filter produces predicted value by taking inputs as the sum of quantizer o/p & previous prediction. This makes prediction close to the actual sampled signal.

we can observe that $e_q(nTs)$ is very small & can be encoded using small number of bits. Thus number of bits per sample are reduced in DPCM.

The quantizer o/p is given as;

$$e_q(nTs) = e(nTs) + q(nTs) \quad —②$$

where, $q(nTs) =$ quantization error

$$x_q(nTs) = \hat{x}(nTs) + e_q(nTs) \quad —③$$

$$n_q(nTs) = \hat{u}(nTs) + e(nTs) + q(nTs) \quad \text{---(iv)}$$

from eq$^n$ ①;

$$e(nTs) = u(nTs) - \hat{u}(nTs)$$
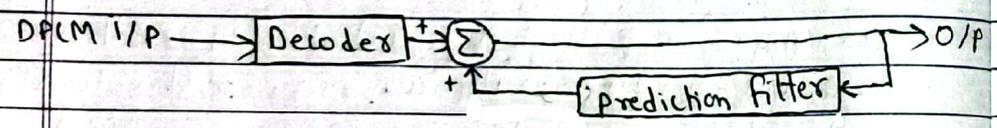
$$\therefore \quad e(nTs) + \hat{u}(nTs) = u(nTs) \quad \text{---(v)}$$

in eq$^n$ (iv), putting ⑤.

$$\boxed{n_q(nTs) = u(nTs) + q(nTs)} \quad \text{---(vi)}$$

Similarly at receiver, the decoder first reconstru-ets the quantized error signal from incoming binary signal. DPCM receiver is shown below:



The prediction filter o/p & quantized error signals are summed up to give quantized version of the original signals. Thus the signal at receiver differs from actual signal by quantization error $q(nTs)$, which is introduced permanently in the reconstructed signal.

# Linear Predictive Coding (LPC): Short notes. 70th [6]

LPC synthesizes the analog signal first. Then the parameters of the waveform synthesizer are encoded and transmitted instead of the actual signal. LPC is well suited for speech synthesis and transmission.

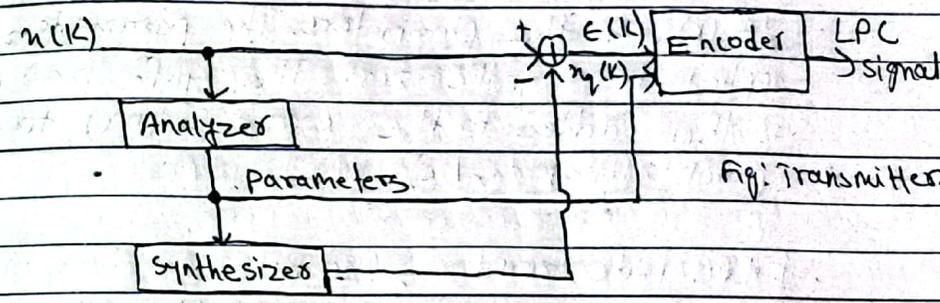Figure below shows the block diagram of LPC Transmitter.



Fig: Transmitter.

The voice input signal is first sampled to obtain signal $x(k)$ at the input of LPC transmitter These sampled values are applied to an analyzer which analyzes them to determine the parameters for synthesizer. The synthesizer reconstructs the approximated speech signal $n_q(k)$. The reconst-ruted speech signal & sampled signal $x(k)$ are compared to produce error $E(k)$. The error signal & parameter values are encoded by the encoder to form a digital

signal which is known as LPC signal

Figure below shows the LPC Receiver.

LPC signal → | Decoder | → $E_q(k)$ → (+) → $n_q(k)$
| Synthesizer | ↑
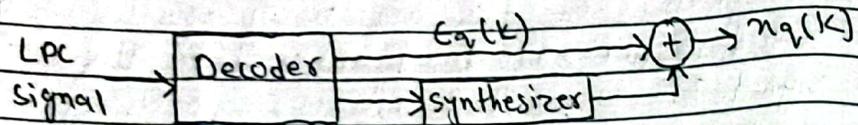
From the received LPC signal, the decoder or demultiplexer separates out the error signal '$E_q(k)$' & the parameters. The parameters are applied to a synthesizer. The synthesizer output is then added to the error signal to produce speech signal $n_q(k)$.

<u>1</u> LPC code word → 80 bits
→ 1 bit: for voiced/unvoiced switch
→ 6 bits: Pitch frequency of the voice
→ few bits: to represent error.

LPC has low bit rate of about 3K to 8 Kbits/sec.

---

* # **T1 carrier system:** T1-digital system is the basic Time division multiplexing scheme utilized to transmit large number of PCM signals over a common channel.
• This system can accomodate 24 voice channels, $S_1$ to $S_{24}$.
• Each signal is band limited to 3.3 KHz.
• sampling rate = 8KHz.

* **Basic block diagram:**

$S_1$ — LPF
$S_2$ — LPF → samples 8000 per second → Commutator SW 1 (switch) → PCM Transmitter → coaxial cable
Voice signals
$S_{24}$ — LPF

Repeater
Repeater
Repeater
Repeater → PCM receiver → $SW_2$ → Decommutator → LPF — $S_1$
LPF — $S_2$
8000 rev/s : Recovered voice signal
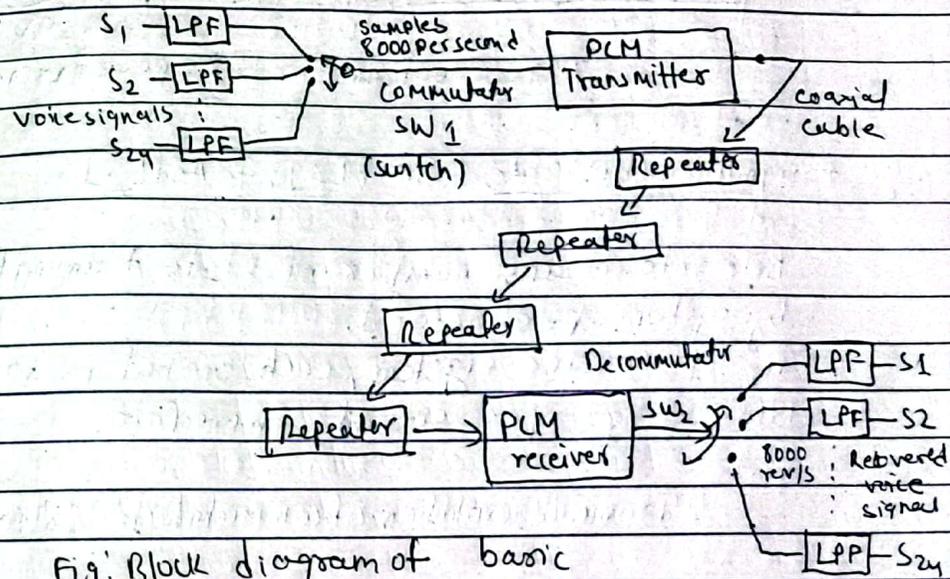LPF — $S_{24}$

Fig: Block diagram of basic PCM-TDM system or T1 carrier system.

\* Frame diagram of T1 : 70 Asd [3]

sampling Rate : 8000 samples / sec = 8000 rps.

1 frame = 1 revolution = 24 channels = 24 × 8 bits
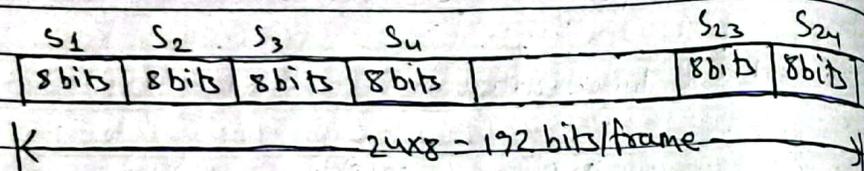$$= 192 \text{ bits/frame}$$

| S1 | S2 | S3 | Su | | S23 | S24 |
|------|------|------|------|------|------|------|
| 8 bits | 8 bits | 8 bits | 8 bits | | 8 bit | 8 bit |

|←      24×8 = 192 bits/frame      →|

Fig. one frame & bits per frame.

\* Signalling (bit) rate : 70 ch [3], 70 Asd [3]

Bit rate means number of bits transmitted by system per second.
In T1 carrier sigstem, each signal is sampled 8000 times per second, therefore,

1 frame (1 revolution of commutator) = 1/8000 = 125 μs.

But 1 frame contains 173 bits. Hence 193 bits are transmitted in 125μs.

∴ number of bits in 1 sec = $\dfrac{173}{125 \times 10^{-6}}$ = 1.544 × 10$^6$

so, bit rate of T1 carrier system = 1.544 M bits/sec

\* Bandwidth requirement : 70 ch [3]

minimum BW = $\dfrac{1}{2}$ (bit rate) = $\dfrac{1}{2}$ × (1.544×10$^6$)

$$= 772 \text{ KHz}.$$

∴ | Bandwidth of T1 system = 772 kHz |

\* Duration of each bit :
193 bits = 125 μs
∴ 1 bit = (125/193) μs = 0.6476 μs.

Shortnotes: 76 ch[5]

71ch[4] E1 - digital hierarchy! explain: 74As[3], 72ch[4], 74ch[4]
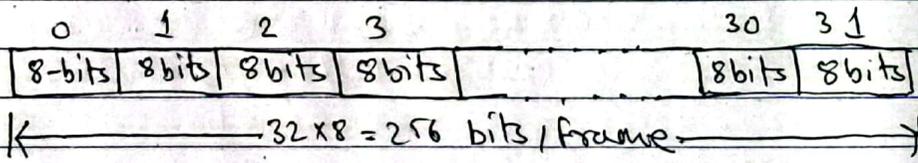69ch[4]

E-1 is the european version of T1. Both T1 &
E-1 are conceptually identical but their capacities
and number of voice channels which they can
carry will be different.
E1 consists of:

- 32 channels: 2 time slots are reserved for
  signalling and synchronization while
  remaining 30 are for voice calls or data
  communication.
- 1 channel = 8 bits
  ∴ Total bits = 8×32 = 256 bits
- Sampling frequency = 8 kHz.
- Each slot of E1 has bitrate of 64 kbps.
  Thus total bitrate = 2048 Kbps = 2.048 Mbps.

Frame diagram:

1 frame = 32 channels = 32×8 = 256 bits/frame.

| 0 | 1 | 2 | 3 | | 30 | 31 |
|---|---|---|---|---|----|----|
| 8-bits | 8bits | 8bits | 8bits | | 8bits | 8bits |

$\longleftarrow$ ─────── 32×8 = 256 bits/frame ──────── $\longrightarrow$

Signalling (bit) rate: 70ch[3] 74 As[5]

Sampling rate = 8000 samples per second.

Time taken to send 1 frame = $\frac{1}{8000}$ = 125 µs.

1 frame has 256 bits.
Thus, 256 bits are transmitted in 125 µs.

∴ number of bits in 1 sec = $\frac{256}{125 \times 10^{-6}}$ = 2.048×10⁶

so, bit rate of E1 carrier system = 2.048 Mbps

Bandwidth requirement: 70 ch[3]

Min. BW = $\frac{1}{2}$ (bit rate) = $\frac{1}{2}$ × 2.048 Mbps.

= 1.024 MHz

∴ BW of E1 system = 1.024 MHz

# chapter 4

Baseband Data communication system (7hours/ 12 Marks)

70 Asd [2] Information:

The amount of information received from the knowledge of occurance of a event may be related to the likelihood or probability of occurance of that event. The message associated with the least likelihood event thus consists of Maximun information. The amount of information in a message depends only upon the uncertainity of the underlying event rather than its actual content.

Entropy: 70 Asd [2)

In practice, we transmit long sequences of symbols from an information source. Thus, we are more interested in average information that a source produces than the information content of a single symbol. However, flow of information can fluctuate because of randomness in selection of symbols. Thus, average information content is important in context of long message.

Entropy is the average information content of a

sequence of symbols. Mathematically, it is given by,

$$H(n) = -\sum_{i=1}^{m} P(x_i) \cdot \log_2 P(x_i) \text{ bits/symbol.} \quad (i)$$

Information Rate: 76th [4]

If the time rate at which source X emits symbols is $r$, the information rate R of the source is given by,

$$R = r H(X) \text{ b/s} \quad (ii)$$

Here, R → information rate
M(X) → Entropy/Average information
r → Rate at which symbols are generated.

Information rate R is represented in average number of bits of information per second. It is calculated as under:

$$R = \left[ r \text{ in } \frac{symbols}{second} \right] \times \left[ H(X) \text{ in } \frac{information \text{ bits}}{symbol} \right] \quad (iii)$$

or, R = Information bits / second, (iv)

# Shannon's channel capacity theorem : 71ch[1], 72ch[2]

According to this theorem, " The capacity of a channel 'C' to transmit information without error is limited by its Bandwidth and noise in the channel."

Mathematically,

$$C = B \log_2 (1 + SNR) \text{ bits/second.} \quad -①$$

In this expression,

$\quad B = $ channel Bandwidth in Hz

$SNR = \dfrac{S}{N} \rightarrow \begin{cases} S = \text{Signal power} \\ N = \text{Noise power} \end{cases}$

$\quad\quad C = $ Channel Bandwidth.

Here, $N = 0$ for noiseless channel
and $C = $ infinite however this
case is impossible in real life.

similarly B & S can be exchanged for one anoth
when $B\uparrow \longrightarrow S\downarrow$ and $S\uparrow \longrightarrow B\downarrow$

We will talk about Noiseless channel & infinite Bandwidth in next section.

# Theoretical limits of Shannon channel Capacity Theorem

71ch[3], 72ch[3]

1) As the noise in the channel tends to zero, the value of SNR will tend to infinity. Subsequently the channel capacity C will tend to infinity. It means that the noiseless channel has an infinite capacity. This type of channel is referred to as ideal channel. This is not possible practically as Noise is always finite in real life.

2) As bandwidth of channel B tend to infinity, the channel capacity reaches to an upper limit $C_{max}$ (instead of $C \rightarrow \infty$) because as the channel BW increases noise power increases correspondingly. Noise power is proportional to channel BW.

70Asd [4]
76ch [6]

$$C = B \log_2 (1 + SNR)$$

$$C = B \log_2 \left( 1 + \frac{S}{N} \right) \quad -①.$$

we have, $N = \eta B$

where, $\eta \rightarrow$ psd of white noise.

Equation ① can be rewritten as,

$$C = \frac{S}{\eta} \cdot \frac{\eta B}{S} \log_2 \left( 1 + \frac{S}{\eta B} \right)$$

$$or, \ C = \frac{S}{\eta} \log_2 \left( 1 + \frac{S}{\eta B} \right)^{\frac{\eta B}{S}}$$

$$or, \ C = \frac{S}{\eta} \log_2 \left( 1 + \frac{S}{\eta B} \right)^{\frac{1}{S/\eta B}} \quad —(ii)$$

let $r = \frac{S}{\eta B}$

as $B \to \alpha$, $r \to 0$

we know that,
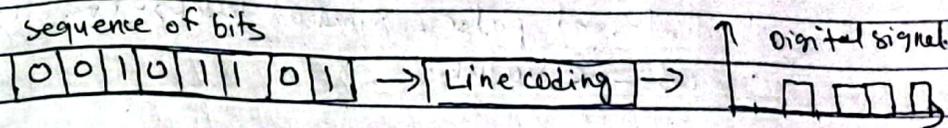
$$\lim_{r \to 0} \log_2 (1+r)^{\frac{1}{r}} = \log_2 e$$

$$\therefore \lim_{B \to \alpha} C = \frac{S}{\eta} \log_2 e$$

$$\therefore \lim_{B \to \alpha} C = 1.44 \frac{S}{\eta} \quad —(iii)$$

$$\boxed{\therefore \lim_{B \to \alpha} C = C_{max} = 1.44 \frac{S}{\eta}} \quad —(iv)$$

Short Notes : Line codes [4] 72kar

Line coding is the process of converting binary data, a sequence of bits to a digital signal.

sequence of bits

| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | → | Line coding | → ⊓ digital signal

There are various Line codes. They are listed below :

① Unipolar RZ & NRZ : In this format, waveform has single polarity. Waveform can have +5V or +12V when high. The waveform is simple ON-OFF.

② Polar RZ & NRZ : In this format, '1' is represented by the voltage polarity while '0' is represented by -ve voltage polarity.

③ Bipolar NRZ : In this format, successive 1 are represented by pulses with alternate polarity & '0' are represented by no pulses.

④ split phase Manchester Format : In this format,

When 'I' is transmitted, then a +ve half
interval pulse is followed by -ve half
interval pulse
When '0' is transmitted, then -ve half
interval pulse is followed by +ve half
interval pulse. Symbol takes +ve as well
as -ve value.

## Intersymbol Interference (ISI): VVVI

In communication system, when the data is
being transmitted in the form of pulses (ie, bits)
the output produced at the receiver due to
other bits or symbols interferes with the
output produced by the desired bit. This
is known as intersymbol interference (ISI).
The isi will introduce errors in the
detected signal.

input binary data → Pulse generator → $x(t)$ → Transmitting filter → Channel → noise

output binary data ← Decision Device ← $y_i(t)$ Sample at time $t = lT_b$ ← $y(t)$ Receiving filter ←
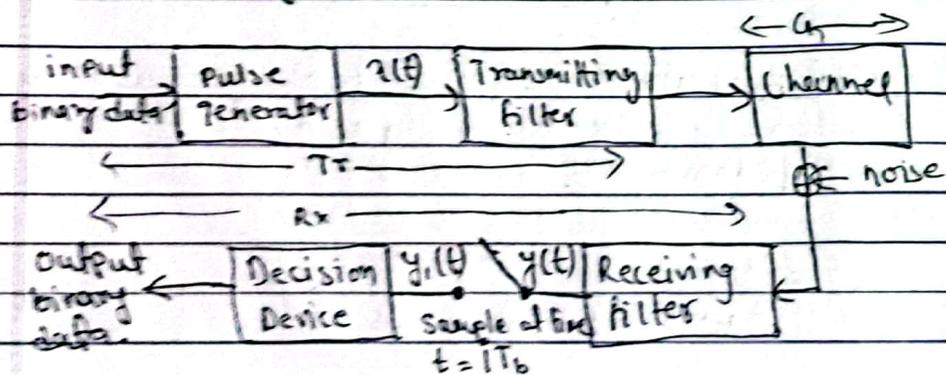
Figure above shows elements of binary PAM system.

i/p → binary data $\{b_k\}$ with bit duration $T_b$ seconds.

This input is applied to pulse generator to produce
discrete PAM signal which is given by,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k V(t - kT_b) \quad -(i)$$

where, $V(t)$ → basic pulse,
normalized such that
$V(0) = 1$

Thus this $x(t)$ is transmitted via a channel which
introduces noise to the signal.

The receiving filter o/p is written as;

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k P(t - kT_b) + n(t) \quad -(ii)$$

where, $\mu$ → scaling factor.
$n(t)$ → noise
$P(t - kT_b)$ → combined impulse
response of receiving
filter

The receiving filter o/p $y(t)$ is sampled at time $t_i = iT_b$ with $i$ taking on integer values to provide following expression:

$$y(t_i) = \mu \sum_{k=-\alpha}^{\alpha} a_k \, P(iT_b - KT_b) + n(t_i) \quad \text{(iii)}$$

when $k = i$

$$y(t_i) = \mu \, a_i \cdot P(0) + n(t_i)$$
$$k = i = \mu a_i$$

$$\therefore y(t_i) = \mu a_i + \mu \sum_{\substack{k=-\alpha \\ k \neq i}}^{\alpha} a_k \, P(iT_b - KT_b) + n(t_i) \quad \text{(iv)}$$

This is the receiver o/p $y(t)$ at instant $t = t_i$

From eqn (iv);

$\mu a_i \rightarrow$ produced by $i^{th}$ transmission bit & theoretically only this should exist, but because of residual effect of all transmitted bits second term is also present. This residual effect is intersymbol interference.

Methods to reduce ISI:

(i) The function which produces a zero isi is a sinc function. Hence, instead of a rectangular pulse if we transmit a sinc pulse then the ISI can be reduced to zero. This is known as Nyquist pulse Shaping. {Fig(a)}

(ii) The fourier transform of a sinc pulse is a rectangular function. Hence to preserve all the frequency components, the frequency response of the filter must be exactly flat in the pass band & zero in the attenuation band {fig b}
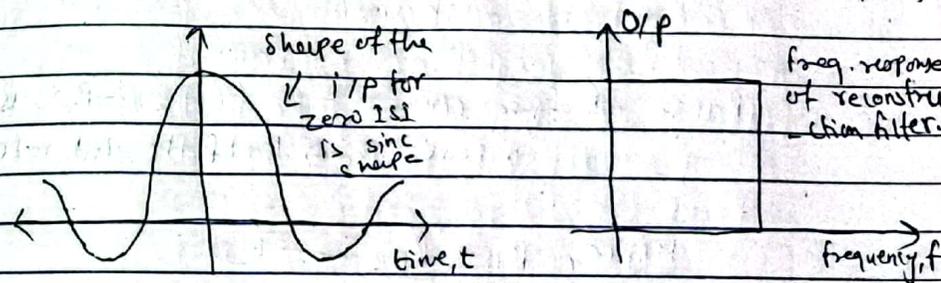


Fig: Ideal pulse shape for zero ISI

Fig: Frequency response of the filter.

# Ideal solution:

The Nyquist criterion for distortionless baseband transmission in absence of noise is given by

$$\sum_{n=-\alpha}^{\alpha} P(f - nR_b) = \frac{1}{R_b} = T_b \quad \text{—(i)}$$

The LHS of eq$^{\underline{n}}$ (i) represents a series of shifted spectrums.

for n=0

LHS = P(f) ; which represents a frequency function with the narrowest band.

Range of frequencies for $P(f) = (-B_0, B_0)$
where, $B_0 \rightarrow$ half the bit rate.

Hence, $B_0 = \frac{R_b}{2}$ —(ii)

$\therefore$ P(f) can be specified in following term,

$$P(f) = \frac{1}{2B_0} rect\left(\frac{f}{2B_0}\right) \quad \text{—(iii)}$$

This is shown graphically in fig (a) below.

This is the spectrum of signal which produces zero ISI. Hence the signal that produces zero ISI can be obtained by taking the IFT of P(f).

$$\therefore P(f) = F^{-1}[P(f)]$$

$$p(f) = F^{-1}\left[\frac{1}{2B_0} rect\left(\frac{f}{2B_0}\right)\right]$$

$$\therefore P(t) = sinc(2B_0 t) \quad \text{—(iv)}$$

Fig (b) shows plot of this function.
fn$^{\underline{n}}$ P(t) $\rightarrow$ impulse response of an ideal LPF with Bandwidth B$_0$.

Thus the shape of a pulse should be sinc rather than being a rectangular one to eliminate ISI.
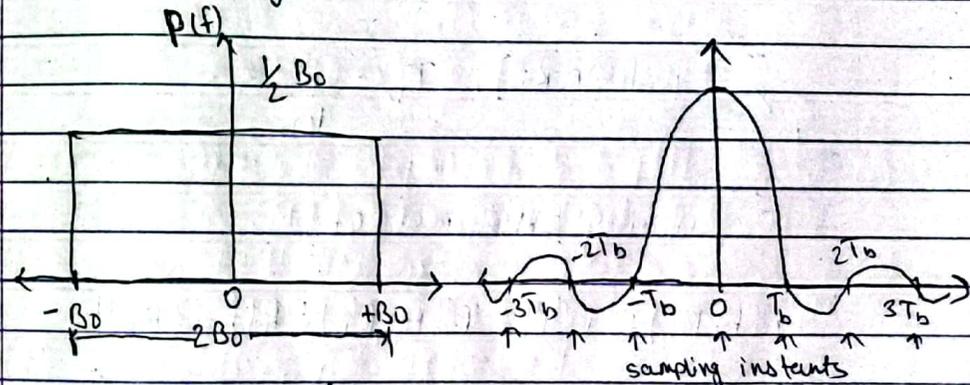


Fig (i): Graphical representation of P(f)    Fig (ii): Time-domain representation

**\* Difficulties of ideal solution:**

① The abrupt transition at $\pm B_0$ from flat to zero is not physically realizable.

② Due to discontinuity of $P(f)$ at $\pm B_0$, there is practically no margin of error in sampling times at receiver end.

**\# Raised Cosine spectrum: [3 marks - 6 Marks]**

Because of two difficulties by ideal Nyquist channel we cannot realize them practically. Instead we can overcome them by increasing the bandwidth from its minimum value $B_0 = R_b/2$ to an adjustable value between $B_0$ and $2B_0$. A condition is put on overall frequency response $P(f)$ to satisfy the given condition.

From (78.1)

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b = \frac{1}{R_b}$$

Expanding LHS we get,

$$\dots + P(f + R_b) + P(f) + P(f - R_b) + P(f - 2R_b) + \dots = T_b \quad \text{—①}$$

But, $B_0 = \dfrac{R_b}{2}$   $\therefore R_b = 2B_0$.

Putting $R_b = 2B_0$, taking only the values having $n = -1, 0, 1$ & restricting frequency band to $(-B_0, B_0)$, we get

$$P(f + 2B_0) + P(f) + P(f - 2B_0) = \frac{1}{2B_0} \quad \{-B_0 \le f \le B_0$$

It is possible to devise several bandlimited functions that satisfy above eq., one of them is raised cosine spectrum which consists of a flat portion and a roll off portion. The raised cosine spectrum is mathematically expressed as:

$$P(f) = \begin{cases} \dfrac{1}{2B_0} & \text{(flat portion)} \quad \dots 0 \le |f| < f_1 \\[2mm] \dfrac{1}{4B_0}\left\{1 - \sin\left[\dfrac{\pi(|f| - B_0)}{2B_0 - 2f_1}\right]\right\} & \dots f_1 \le |f| < 2B_0 - f_1 \\[2mm] 0 & \dots |f| \ge 2B_0 - f_1 \end{cases}$$

frequency parameter $f_1$ & BW $B_0$ are related as:

$$a = 1 - \frac{f_1}{f_m} \quad \text{, where, } a = \text{roll off factor}$$

indicates excess BW over ideal sol.$^n$ $B_0$.

Fig: Frequency response.
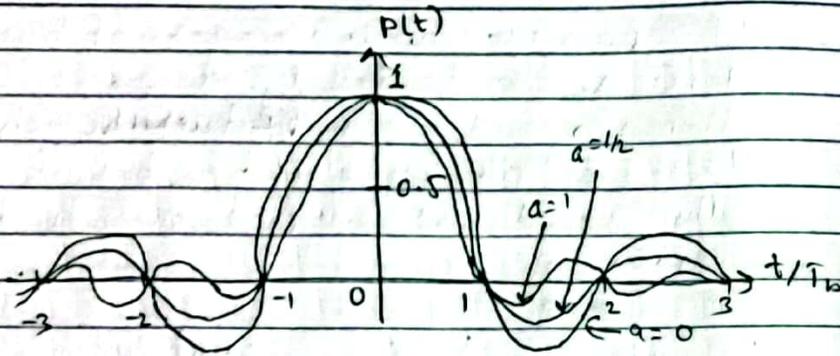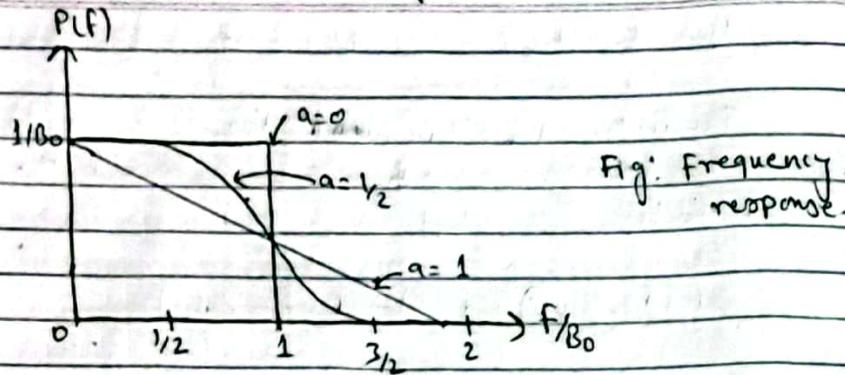


Fig: Time response.

observations:

① for a= 0.5 & 1, P(t) changes gradually wrt frequency. Hence it is Practically realizable.

② Time response has sinc shape & all sinc f" pass through zero at t = ±Tb, ±2Tb, ....

③ Amplitude of sidelobes ↑ when a ↓.

(i) with a= 0, BW requirement is max equal to 2 Bo

# Correlative coding:

a) Explain one type of correlative coding with its impulse response and transfer function. 73ch[6]

→ Normally we consider ISI as an undesirable phenomenon which degrades the system performance. But by adding the ISI to the transmitted signal in a controlled manner it is possible to achieve a bit rate of 2B bits per second in a channel of bandwidth B Hz. Such schemes are called as correlative coding or partial response signalling schemes. These schemes are based on assumption that ISI introduced to transmitted signal is known, therefore its effect at the receiver can be compensated. Thus using correlative coding it is possible to achieve the theoretical maximum signalling rate of 2B bits per second for a channel bandwidth of B Hz. It was predicted by Nyquist and can be achieved by using realizable & perturbation tolerant filter.

One type of correlative coding is duo-binary signalling. It's basic scheme is shown in figure below:



↳ Duo binary conversion filter.
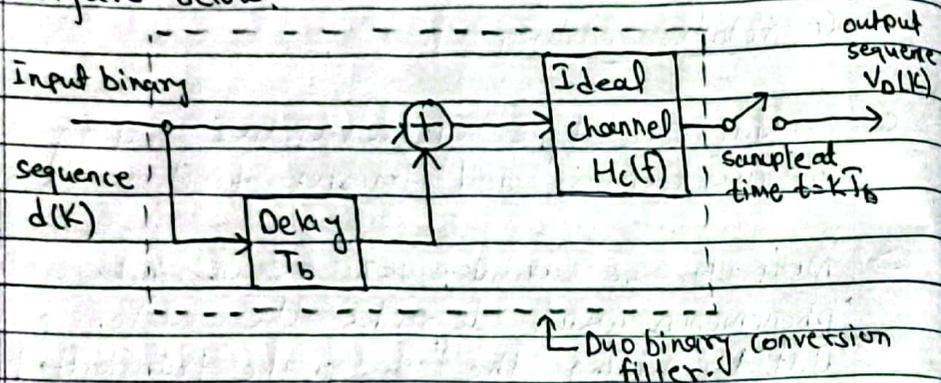
The binary signal d(k) is first passed through a simple filter which consists of a single delay element. The present signal d(k) & its delayed version d(k-1) are added to get the duobinary signal at the output of the coder. This is mathemat-ically expressed as:

$$V_0(k) = d(k) + d(k-1) \quad — ①$$

Here, d(k) sequence of the uncorrelated binary digits at the input is converted into a sequence $V_0(k)$ of correlated digits. This correlation bet? adjacent pulses is equivalent to introducing ISI which is in control of designer.

Let, delay element is ideal, thus its Transfer Function is given by,

$$H_0(f) = e^{-j2\pi f T_b} \quad — ②$$

TF of filter containing delay & adder as shown in figure will be,

$$1 + H_0(f) = 1 + e^{-j2\pi f T_b} \quad — ③$$

∴ overall TF of this basic filter cascaded with an ideal channel is given by,

$$H(f) = [1 + H_0(f)][H_c(f)]$$
$$H(f) = H_c(f)[1 + e^{-j2\pi f T_b}] \quad — ④$$

Substituting $1 = e^{+j\pi f T_b} \cdot e^{-j\pi f T_b}$ and
$$e^{-j2\pi f T_b} = e^{-j\pi f T_b} \cdot e^{-j\pi f T_b}$$

$$H(f) = H_c(f)\left[e^{+j\pi f T_b} \cdot e^{-j\pi f T_b} + e^{-j\pi f T_b} \cdot e^{-j\pi f T_b}\right]$$

$$H(f) = 2H_c(f) \, e^{-j\pi f T_b} \left[\frac{e^{+j\pi f T_b} + e^{-j\pi f T_b}}{2}\right]$$

$$\boxed{H(f) = 2H_c(f) \, e^{-j\pi f T_b} \cos(\pi f T_b)} \quad — ⑤$$ is the reqd TF of duobinary system.

For impulse response, we need to take IFT of H(f).

$$\therefore h(t) = IFT[H(f)]$$

$$h(t) = IFT\left[2 \quad e^{-j\pi fT_b} \cos(\pi f T_b)\right] \quad |f| \leq \frac{R}{2}$$

we can imagine cosine function is being multiplied by rectangular function $(-R/2, R/2)$

$$\therefore h(t) = IFT\left[2e^{-j\pi fT_b} \cos(\pi f T_b) \cdot rect(f/R)\right]$$

The corresponding impulse response is given by,

$$h(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t-T_b)/T_b]}{\pi(t-T_b)/T_b}$$

$$or, \quad h(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b}$$

$$h(t) = \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)} \quad \text{——(vi)}$$

Hence impulse response is a sinc function as depicted in figure below.
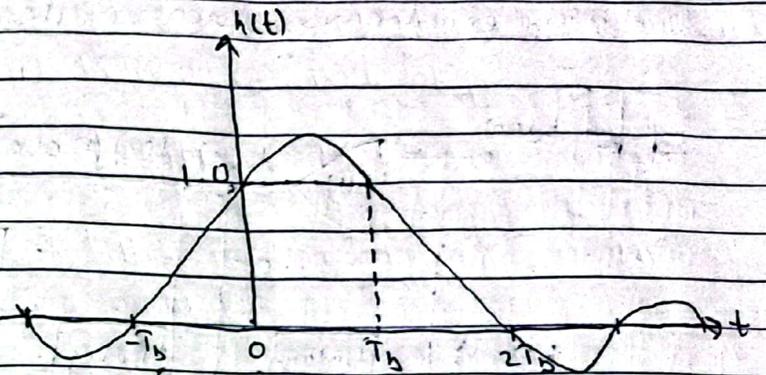
Fig: Impulse response of duobinary filter.

# Duobinary Encoding: 72ch[4], 71sh[6]

BW of a communication system depends on the max$^m$ frequency $(f_m)$ in the spectrum of the modulating baseband signal. The BW will reduce with decrease in $f_m$. The duobinary encoding is a mode of encoding a binary bit stream in which the max$^m$ frequency $f_m$ will decrease as compared to that in the unencoded data. Thus if a carrier is modulated by a duobinary encoded waveform, then the bw of the modulated signal will be smaller than if the unencoded data were used.

Duobinary encoder is shown in figure below



Fig: Duobinary encoder
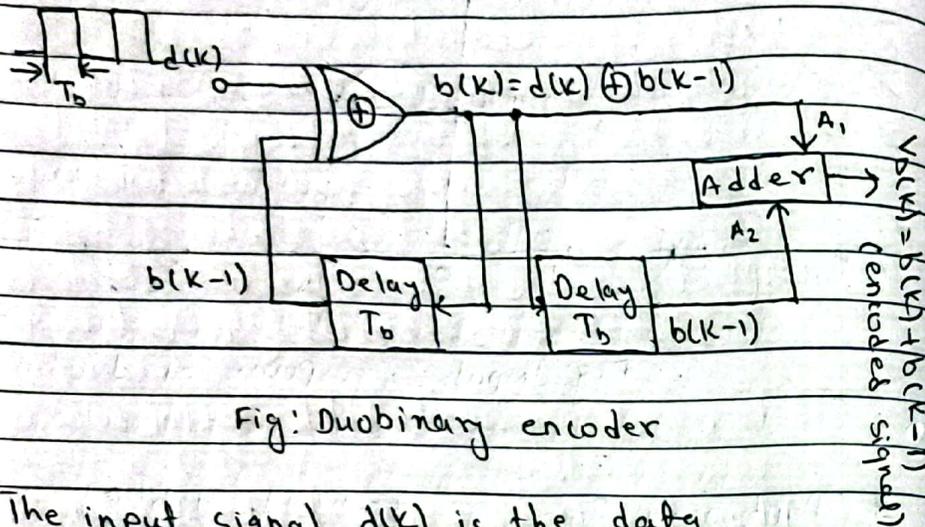
The input signal d(k) is the data bit stream with a bit duration equal to $T_b$. It changes its values between 0 & 1 levels.

Lets assume '0' → -1V
'1' → +1V

b(k) → O/P of Ex-OR gate given by

$$b(k) \sim d(k) \oplus b(k-1) \quad —①$$

This is further delayed by one bit & b(k) & b(k-1) are applied to Adder to produce encoded signal $V_D(k)$.

$$V_D(k) = b(k) + b(k-1)$$
$$= [d(k) \oplus b(k-1)] + [d(k-1) \oplus b(k-2)] \quad —②$$

d(k) → changes bet$^n$ +1V & -1V.
b(k) → changes bet$^n$ +1V & -1V.

Hence, b(k-1) will change bet$^n$ ±1V.

∴ $V_D(k)$ will change as under:

$$V_D(k) = +2V \quad \{b(k) = +1V, \ b(k-1) = -1V\}$$
$$= 0 \quad \{b(k) = b(k-1)\}$$
$$= -2V \quad \{b(k) = -1V, \ b(k-1) = +1V\}$$

Since $V_D(k)$ depends on b(k) & b(k-1) There is correlation bet$^n$ values of $V_D(k)$. The name of this coding is appropriate because in each bit interval, the encoded voltage $V_D(k)$ results from combination of two bits.

Figure below shows a duobinary decoder.
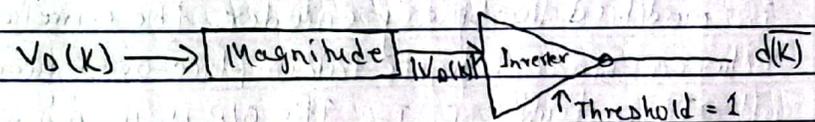


O/P 1 when magnitude is +1V or greater
O/P 0 when magnitude is 0V.

* Problems in duobinary encoder: 71Ch[6], 73 Ch[2]

1) If a carrier is modulated by a duo binary encoded waveform, then the bandwidth of the modulated signal will be smaller than if the unencoded data were used.

2) If there is an error in the received signal then it will propagate in the system.

To solve above difficulties:

To eliminate first issue, we should modify the duo binary encoder as follows.
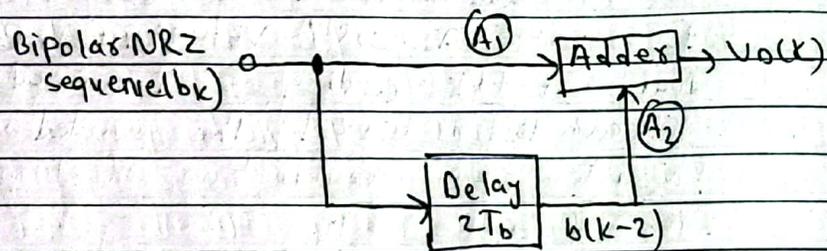


In modified duobinary encoder the correlation between binary digits takes place over duration of $2T_b$ instead of $T_b$. It's O/P is given by

$$V_0(k) = b(k) - b(K-2)$$

However even in this method the error is not eliminated. The error present in the received signal may be propagated through the whole system.

To eliminate the error, it must be precoded. Hence a modified duobinary encoder with precoder can solve the above two major difficulties. It will be discussed in detail in next section.

# Duobinary-Encoder with precoder (Differential Encoder)
74ch[6], 73sh[6]

The modified duobinary system discussed above has an important limitation, which is that if there is an error in the received signal then it will propagate in the other values of $b(k)$.
The duo-binary system with precoder is free from such a problem. It's block diagram is shown below
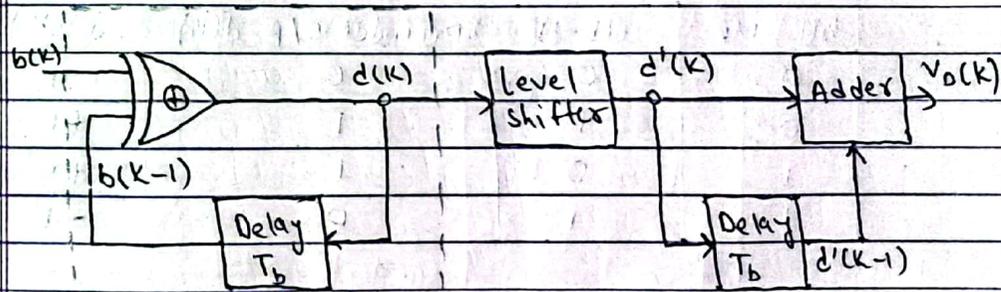


Precoder or differential encoder          |← Modified duo binary →|
                                                Encoder

The precoder is added to the modified duobinary encoder to obtain the duobinary encoder with precoder as shown in figure above. The precoder consists of an Ex-OR gate along with a delay unit that introduces a delay of one bit duration $T_b$.

The precoder output d(k) is given by,

$$d(K) = b(K) \oplus d(K-1) \quad —— ①$$

where, b(k) = unipolar stream of binary digits.

Eqn shows that precoder o/p is equal to zero, if both its i/p are alike (both 0 or 1) and it is 1 if any one of the input is 1.

operation of Precoder.

| b(k) i/p | d(k-1) | Precoder o/p d(k) | d'(k) |
|---|---|---|---|
| 0 | 0 | 0 | -1 |
| 0 | 1 | 1 | +1 |
| 1 | 0 | 1 | +1 |
| 1 | 1 | 0 | -1 |

The precoder o/p is then applied to

a level shifter which converts it into a bipolar signal having values +1 or -1 as follows:

$$d'(k) = \begin{cases} +1 & \text{if } d(k)=1 \\ -1 & \text{if } d(k)=0 \end{cases} \quad —— ②$$

d'(k) is applied to basic duobinary encoder. Its o/p i.e, $V_0(k)$ is given by,
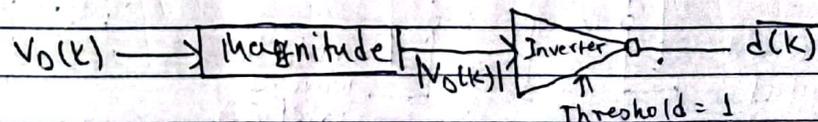
$$V_0(k) = d'(k) + d'(k-1) \quad —— ③$$

As d'(k-1) is delayed version of d'(k) it will also be a bipolar signal having value ±1

$$\therefore V_0(k) = \begin{cases} \pm 2 & \text{if } d'(k) = d'(k-1) \\ 0 & \text{if } d'(k) \neq d'(k-1) \end{cases} \quad —— ④$$

As this is duobinary signal,

$$V_0(k) = \pm 2 \quad \text{when } d(k)=0$$
$$V_0(k) = 0 \quad \text{when } d(k)=1 \quad —— ⑤$$

Decoder Block diagram

$$V_0(k) \longrightarrow \boxed{\text{Magnitude}} \; |V_0(k)| \; \triangleright \text{Inverter} \triangleright \circ \longrightarrow d(k)$$
$$\text{Threshold} = 1$$

## Eye Diagram: 78Bh[5], SN: 72ch[5], 76ch[5], 69ch[4]

Eye pattern is a pattern displayed on the screen of a cathode ray oscilloscope (CRO). The shape of this pattern resembles with the shape of human eye. Hence it is known as eye pattern. Eye pattern is a practical way to study the intersymbol interference (ISI) & its effects on a PCM or data communication system.

The eye pattern is obtained on the CRO interface by applying the received signal to vertical deflection plates (Y-plates) of the CRO and a sawtooth wave at the transmission symbol rate i.e, $(1/T_0)$ to the horizontal deflection plates (x-plates) as shown in figure (a) below. The resulting oscilloscope display is shown in figure (b) is called as the eye pattern which resembles human eye.



Received digital signal

Sawtooth wave at rate $(1/T_0)$

Fig: a

Fig: b

eye opening

$T_0$
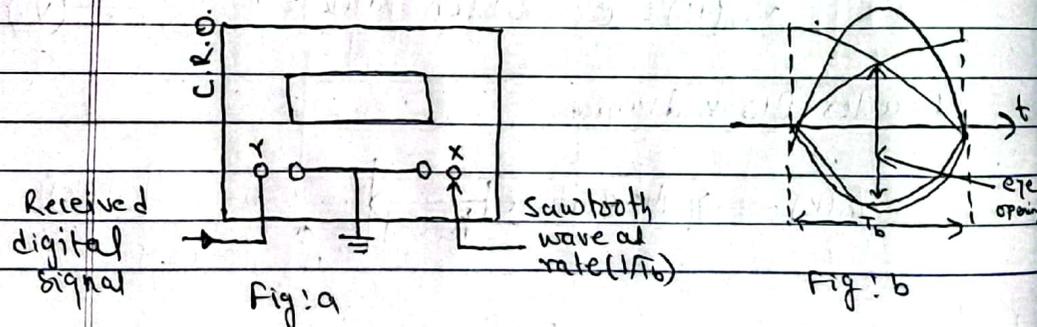
The interior region of the eye pattern is known as the eye opening. The eye pattern provides great deal of information about the performance of the system.
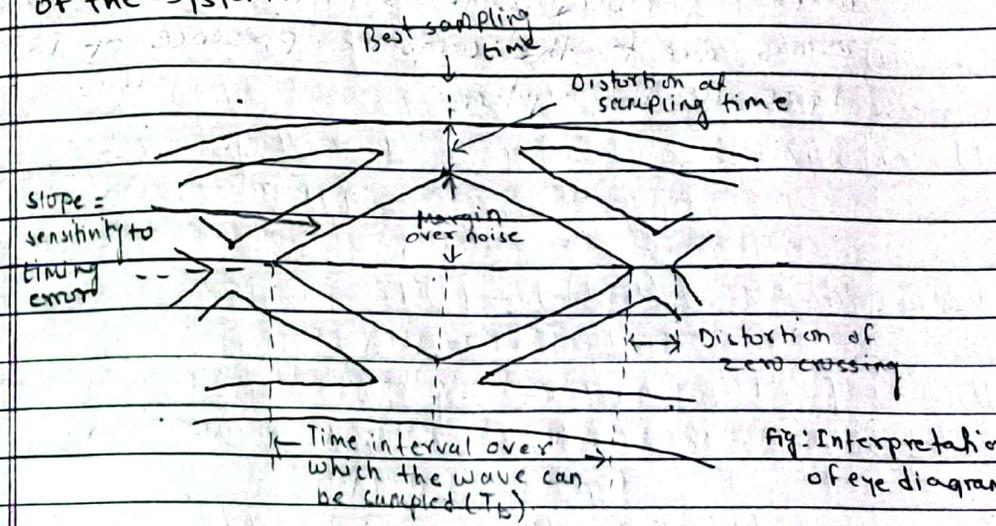


Best sampling time

Distortion at sampling time

slope = sensitivity to timing error

margin over noise

Distortion of zero crossing

Time interval over which the wave can be sampled $(T_b)$

Fig: Interpretation of eye diagram

### information obtained from eye pattern:

(i) The width of the eye opening defines the time-interval over which the received wave can be sampled, without an error due to ISI. The best time for sampling is when eye is open widest.

(ii) The sensitivity of the system to the timing error is determined by the rate of closure of the eye as the sampling rate is varied. The height of eye opening at a specified sampling time defines the

margin over noise.

(vii) When the effect of ISI is severe, the eye is completely closed & it is impossible to avoid errors due to the combined presence of ISI & noise in the system.

Bandpass (Modulated) data communication systems (4H/8%

No chapter 5 in examquestions → 74ch, 74As, 73Sh,71ch

Q) What are the design goals of digital Modulation techniques? 73ch[2]

= To send baseband signals over radio links or satellites is not possible because of impractically large antennas that have to be used. Hence the spectrum of the message signal has to be shifted to higher frequencies. This is achieved by using the baseband digital signal to modulate a sinusoidal carrier. This is called as digital carrier Modulation. There are various design goals of digital modulation techniques. They are:

(i) Binary & M-ary schemes: In binary we send any one of two possible signals during each signalling interval of duration $T_b$. Similarly in M-ary we can send any one of M possible signals during each signal interval of duration $T_b$. M-ary scheme needs less BW but error performance is poor compared to binary schemes.

(ii) **Probability of Error (Pe):** The value of Pe of a system indicates its performance in presence of AWGN (additive white Gaussian Noise). Pe should be as small as possible.

(iii) **Power spectra:** It is a graph of power spectral density plotted on Y axis vs frequency plotted on x axis. It gives information about the bandwidth requirement & co-channel interference.

(iv) **BW efficiency:** It is defined as the ratio of the data rate (bits/sec) to the effectively utilized channel BW. It is denoted by $\rho$.

$$\rho = \frac{R_b}{B} \text{ bits/Hz.}$$

It depends on Multichannel encoding & spectral shaping.

(V) **Passband Transmission Model:** The passband transmission model consists of 3 parts: Transmitter, communication channel & receiver.

**Coherent Modulation:** This technique employs coherent detection. Here, receiver carrier is phase locked with the transmitter carrier. It's design is complex but provides low error probability.

**Non-coherent Modulation:** Here, receiver carrier is not phase locked with transmitter carrier. Its design is simple but error probability increases.

\# **Binary Phase shift keying:**

- Modulation technique  ⎫
- Demodulation technique  ⎬ 73ch[8]
- Signal space diagram  ⎭

It falls under the coherent modulation technique. It is the most efficient of the 3 digital modulation (ASK, FSK & PSK). Hence, BPSK is used for high bit rates. In BPSK, phase of the sinusoidal carrier is changed according to the data bit to be transmitted. Also, a bipolar NRZ signal is used to represent the digital data coming from the digital source.

In a binary phase shift keying (BFSK), the binary symbols '1' and '0' modulate the phase of the carrier.

BPSK signal can be defined as;

$$s(t) = b(t)\sqrt{2P}\;\cos(2\pi f_c t) \quad\underline{\quad}① $$

where, $b(t) = +1$ when binary '1' is transmitted
$= -1$ when binary '0' is transmitted

BPSK signal may be generated by applying carrier signal to a balanced modulator. The binary data signal (0s & 1s) is converted to NRZ bipolar signal by an NRZ encoder. Here, the bipolar signal $b(t)$ is applied as a modulating signal to the balanced modulator.
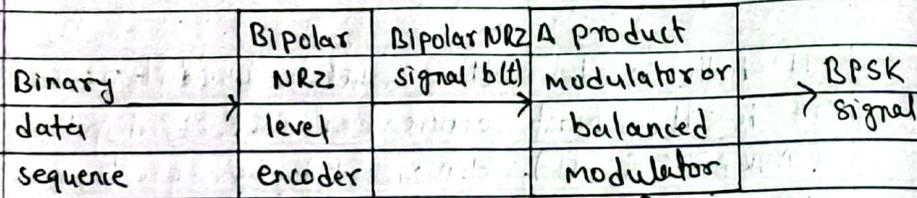
| Binary data sequence | Bipolar NRZ level encoder | Bipolar NRZ signal b(t) | A product Modulator or balanced modulator | BPSK Signal |
|---|---|---|---|---|

Carrier signal
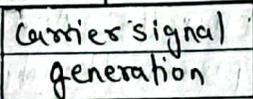
Carrier signal generation

Fig: Generation of BPSK

A NRZ level encoder converts the binary data sequence into bipolar NRZ signal. The table below shows i/p digital & corresponding bipolar NRZ signal.

| S.N. | i/p digital signal | Bipolar NRZ signal b(t) | BPSK O/p signal |
|---|---|---|---|
| 1. | Binary '0' | $b(t) = -1$ | $-\sqrt{2P}\,\cos w_c t$ |
| 2. | Binary '1' | $b(t) = +1$ | $+\sqrt{2P}\,\cos w_c t$ |

Here,

$$P = \frac{E_b}{T_b} \quad ; \qquad E_b \to \text{Signal energy}$$
$$T_b \to \text{bit duration}$$
$$w_c \to 2\pi f_c$$

Figure below shows the block diagram of the scheme to recover baseband signal from BPSK signal. The Transmitted BPSK signal is given as:
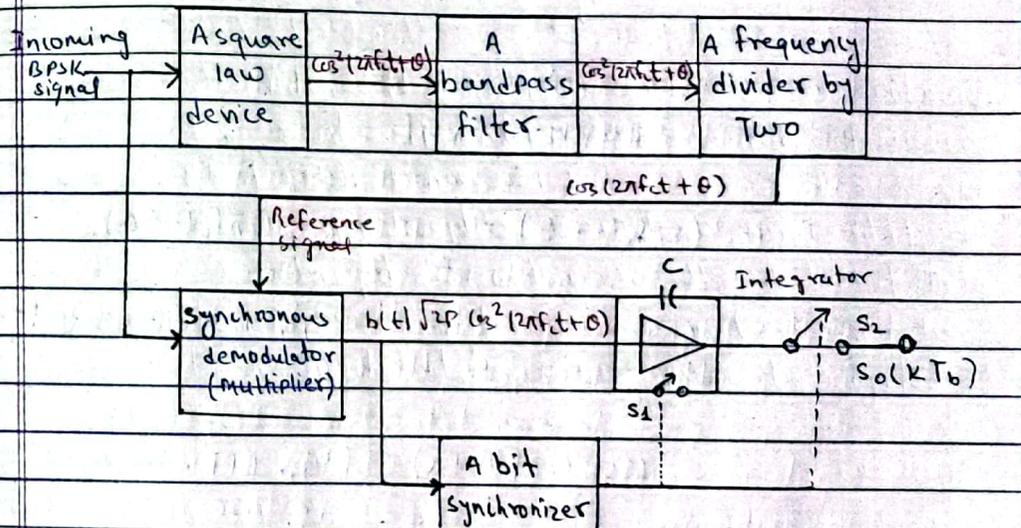
$$s(t) = b(t)\sqrt{2P}\;\cos(2\pi f_c t)$$



Fig: Reception of baseband signal in BPSK signal

The transmitted BPSK, signal undergoes, phase change depending upon timedelay from transmitter end to receiver end. This phase change is phase shift in transmitted signal, $\theta$. Thus, the signal at input of receiver can be written as;

$$S(t) = b(t) \sqrt{2P} \cos(2\pi f_c t + \theta) \quad \text{—①}$$

Since it is a coherent detection, the _____ carrier is separated.

The received signal is passed through square law device. Thus we get

$$\cos^2(2\pi f_c t + \theta) \quad \begin{cases} \text{we have neglected amplitude} \\ \text{as we are interested in carrier} \end{cases}$$

we know that,

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\therefore \cos^2(2\pi f_c t + \theta) = \frac{1}{2} + \frac{1}{2}\cos 2(2\pi f_c t + \theta)$$

This signal is passed through Bandpass filter. thus we get the O/P

$$\cos 2(2\pi f_c t + \theta) \quad \text{from BPF.}$$

This signal has frequency equal to $2f_c$ thus it is

passed through frequency divider by two. whose O/P will be $\cos(2\pi f_c t + \theta)$

The coherent demodulator multiplies the i/p signal & the recovered carrier signal. Hence at O/P of multiplier, we get:

$$b(t)\sqrt{2P}\cos(2\pi f_c t + \theta) \times \cos(2\pi f_c t + \theta) = b(t)\sqrt{2P}\cos^2(2\pi f_c t + \theta)$$

$$\Rightarrow b(t)\sqrt{\frac{2P}{4}}[1 + \cos 2(2\pi f_c t + \theta)]$$

$$\Rightarrow b(t)\sqrt{\frac{P}{2}}[1 + \cos 2(2\pi f_c t + \theta)] \quad \text{—③}$$

This signal is applied to bit synchronizer and integrator. The integrator integrates the signal over one bit period. The bit synchronizer takes care of starting & ending times of bit. By closing and opening $S_1$ & $S_2$ (switches) integrator performs integration causing phase changes and sends the values generated to output of Receiver.

The output $S_0(kT_b)$ generated in receiver depends upon the value $b(kT_b)$ given on the input side.

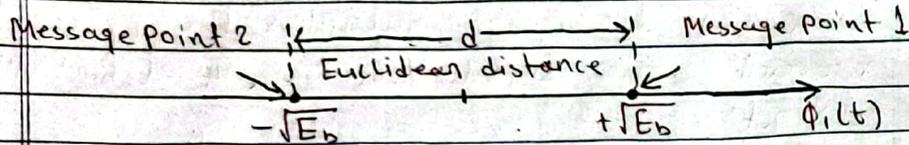The following figure shows the signal space diagram of BPSK signal



Fig: SS diagram of BPSK signal.

where,

Signal energy, $E_b = P T_b$

$P = $ power
$T_b = $ bit duration

**Drawback:** In the detection of BPSK, we square the $b(t) \sqrt{p} \cos(2\pi f_c t)$ to regenerate the carrier. However if the received signal is

$-b(t) \sqrt{2P} \cos(2\pi f_c t + \theta)$

the squared signal remains same as before. Hence, the recovered carrier is unchanged even if the i/p signal has changed its sign. Therefore it is not possible to determine whether the received signal is equal to $b(t)$ or $-b(t)$, which results in ambiguity. Its remedy is we can solve this problem by using DPSK.

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\# Coherent Binary Frequency shift keying [BFSK].

- Modulation
- Demodulation
- Signal space diagram
- Non - coherent detection

76ch[8]
72ch[6]
69ch[8]

In binary frequency shift keying (BFSK), the frequency of a sinusoidal carrier is shifted according to the binary symbol. In other words, the frequency of a sinusoidal carrier is shifted between two discrete values. However the phase of the carrier is unaffected. This means we have two different frequency signals according to binary symbols. Let there be frequency shift by $\Omega$ then we can write following equation-

$\text{If } b(t) = '1' \text{ then } S_H(t) = \sqrt{2P_s} \cos(2\pi f_c + \Omega)t \quad —①$
$\text{If } b(t) = '0' \text{ then } S_L(t) = \sqrt{2P_s} \cos(2\pi f_c - \Omega)t \quad —②$

Hence there is increase or decrease in frequency by $\Omega$. Let us we the following conversion table to combine above eq.

| b(t) Input | d(t) | $P_H(t)$ | $P_L(t)$ |
|---|---|---|---|
| 1 | +1V | +1V | 0V |
| 0 | -1V | 0V | +1V |

The equation ① & ② can combinely be written as:

$$s(t) = \sqrt{2P_s} \cos\left[\left(2\pi f_c + d(t)\Omega\right)t\right] \quad \text{——(iii)}$$

when 1 is transmitted.

carrier frequency, $f_H = f_c + \dfrac{\Omega}{2\pi}$ ——(iv)

when '0' is transmitted

carrier frequency, $f_L = f_c - \dfrac{\Omega}{2\pi}$ ——(v)

It may be observed from table above that $P_H(t)$ is same as b(t) but $P_L(t)$ is inverted of b(t). The block diagram of BFSK generation is shown below:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_1 t)$$



Fig: BD of BFSK generation
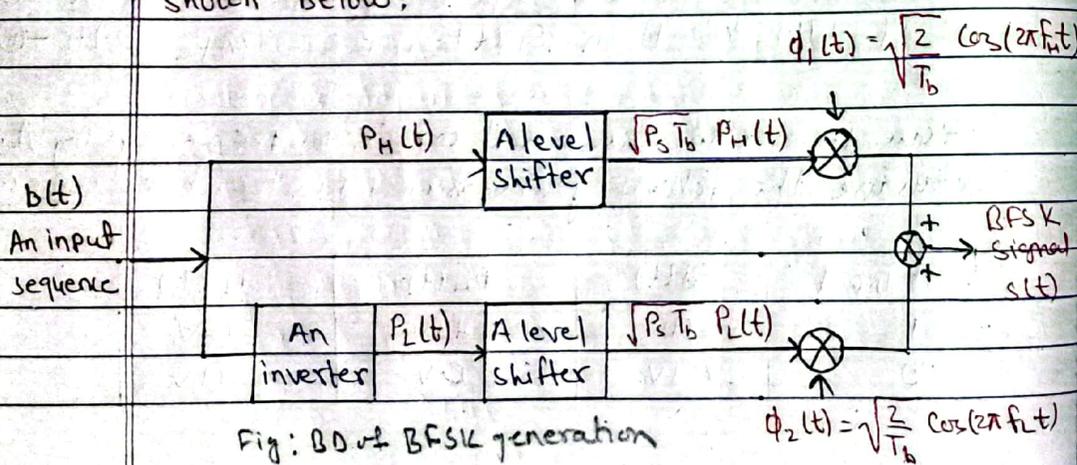
We know that i/p sequence b(t) is same as $P_H(t)$. An inverter is added after b(t) to get $P_L(t)$. The level shifter converts +1 level to $\sqrt{P_s T_b}$ while zero is unaffected. The transmitted signal will have a frequency of either $f_H$ or $f_L$.

Further, there are product modulators after level shifter. The two carrier signals - $\phi_1(t)$ & $\phi_2(t)$ are used. $\phi_1(t)$ & $\phi_2(t)$ are orthogonal to each other.

output from both modulators are not possible at a time since $P_H(t)$ & $P_L(t)$ are complementary to each other.

Similarly figure below shows the block diagram of demodulation of BFSK using coherent detection technique.

The detector consists of two correlators that are individually tuned to two different carrier frequencies, to represent symbols '1' and '0'. A correlator consists of a multiplier followed by an integrator. Then, the received binary fsk signal is applied to the multipliers of both the correlators. To the other input of the multipliers, carriers with frequency $f_{c1}$ and $f_{c2}$ are applied
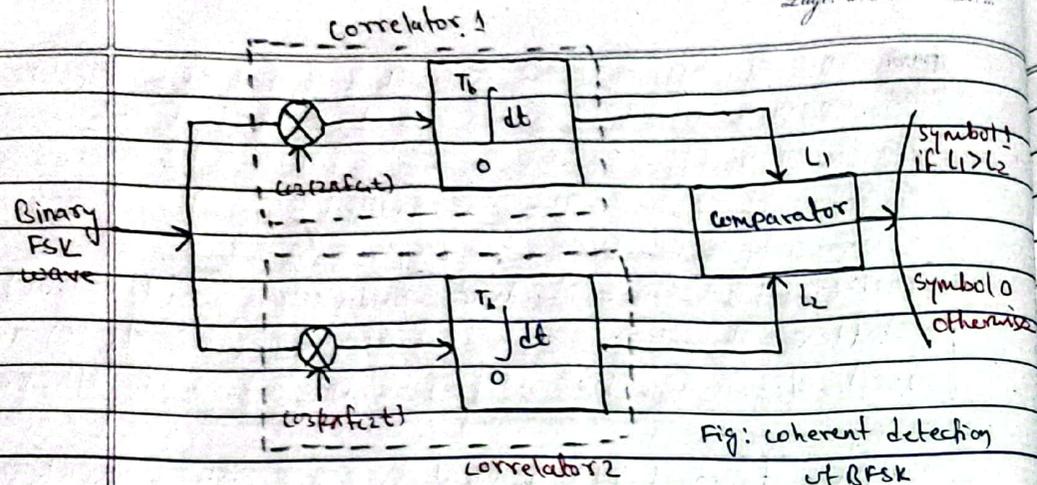
Correlator 1



Fig: coherent detection
of BFSK

The multiplied output of each multiplier is subsequently passed through integrators generating output $l_1$ & $l_2$ in the two paths. The output of the two integrators are fed to the decision making device,

If $l_1 > l_2 \longrightarrow$ output is symbol 1
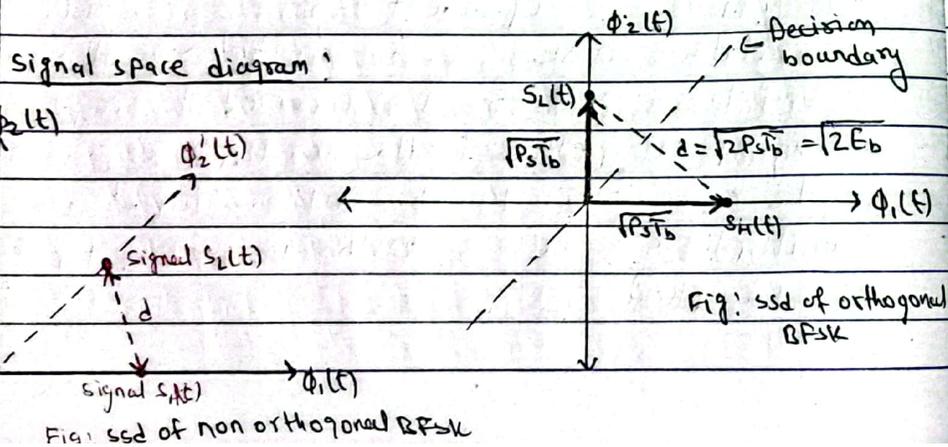If $l_1 < l_2 \longrightarrow$ output is symbol 0.

signal space diagram:



Fig: ssd of orthogonal BFSK

Fig: ssd of non orthogonal BFSK

Binary FSK waves may be demodulated non-coherently using envelope detector. The received FSK signal is applied to bank of two bandpass filters, one tuned to frequency $f_{c1}$ and the other tuned to $f_{c2}$. Each filter is followed by an envelope detector. The resulting outputs of the two envelope detectors are sampled and then compared to each other.
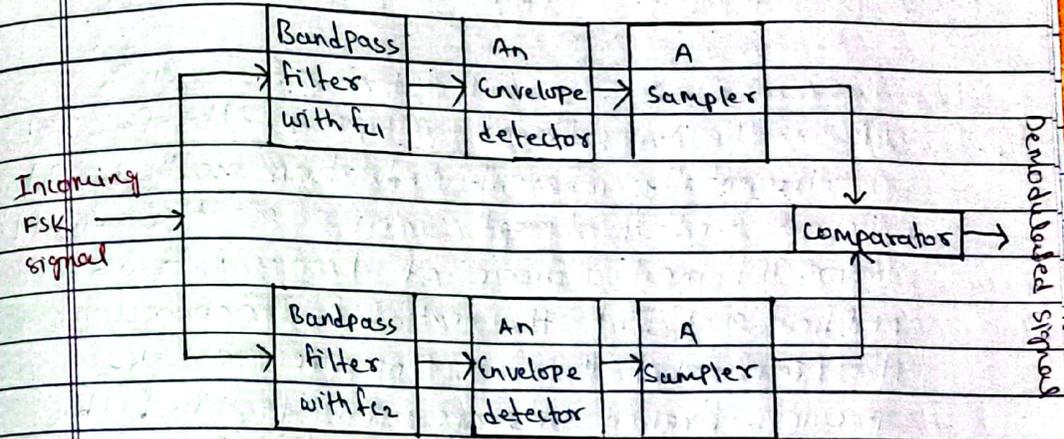


Fig: Non-coherent detection of FSK binary signal

A decision is made in favour of symbol '1' if the envelope detector output derived from the filter tuned to frequency $f_{c1}$ is larger than that derived from the second filter. Otherwise, a decision is made in favour of the symbol '0'.

## Differential phase shift keying (DPSK):

- what do you understand by differential coding ? 71sh[2]
- Modulation : 78Bh[6], 72 kar[2], 71sh[5]
- demodulation : 78Bh[6], 72 kar[2], 71sh[5]
- Signal space diagram : 72 kar[1]
- need : 78Bh[5]
- why DPSK is preferred over PSK. 72 kar[5]

DPSK is the non-coherent version of the PSK. It is differentially coherent modulation method. It doesn't need a coherent carrier at the demodu--lator. The input sequence of binary bits is modified such that the next bit depends upon the previous bit. Therefore, in receiver, the previous received bits are used to detect the present bits.

In DPSK, the input binary sequence is first differentially encoded and then modulated using a product modulator as shown in figure below. The differentially encoded sequence $\{d(k)\}$ is generated from the input binary sequence $\{b(k)\}$ by complementing the modulo-2 sum of $b(k)$ and $d(k-1)$. The effect is to leave the symbol

$d(k)$ unchanged from the previous symbol if the incoming binary signal $b(k)$ is 1 & to toggle $d(k)$ if $b(k)$ is 0.

$$\therefore d(K) = b(K) \oplus d(k-1)$$

The block diagram of DPSK transmitter is shown below. It consists of one bit delay element and encoder circuit interconnected so as to generate differentially encoded sequence from the input binary sequence. The output is passed through product modulator to obtain DPSK signal.
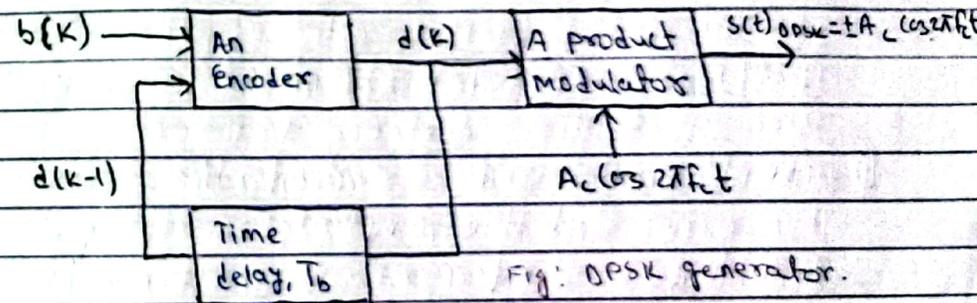


Fig: DPSK generator.

At receiver, the original sequence is recovered from the demodulated differentially encoded signal through a complementary process as shown in figure below. The received DPSK signal and its time delayed version is applied to each one of the input of the multiplier.

The output of the multiplier is then given to the integrator which produces output value.



Fig: DPSK receiver.
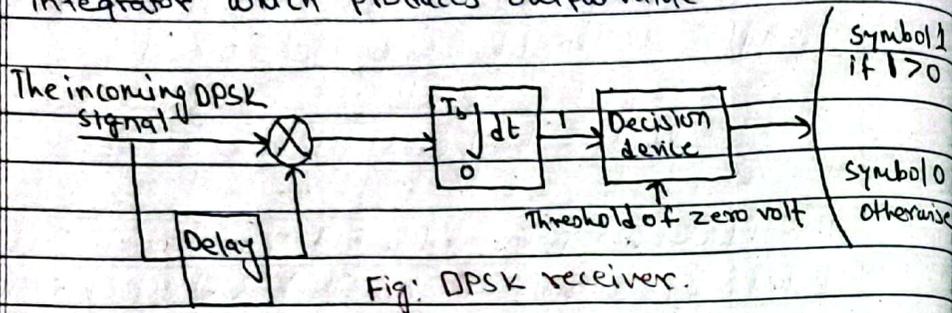
By comparing the integrator output with decision level of zero volt, the decision device can reconstruct the binary sequence by assigning a symbol 'o' for negative output & a symbol '1' for positive output.

Need/Features/Advantages of DPSK

(i) DPSK does not need carrier at the receiver end. This means that the complicated circuitry for generation of local carrier is not required.

(ii) The Bandwidth requirement of DPSK is lower than BPSK.

# Quadrature Phase shift Keying (QPSK)

- Modulation
- Demodulation        } 7o ch [8]
- SS Diagram
- Relevant derivations

In communication systems, we have two main resources, transmission power and channel Bandwidth. Since channel bandwidth depends upon information bit rate, if two or more bits are combined into a symbol then the signalling rate will reduce and thus transmission channel Bandwidth. In QPSK, two successive bits in data sequence are grouped together. This reduces bit rate or signalling rate and thus reduces channel bandwidth requirement. It has twice the bandwidth efficiency of BPSK.

The phase of the carrier takes on one of four equally spaced values such as $0, \pi/2, \pi$ and $3\pi/2$, where each value of phase corresponds to a unique pair of message bits.

$$S_{QPSK}(t) = \sqrt{\frac{2E_S}{T_S}} \cos\left[2\pi f_c t + (i-1)\frac{\pi}{2}\right] \qquad 0 \leq t \leq T_S$$
$$i = 1, 2, 3, 4$$

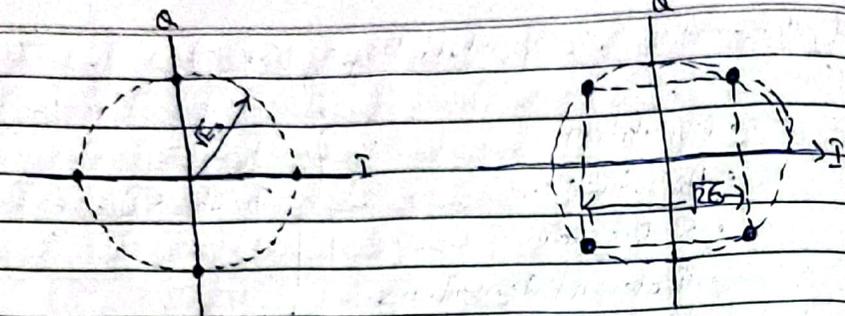where, $T_S$ = Symbol duration & is twice the bit period.

Fig: QPSK Constellation
diagram where carrier
phases are $0, \pi/2, \pi, 3\pi/2$

Fig: QPSK constellation
where carrier
Phases are $\pi/4, 3\pi/4,$
$5\pi/4, 7\pi/4.$

Here, the above are the constellation diagram
of QPSK signal. Different QPSK signal sets can
be derived by simply rotating the constellation.
From the constellation diagram, it can be
seen that the distance between adjacent
Points in the constellation is $\sqrt{2E_s}$.
Since each symbol corresponds to two bits,
then $E_s = 2E_b$, thus the distance between
two neighboring points in constellation is
equal to $2\sqrt{E_b}$.

QPSK Transmission: The unipolar binary message
stream has bit rate $R_b$ and is first converted
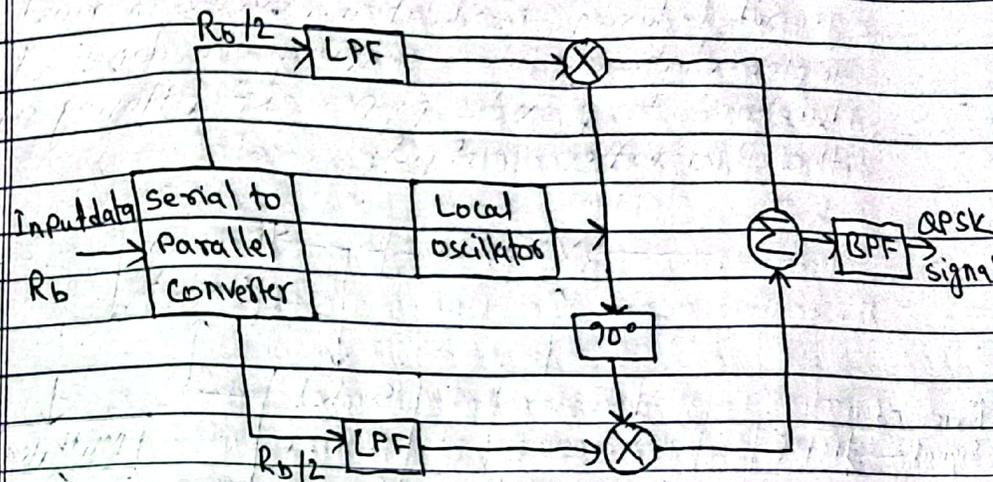into a bipolar NRZ sequence using unipolar to

bipolar converter.



Fig: Block diagram of QPSK Transmitter.

The bit stream $m(t)$ is split into two bit streams
$M_i(t)$ & $M_0(t)$, each having bit rate of $R_s = R_b/2$.
The bit stream $M_I(t)$ is called even stream & $M_0(t)$
is called odd stream. These two binary sequences
are separately modulated by two carriers $\phi_1(t)$ and
$\phi_2(t)$, which are in quadrature. The two modulated
signals, each of which are considered to be
BPSK signal, are summed to produce a QPSK
signal. The filter at the output of the modulator
confines the power spectrum of the QPSK signal
within the allocated band. This prevents from
spill-over of signal energy into adjacent channels

and also removes out of band spurious signals generated during modulation Process. In most implementations, Pulse shaping is done at baseband to provide proper RF filtering at the transmitter output.
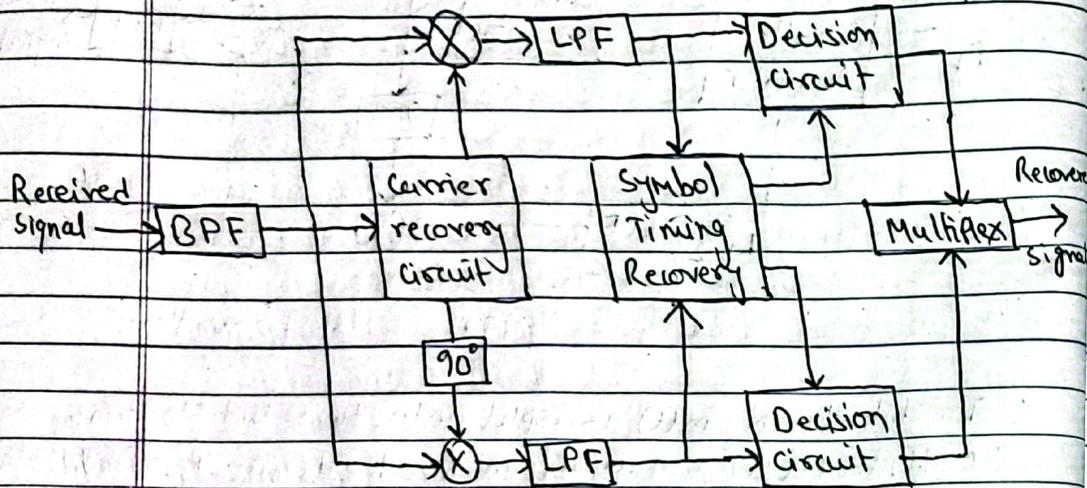


Fig: Block diagram of a QPSK receiver.

The frontend BPF removes the out of band noise and adjacent channel interference. The filtered output is split into two parts, & each part is coherently demodulated using the in-phase & quadrature carriers. The coherent carriers used for demodulation are recovered from the received signal using carrier recovery circuits. The O/Ps

of demodulators are passed through decision circuits which generate the inphase and quadrature binary streams. The two components are then multiplexed to reproduce the original binary sequence.

M-ARY data communication system: SN: 72ch[2.5]

In an M-ary signalling scheme, we can send one of the M possible signals such as $S_1(t), S_2(t), \dots S_m(t)$, during each signalling interval of duration T seconds. The number of possible signals, ie, M is given by,

$$M = 2^N \quad —①$$

where, N = integer.

Advantages of M-ary scheme over binary schemes is that they achieve better bandwidth efficiency. However they have their own drawbacks like increasing in transmitted power & increase in error probability.

Q) What is MODEM? Discuss the modes of operation of MODEM. 7OASD[4]

= MODEM (Modulator-Demodulator) is an electronic device used to transmit & receive digital data

for transmission over public telephone lines. A modulator translates the data pulses into voice band analog signals at the transmitting end and at the receiving end, the analog signals are demodulated to recover the digital information. Initially, MODEMS were used to connect terminals located in remote places, to a central computer. The following are the Modes of operation of MODEM.

① Simplex: In this mode, data transmission is done only in one direction & no signalling path is available from receiver to transmitter. Therefore, there is no possibility of retransmission and error connection. This mode has limited use.

⑪ Half-Duplex: In this mode, reverse channel is available but in turn, not simultaneously. The transmission speed is reduced because of the need to wait for the transmission or reception.

⑬ Full-Duplex: In this mode, data can be transmitted and received in both direction at same time. It requires two independent channel.

Random signals and noise in communication systems.
(7 Hours / 12 Marks)

This chapter was not asked on 2076 ch.

Q) Define Moment and central Moment.
= The $n^{th}$ moment of a random variable is defined as the mean value of $x^n$. Thus, the expression for the $n^{th}$ moment of X is given by,

$$n^{th} \text{ moment of } X = E[X^n] = \overline{x^n} = \int_{-\infty}^{\infty} x^n f_x(x)dx$$

'Central moment' is the expected value of the difference between the random variable X & its mean value $M_x$. Thus, $n^{th}$ central moment is defined as under!

$$n^{th} \text{ central moment of } X = E[(X - M_x)^n]$$
$$= \int_{-\infty}^{\infty} (x - M_x)^n f_x(x)dx$$

Show that 1st central moment is always zero.

From eqⁿ (ii)

$m_x$ = average value of X.

Now, for 1st central moment, n=1

1st central moment of $x = E[(x - m_x)^1]$

$$= E[(x)] - E[m_x] \quad \text{——(iii)}$$

Now, since 1st moment of x is given as

$$E[x] = \bar{x} = \int_{-\infty}^{\infty} x \, f_x(x) \, dx = m_x$$

Thus from eqⁿ (iii)

1st central moment of $x = E[x] - E[m_x]$

$$= m_x - E[m_x]$$

$$= m_x - m_x$$

$$= 0$$

Hence, 1st central moment of continuous random variable is always zero.

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72 Kar [2]
71Sh [2]
69Ch [4]

**Random Process:** Let there be a random experiment E having outcome λ from the sample space S. Thus every time an experiment is conducted, the outcome λ will be one of the sample point in sample space. If this outcome λ is associated with time, then a function of λ and time t is formed ie, X(λ, t). Then the function X(λ, t) is known as random process. Hence when any random experiment E is given a time dimension, then each outcome appears at some certain time & the random experiment will be converted into Random Process. A random process is the function of two variables λ & t.

Q) Explain Auto-correlation function. 74Ch [3]

= For carrying out signal analysis, one of the most important statistical characteristics of a random process is its auto-correlation function. This provides the spectral information about the random process. The auto correlation function of a random process is defined as under:

$$R_x(\tau) = E[X(t+\tau) X(\tau)] \quad \text{——①}$$

Figure below illustrates the physical significance of the autocorrelation function.

$R_x(\tau)$



slowly changing
Random process
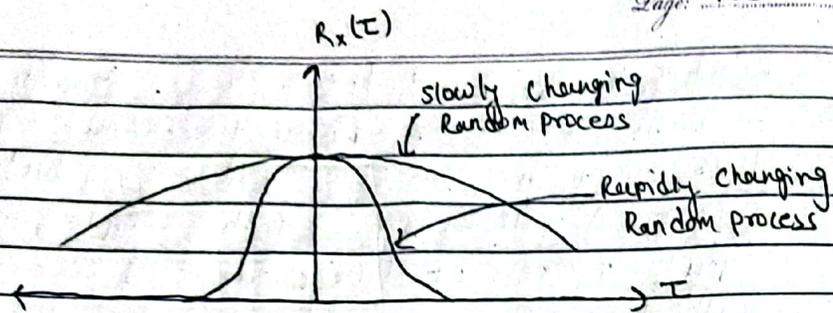
Rapidly changing
Random process

$\tau$

Fig: Physical significance of Ac function.

The physical significance of $R_x(\tau)$ is that it is a measure of interdependence of two random variables obtained by observing a random process $X(t)$ at times $\tau$ seconds apart.

Therefore, if the random process $X(t)$ changes very rapidly with time, the AC function will decrease rapidly to zero as shown in figure above. The decorrelation time $\tau_0$ characterizes the reduction in $R_x(\tau)$ from $R_x(0)$ to 0. If $\tau > \tau_0$, then, the magnitude of the ac function $R_x(\tau)$ remains below some prescribed value. Therefore, the decorrelation time $\tau_0$ of a wide-sense stationary random process $X(t)$ of a zero mean as the time taken for the magnitude of ac function $R_x(\tau)$ to decrease to 1 percent of the maximum value $R_x(0)$.

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Q) What do you mean by Ergodic stochastic process? 74ch[2]

= A random process is known as ergodic process if the time-averages are equal to ensemble averages. Hence for a ergodic process, we have

$$m_x = \langle M_x \rangle$$
$$R_x(t_1, t_2) = \langle R_x(\tau) \rangle$$

For any ergodic process, all ensemble averages will be equal to corresponding time averages for any particular sample functions. The time. averages are not a function of time. When time & ensemble averages are same, it implies that ensemble averages also are not a function of time.

Thus an ergodic process is always stationary but converse is not true.

\# Transmission of a Random process through a Linear filter. 74ch[10], 70Asd[5].

Let us assume that a linear process $X(t)$ is applied as input to a linear time invariant (LTI) filter. Let the impulse response of such filter be $h(t)$. At the output of this filter, let a new random process $Y(t)$ be produced as shown in

figure below:



$$X(t) \longrightarrow \boxed{\begin{array}{c} \text{Impulse} \\ \text{response} \\ h(t) \end{array}} \longrightarrow Y(t)$$
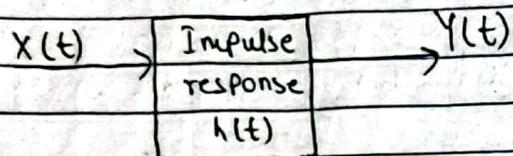
Fig: Transmission of random process
through an LTI filter.

Even if the distribution of $X(t)$ is known to us, it is difficult to describe the probability of distribution of $Y(t)$. Let us obtain the mean and autocorrelation function of the output process $Y(t)$. We assume that $X(t)$ is a wide sense stationary process.

* The first mean of $Y(t)$ is defined as under:

$$m_y(t) = E[Y(t)] = E\left[ \int_{-\alpha}^{\alpha} h(\tau_1) X(t-\tau_1) d\tau_1 \right] \quad \text{—(i)}$$

where, $\tau_1 \rightarrow$ dummy variable.

we can interchange order of expectation & integration wrt $\tau_1$ provided that $[X(t)]$ is finite for all $t$ & system is stable

$$m_y(t) = \int_{-\alpha}^{\alpha} h(\tau_1) E[X(t-\tau_1)] d\tau_1$$

$$m_y(t) = \int_{-\alpha}^{\alpha} h(\tau_1) m_x(t-\tau_1) d\tau_1 \quad \text{—(ii)}$$

If i/p $X(t)$ is wide sense stationary, then $m_x(t)$ is a constant equal to $m_x$. Thus from (ii)

$$m_y(t) = m_x \int_{-\alpha}^{\alpha} h(\tau_1) d\tau_1 \quad \text{—(iii)}$$

$$\therefore m_y(t) = m_x H(0) \quad \text{—(iv)}$$

where, $H(0) \rightarrow$ zero frequency (dc) response of system.

Eq. (iv) shows mean of $Y(t)$ is equal to product of mean $X(t)$ & dc response of system.

* Similarly the autocorrelation function of $Y(t)$ is given by:

$$R_y(t,u) = E[Y(t) Y(u)]$$

where; $t, u \rightarrow$ two values of time at which o/p is observed

The AC function of $Y(t)$ can be obtained by using the convolution integral as under:

$$R_Y(t,u) = E\left[\int_{-\infty}^{\infty} h(\tau_1) X(t-\tau_1)\, d\tau_1 \int_{-\infty}^{\infty} h(\tau_2) X(u-\tau_2)\, d\tau_2\right] \quad —(v)$$

If $x(t) \to$ stable &
$\qquad E[X^2(t)] \to$ finite for all $t$,
we can interchange order of expectation & integration.

$$R_Y(t,u) = \int_{-\infty}^{\infty} d\tau_1\, h(\tau_1) \int_{-\infty}^{\infty} d\tau_2\, h(\tau_2)\, E\left[X(t-\tau_1) X(u-\tau_2)\right]$$

$$= \int_{-\infty}^{\infty} d\tau_1\, h(\tau_1) \int_{-\infty}^{\infty} d\tau_2\, h(\tau_2)\, R_X(t-\tau_1, u-\tau_2) \quad —(vi)$$

when i/p process $X(t) \to$ wide sense stationary process the ac function of $X(t)$ is only a function of the difference $(t-\tau_1)$ & $(u-\tau_2)$. Hence, substituting $\tau = (t-u)$ in eq^n (vi), we get.

$$R_Y(\tau) = \int_{-\infty}^{\infty} h(\tau_1)\, h(\tau_2)\, R_X(\tau - \tau_1 + \tau_2)\, d\tau_1 d\tau_2 \quad —(vii)$$

If we look at the expressions of $m_Y$ and $R_Y(\tau)$, then we can conclude that if the input to a stable LTI filter is a wide sense stationary process, then the output of the filter is also a wide sense stationary random process.

White noise: It is the noise whose power spectral density is uniform over the entire frequency range of interest as shown in figure below:
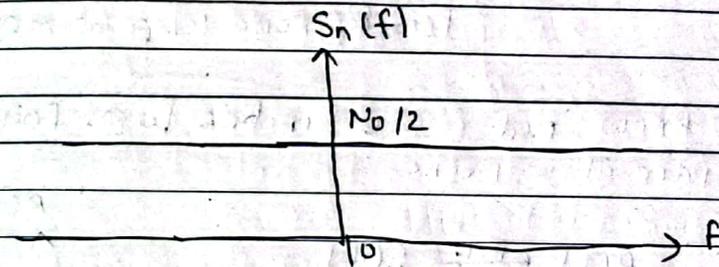


Fig: psd of white noise.

white noise has all the frequency components in equal proportion. It has a gaussian distribution which means PDF of white noise has the shape of gaussian PDF. Thus it is also known as gaussian noise.

From figure above, psd of white noise is given by,

$$S_n(f) = \frac{N_o}{2} \qquad \text{—①.}$$

This eq^n shows psd of white noise is independent of frequency.

As $N_o$ is constant, psd is uniform over entire frequency range.

$$N_o = K T_e$$
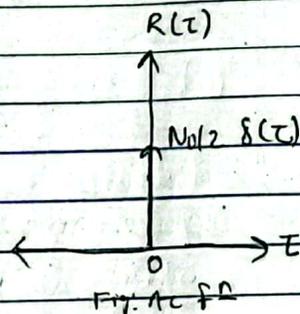
where,

$$K = \text{Boltzmann's constant}$$
$$T_e = \text{Eq. noise Temperature of system.}$$

We know psd & ac function form Fourier transform pair. This means,

$$R(\tau) \xleftrightarrow{F.T.} S(f)$$

i.e, $FT\{R(\tau)\} = S(f)$

or, $R(\tau) = IFT\{S(f)\}$



Fig. Ac f^n

for white noise,

$$R(\tau) = IFT\{S_n(f)\} = IFT\{N_o/2\}$$

$$\therefore R(\tau) = \frac{N_o}{2}\delta(t) \quad \begin{cases} AC\ f^n\ of \\ white\ noise \end{cases}$$

Noise Equivalent Bandwidth: 73ch[2], 72ch[2-5], 70Asd[3]

Assume that a white noise is present at the input of a receiver (filter). Let the filter have a TF H(f) as shown in figure below. This filter is being used to reduce the noise power actually passed on to the receiver. Now consider an ideal (rectangular) filter as shown by dotted plot in figure below. The center frequency of this ideal filter is also fo.
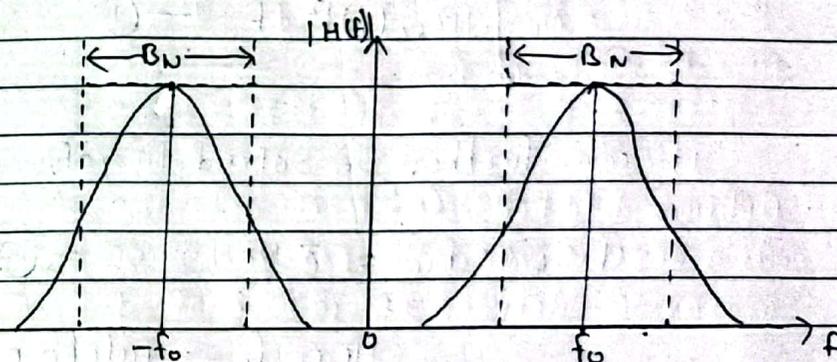


fig: Noise BW of a filter

Let the bandwidth $B_N$ of the ideal filter be adjust in such a way that the noise output power of the ideal filter is exactly equal to the noise output power of a real R-C filter. Then $B_N$ is called as the noise bw of the real filter.

Q) Determine the noise equivalent bandwidth of RC-LPF and that of ideal LPF of zero frequency response. Also, find o/p noise power of this RC-LPF when input is white noise.

= The noise power at the o/p of a RC LPF is theoretically infinite & is defined only by transfer function of RC filter.

$$P_{NRC} = \frac{N_0}{2} \int_{-\infty}^{\infty} |H_{RC}(f)|^2 df$$

$$= N_0 \int_{0}^{\infty} |H_{RC}(f)|^2 df \quad —①$$

It can further be solved to get,

$$P_{NRC} \text{ or } P_{no} = \frac{\pi N_0 f_c}{2} \quad —②$$

Here, $f_c \to$ cutoff filter.

Noise power for ideal LPF is finite & proportional to BW of filter.

$$P_{N ideal} \text{ or } P_{no} = \frac{N_0}{2} \times 2B = N_0 B \quad —③$$

To generalize the noise power, we need to define some standard parameter called noise equivalent bandwidth (BW) that can be used to calculate average noise power.

In case of RC filter with an ideal LPF having TF H(0) and Bandwidth equal to $B_N$. so that the noise power at an output of this idealized filter & RC filter are equal.

$$P_{N ideal} = N_0 B_N H^2(0) \quad —④$$

$$\& \ P_{N real} = N_0 \int_{0}^{\infty} |H(f)|^2 df \quad —⑤$$

Thus we get,

$$B_N = \frac{\int_{0}^{\infty} |H(f)|^2 df}{|H^2(0)|} \quad —⑥$$

Similarly for Bandpass filter Noise equivalent BW equals,

$$B_N = \frac{\int_{0}^{\infty} |H(f)|^2 df}{|H^2(f_c)|^2} \quad —⑦$$

In general equivalent noise bandwidth is expressed as,

$$B_N = \frac{1}{g_a} \int_0^\alpha |H(f)|^2 df \quad \text{(viii)}$$

where, $g_a \rightarrow$ max$^m$ available power gain of filter

∴ In terms of equivalent noise BW the noise power at the o/p of any filter is

$$P_N = N_0 \, B_N \, g_a \quad \text{(ix)}$$

Optimum detection: Optimum detector of the pulse is a device that has least probability of errors in making decision in favour of 1 or 0.

The basic idea is to pass received signal through the filter that supresses the noise & give sharp peak to the signal at the detection making instance, thus creating max$^m$ SNR at decision making instance.

noisy i/p → | Optimum filter | → | Decision Maker | → o/p y(t)
[x(t)+n(t)]

$y(t)+n_0(t)$
$t=t_m$
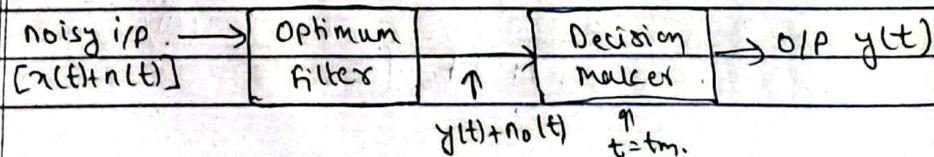
# Matched filter! In optimum filter we consider the noise as generalized Gaussian noise. When this noise is white Gaussian noise, then the optimum filter is known as matched filter. For white gaussian noise, PSD is given as,

$$S_{ni}(f) = \frac{N_0}{2} \quad \text{(i)}$$

* Realization of matched filter:

f(t)=x(t)+n(t) → | Matched filter h(t) | y(t) → ✗ t=t_m → y(t_m)
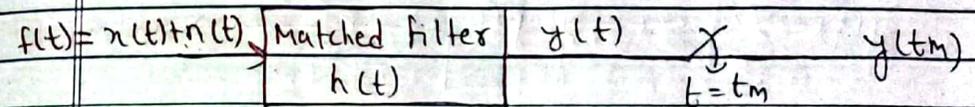
Here, f(t) is noisy input signal.
$$f(t) = x(t) + n(t) \quad \text{(ii)}$$

The output of signal from above figure is;

$$y(t) = f(t) * h(t)$$
$$= \int_{-\alpha}^{\alpha} f(s) h(t-s) ds \quad \text{(iii)}$$

for matched filter, $h(t) = x(t_m - t)$
for time shifted version of h(t);
$$h(t-s) = x(t_m - t + s)$$

$$\therefore y(t) = \int_{-\alpha}^{\alpha} f(s) \, x(t_m - t + s) \, ds \quad \text{---} \quad \text{④}$$

at $t = t_m$

$$y(t_m) = \int_{-\alpha}^{\alpha} f(s) \cdot x(s) \, ds \quad \text{---} \quad \text{⑤}$$

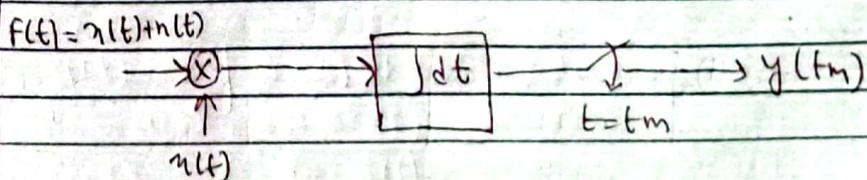This eq² can be represented in block diagram as:

$$f(t) = x(t) + n(t)$$



Fig : BD of correlator.

The signal $f(t)$ is multiplied by locally generated replica of i/p signal $x(t)$. Result is integrated & sampled at $t = t_m$. The above arrangement is called time correlator which is synchronous detector.

Matched filter for Rectangular Pulse:

= Impulse response of matched filter for rectangular pulse is,

$$H_{MF}(t) = \begin{cases} A & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

It's transfer function is,

$$H_{MF}(f) = \text{sinc}(FT) \exp(-j\pi f T)$$
$$\text{Assuming } AT = 1$$

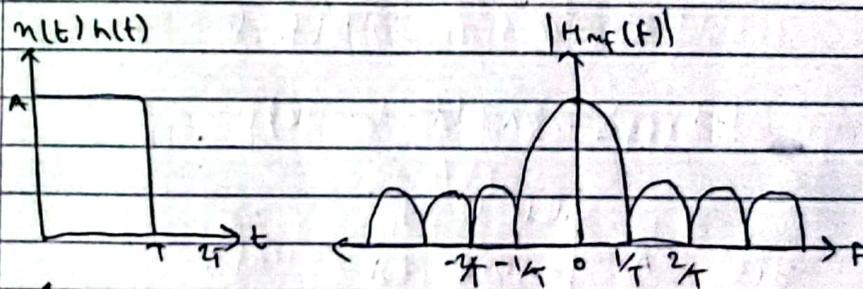But TF of an ideal LPF is a rectangle.



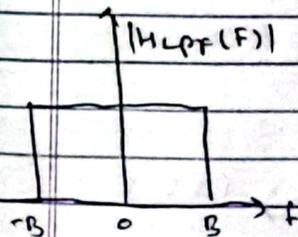Fig : Amplitude response of filter matched to rectangular pulse.

Fig : Amplitude response of ideal LPF approx the MF.

Impulse response of ideal LPF that works as distortionless transmission is,

$$h(t) = \frac{\sin 2\pi B(t-t_0)}{\pi(t-t_0)} = 2B \sin c\left[2B(t-t_0)\right]$$

when a rectangular pulse of amplitude A & duration T is passed through an ideal LPF its response is given by,

By convolution,

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \, h(t-\tau) \, d\tau$$

$$= 2BA \int_{-T/2}^{T/2} \frac{\sin 2\pi B (t-t_0-\tau)}{2\pi B(t-t_0-\tau)} \, d\tau$$

Let, $\lambda = 2\pi B(t-t_0-\tau)$

$$y(t) = \frac{A}{\pi} \int_{-2\pi B(t-t_0-\tau)}^{2\pi B(t-t_0-\tau)} \frac{\sin \lambda}{\lambda} \, d\lambda$$

$y(t)$ is maximum at $t = T/2$ for $BT \geq 1$

$$y(t)\Big|_{max} = \frac{2A}{\lambda} \, \text{sinc}(\pi BT)$$

where, sinc signal is defined by,

$$\text{sinc}(u) = \int_0^u \frac{\sin \lambda}{\lambda} \, d\lambda$$

So if LPF will satisfy first condition of realization aspect of MF (if condition is met)

Noise power at o/p of ideal LPF is $\frac{N_0}{2} \times 2B$

$$= B N_0.$$

So, maxm SNR at o/p of ideal LPF is,

$$SNR_{LPF} = \frac{\left(\frac{2A}{\lambda}\right)^2 si^2 (\pi BT)}{B N_0}$$

Maxm SNR at o/p of MF is,

$$SNR_{MF} = \frac{2A^2 T}{N_0}$$

Ratio of SNR: $\dfrac{SNR_{LPF}}{SNR_{MF}} = \dfrac{2}{\pi^2 BT} si^2(\pi BT)$,

**Impulse response of Matched filter,**

For white gaussian noise, the power spectral density is given as,

$$S_{ni}(f) = \frac{N_0}{2} \qquad \text{(i)}$$

Now, for the IR. of matched filter:

The transfer function of an optimum filter is expressed as,

$$H(f) = \frac{K \cdot X^*(f)}{S_{ni}(f)} e^{-j2\pi fT} \qquad \text{(ii)}$$

Substituting (i) in (ii)

$$H(f) = \frac{K \cdot X^*(f) e^{-j2\pi fT}}{N_0/2}$$

$$= \frac{2K \cdot X^*(f) e^{-j2\pi fT}}{N_0} \qquad \text{(iii)}$$

From the property of Fourier Transform, we know that,

$$X^*(f) = X(-f)$$

using this property we can write eqⁿ (iii) as:

$$H(f) = \frac{2K \cdot X(-f) e^{-j2\pi fT}}{N_0}$$

Now, the IR can be calculated by taking Inverse fourier transfer of above eqⁿ,

$$h(t) = IFT[H(f)]$$

$$= IFT\left[\frac{2K \cdot X(-f) e^{-j2\pi fT}}{N_0}\right] \qquad \text{(iv)}$$

The inverse FT. of $X(-f)$ i $X(-t)$ and $e^{-j2\pi fT}$ represents time shift of T seconds

Hence, we have:

$$FT[x(-t)] = X(-f)$$
$$\& \quad FT[x(T-t)] = X(-f) e^{-j2\pi fT}$$

Using these above properties, the eqn
(v) can be written as,

$$h(t) = \frac{2k}{N_0} x(T-t) \quad \text{—} \quad ⑤$$

if $x(t) = x_1(t) - x_2(t)$ then,

$$h(t) = \frac{2k}{N_0} \left[ x_1(T-t) - x_2(T-t) \right] \quad \text{—} \quad ⑥$$

These two eqns ⑤ & ⑥ give reqd ir of matched filter.

Thus this Also proves that impulse response of MF is time reverse delayed version of the input signal.

Noise performance of band-pass (modulated) communication systems. (8 Hours / 14 Marks)

## Analysis of DSB-FC, DSB-SC & SSB:

• Detecting gain? : 74ch [8], 78 Bh [1]

73ch[8] • Detection gain of DSB-FC, DSB-SC, SSB./compare.
• Prove that for 100% modulation of DSBAM, the detection gain is less than 1. 74ch[8]

72ch[8] • Noise performance of DSBAM, BSB-SC, SSB /compare
• Threshold effect in envelop detector for DSB FC. 72 kar[7]

6M • DG of SSBSC & compare with DSB-SC. 73sh, 71sh
• gain parameter in DSB-FC with envelop detector 70 Asd [5]

\# Detection gain: Detection gain is the ratio of output signal to noise ratio ($SNR_0$) to the (channel)/input signal to noise ratio ($SNR_c/SNR_i$). It is also known as gain parameter of figure of merit and is written as:

Detection gain / figure of merit $(Y) = \dfrac{SNR_0}{SNR_c} = \dfrac{SNR_0}{SNR_r}$

It's value can be less than, greater than or

equal to 1 depending on the type of Modulation.
The figure of merit should be as high as
possible because higher value of figure of merit
indicates better noise performance of the receiver

Noise performance of DSB-AM, SSB-SC & DSB-SC
can be compared according to their gain parameter
individually. The detection method having high
gain $\gamma$ has better noise performance. So lets
find out $\gamma$ parameter for each.

DSB-AM:

The input to the demodulator is the sum of signal
noise.

$$x_i(t) = \{A_c + x_m(t)\} \cos w_c t + n_i(t) \quad —(i)$$

Then signal power at i/p is given by

$$P_{si} = \frac{A_c^2}{2} + \frac{\overline{x_m^2(t)}}{2} \quad —(a)$$

Similarly, noise power at input is
$$P_{Ni} = \overline{n_i^2(t)} \quad —(b)$$

SNRi

Now, using synchronous detection, the received
Am signal is multiplied by $\cos w_c t$ and then
passed through LPF.

$$x_i(t)\Big|_{LPF} = x_i(t) . \cos w_c t$$
$$= \frac{1}{2} x_m(t) + \frac{1}{2} n_c(t)$$

Now, the O/P signal power is

$$P_{so} = \frac{\overline{x_m^2(t)}}{4} \quad —(a)$$

& O/P noise power is,

$$P_{NO} = \frac{\overline{n_c^2(t)}}{4} \quad —(b)$$

So, SNR at O/P is,

$$SNR_O = \frac{P_{so}}{P_{NO}} = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \quad —(ii)$$

Similarly from eqn (a) & (b) we can find
SNR at input,

$$SNR_i = \frac{P_{si}}{P_{Ni}} = \frac{\frac{A_c^2}{2} + \frac{\overline{n_m^2(t)}}{2}}{\overline{n_i^2(t)}}$$

$$SNR_i = \frac{A_c^2 + \overline{n_m^2(t)}}{2\overline{n_i^2(t)}} \quad \text{(iii)}$$

Now, the gain parameter is expressed as

$$Y = \frac{SNR_o}{SNR_i} = \frac{\overline{n_m^2(t)}}{\overline{n_c^2(t)}} \Bigg/ \frac{A_c^2 + \overline{X_m^2(t)}}{2\overline{n_i^2(t)}}$$

$$\left\{ \overline{n_c^2(t)} = \overline{n_i^2(t)} \right\}$$

$$\therefore Y = \frac{\overline{n_m^2(t)}}{\overline{n_c^2(t)}} \times \frac{2\overline{n_c^2(t)}}{A_c^2 + \overline{X_m^2(t)}}$$

$$Y = \frac{2\overline{n_m^2(t)}}{A_c^2 + \overline{n_m^2(t)}} \quad \text{(iv)}$$

$\therefore$ from above eqⁿ, it is clear that as Y increases
Ac decreases.

since for distortionless transmission,

$$A_c \geq |n_m(t)|_{max}$$

$$\& \quad \eta = \frac{\overline{n_m^2(t)}}{A_c^2 + \overline{n_m^2(t)}}$$

$$\therefore \text{system gain, } Y = 2\eta \quad \text{(v)}$$

when modulation index is 100% & modulating
signal is sinusoidal,

$$n = \frac{1}{3}$$

$$\text{or, } Y = 2 \times \frac{1}{3} < 1 \quad \text{(\textasteriskcentered)}$$

so, for 100% modulation, Y is less than 1.

The above was for synchronous detection of DSB-AM.
Now let's study about envelope detection of DSBAM.

The i/p signal can be expressed as

$$n_i(t) = \{A_c + n_m(t)\} \cos(w_c t) + n_c(t) \cos w_c t + n_s(t) \sin w_c t$$

$$= \{A_c + n_m(t) + n_c(t)\} \cos w_c t + n_s(t) \cos w_c t$$

$$n_i(t) = V(t) \cos\{w_c t + \phi(t)\} \quad \text{(vi)}$$

where, $V(t) = \sqrt{(A_c + n_m(t) + n_c(t))^2 + n_s^2(t)}$

$$\phi(t) = -\text{Tan}^{-1} \boxed{\frac{n_s(t)}{A_c + n_m(t) + n_c(t)}}$$

Lets assume that noise is very small

$$[A_c + n_m(t)] \gg n_i(t)$$
or, $[A_c + n_m(t) + n_c(t)]^2 \gg n_s^2(t)$
$$\therefore V(t) = A_c + n_m(t) + n_c(t)$$
$$\& \quad \phi(t) = 0$$

$\therefore$ The output of the ideal envelope detector is

$$V(t) = A_c + n_m(t) + n_c(t)$$

LPF filters out the dc component ie, $A_c$.
so, O/P of LPF is

$$n_m(t) + n_c(t)$$

$\therefore$ O/P signal power $P_{so} = \overline{n_m^2(t)}$

O/P noise power $P_{no} = \overline{n_c^2(t)}$

$$\therefore SNR_o = \frac{\overline{n_m^2(t)}}{\overline{n_c^2(t)}} \quad \text{(vi)}$$

from (vi) & (iii), we can find detection gain,

$$Y = \frac{SNR_o}{SNR_i} = \frac{2\overline{n_m^2(t)}}{A_c^2 + \overline{n_m^2(t)}} \quad \text{(vii)}$$

Again, we know that,

efficiency, $\eta = \frac{\overline{n_m^2(t)}}{A_c^2 + \overline{n_m^2(t)}}$

$$\therefore Y = 2\eta$$

since Max$^m$ efficiency is $\frac{1}{3}$ for 100% in

$\cap$ SBFC AM,

$$\therefore Y_{max} = 2 \times \frac{1}{3} = \frac{2}{3} < 1.$$

$SNR_o < SNR_i \longrightarrow$ degrades the signal.

Detection gain for DSB-SC: The detection method here is always synchronous.

The i/p signal is given by,

$$n_i(t) = n_m(t) \cos w_c t + n_i(t) \quad \text{——(viii)}$$

The signal i/p power is,

$$P_{si} = (n_m(t) \cos w_c t)^2 = \frac{n_m^2(t)}{2} \quad \text{——(a)}$$

The noise power is

$$P_{Ni} = \overline{n_i^2(t)} \quad \text{——(b)}$$

∴ SNR at i/p will be,

$$SNR_i = \frac{n_m^2(t)}{2 \overline{n_i^2(t)}} \quad \text{——(ix)}$$

Similarly at the output of demodulator the useful signal component is $n_m(t)$

$$\frac{n_m(t)}{2}$$

$$\therefore P_{so} = \frac{n_m^2(t)}{4} = \frac{P_{si}}{2} \quad \text{——(x)}$$

similarly, $n_i(t)$ is multiplied by $\cos w_c t$ during process of detection

$$\therefore n_{det}(t) = n_i(t) \cdot \cos w_c t$$

$$= (n_c(t) \cos w_c t + n_s(t) \sin w_c t) \cos w_c t$$

$$= n_c(t) \cos^2 w_c t + n_s(t) \sin w_c t \cos w_c t$$

$$= \frac{1}{2} \{ n_c(t) + n_c(t) \cos 2w_c t + n_s(t) \sin 2w_c t \}$$

All the components except $\frac{n_c(t)}{2}$ is filtered by LPF.

$$\therefore P_{No} = \frac{n_c^2(t)}{4} \quad \text{——(g)}$$

∴ SNR at o/p will be,

$$SNR_o = \frac{4 n_m^2(t)}{4 n_c^2(t)} = \frac{n_m^2(t)}{n_c^2(t)} \quad \text{——(x)}$$

∴ gain parameter, γ is expressed as,

$$Y = SNR_0 = \frac{\eta_m^2(t)}{\overline{\eta_i^2(t)}} \times \frac{2\eta_i^2(t)}{\overline{\eta_m^2(t)}} \qquad \left\{ \eta_i^2(t) = \eta_i(t) \right\}$$

$$\therefore Y = 2 \qquad \qquad (**)$$

The gain is 2, because the i/p signal consists of 2 sidebands & noise is distributed to both side bands.

Detection gain for SSB! It also supports only synchronous detection.

The input signal to the demodulator is

$$\eta_i(t) = \eta_m(t) \cos w_c t + \hat{\eta}_m(t) \cos w_c t + \eta_c(t) \cos w_c t + \eta_s(t) \sin w_c t \qquad (xi)$$

Input signal power is:

$$P_{si} = \frac{\eta_m^2(t)}{2} + \frac{\hat{\eta}_m^2(t)}{2}$$

$$= 2 \frac{\eta_m^2(t)}{2} \qquad \left\{ \therefore \hat{\eta}_m(t) = \hat{\eta}_m(t) \right\}$$

$$P_{si} = \eta_m^2(t) \qquad \qquad (a)$$

Similarly noise power at input is,

$$P_{NI} = \overline{\eta_i^2(t)} \qquad \qquad (b)$$

$$\therefore SNR \text{ at input, } SNR_i = \frac{P_{si}}{P_{Ni}} = \frac{\eta_m^2(t)}{\overline{\eta_i^2(t)}} \qquad (xii)$$

After synchronous demodulation the output will be,

$$\eta_0 = \eta_m(t) \quad \text{or, } \hat{\eta}_m(t) + \eta_c(t) \qquad (xiii)$$

The signal power at o/p is;

$$P_{so} = \overline{\eta_m^2(t)} \qquad \qquad (x)$$

Noise power at output is,

$$P_{NO} = \overline{\eta_c^2(t)} = \overline{\eta_i^2(t)} \qquad (0)$$

$$\therefore SNR_0 = \frac{\eta_m^2(t)}{\overline{\eta_i^2(t)}} \qquad (xiv)$$

$$\therefore \text{Detection gain, } Y = \frac{SNR_0}{SNR_i} = \frac{\eta_m^2(t)}{\overline{\eta_i^2(t)}} \bigg/ \frac{\hat{\eta}_m^2(t)}{\overline{\eta_i^2(t)}}$$

$$\therefore Y = 1 \qquad (***)$$

∴ SSB doesnt provide noise improvement in noise performance.

DSBSC has highest value of Y thus it has better noise performance followed by SSB then by DSB-AM which has the worst noise perfor--mance out of three.

If we compare DSBSC & SSB, Y is better in DSBSC thus noise performance is better but DSBSC has input noise power twice than in case of SSB. Thus DSB-SC & SSB are identical in noise Improvement point of view.
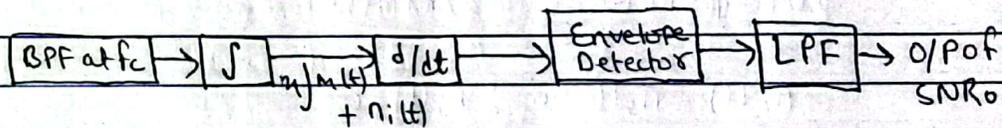
## FM Detector:

- Capture effect : 78Bh[1], 70Asd[2]
- Gain parameter / figure of merit of non coherent FM detector: VVI !73Sh, 71Ch, 74As [6 to 8 M]
- FM threshold effect : 74As[2], 76Ch[4], 69ch[4], 70ch[3]
- Comparision of AM & FM in terms of power efficiency, Bandwidth efficiency & system complexity : 70Asd[3]
- Gain parameter of FM system with limiter discrimination method : 78Bh[7]
- Remedy of threshold effect : 70ch[3], 69Ch[3]

## Capture effect:

If there is interference in the form of other stations within compatible carrier frequencies, the fm station will enhance one of the sections, depending upon the intensities of the signal at the receivers. If the level of interference is low compared to the desired signal then FM captures the desired signal & remains locked. But if the level of interference becomes greater than desired signal then the FM captures the interference. This effect of capturing the strong station is called the capture effect.

## Gain Parameter of FM detector:

Consider the standard Limiter discriminator FM demodulation:

$$BPF\ at\ f_c \rightarrow \int \xrightarrow[+\ n_i(t)]{_{\pi/M(t)}} \boxed{d/dt} \rightarrow \boxed{\text{Envelope Detector}} \rightarrow \boxed{LPF} \rightarrow O/P\ of\ SNR_O$$

The input signal of Demodulator is;

$n_i(t) = n_{FM}(t) + n_i(t)$

$n_i(t) = A_c \cos\left[w_c t + 2\pi k_p \int_0^t n_m(t)\, dt\right] + n_i(t)$ ——①

so, input signal to noise power is,

$P_{si} = \dfrac{A_c^2}{2}$

Similarly $P_{ni} = n_i^2(t) = 2B \times N_0$ ——①①

where, $B = BW$ of message signal
$N_0 = PSDF$ of white noise

$N_c(f) = N_s(f) = N_0$

$\therefore SNR_i = \dfrac{P_{si}}{P_{ni}} = \dfrac{A_c^2}{2} \Bigg/ 2B \times N_0 = \dfrac{A_c^2}{4BN_0}$ ——①①①

To find output SNR, let's first consider the case when i/p noise is absent. i.e,

$n_c(t) = n_s(t) = 0 \; ; \; n_i(t) = 0$

Then, eqⁿ ① will be

$n_i(t) = A_c \cos(w_c t + \phi(t))$ ——①v

where, $\phi(t) = \left[2\pi k_f \int_0^t n_m(t)\, dt\right]$

output of discriminator is :

$\dfrac{d n_i(t)}{dt} = A_c\left(w_c + d\,\phi(t)\right) \sin(w_c t + \phi(t))$

Envelope detector gives envelope of above signal at output and LPF removes dc components centered at $w_c$.

so,

O/P of LPF is $A_c \cdot \dfrac{d\phi(t)}{dt} = A_c\, 2\pi\, k_f\, n_m(t)$

$\therefore P_{so} = 4\pi^2 A_c^2\, k_f^2\, n_m^2(t)$ ——v

Now, assuming message signal is absent in input then $n_m(t) = 0$. So input is only unmodulated carrier and noise then,

$n_i(t) = A_c \cos w_c t + n_i(t)$
$= \{A_c + n_c(t)\} \cos w_c t + n_s(t) \sin w_c t$

Let, $R_n(t) = \sqrt{\{A_c + n_c(t)\}^2 + (n_s(t))^2}$

$$\theta(t) = -\tan^{-1}\left[\frac{n_s(t)}{A_c + n_c(t)}\right]$$

$$\therefore n_i(t) = R_n(t)\cos(w_c t + \theta(t)) \quad\text{—①}$$

we can assume that $R_n(t)$ will be removed by limiter discriminator.

Assume $A_c \gg n_c(t)$

then, $\theta(t) = -\tan^{-1}\left[\frac{n_s(t)}{A_c}\right] \simeq \frac{n_s(t)}{A_c}$

so, input to the discriminator is

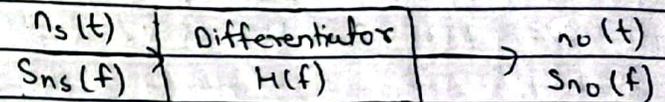$$\therefore n_{dis}(t) = \cos\left(w_c t + \frac{n_s(t)}{A_c}\right) \quad\text{—②}$$

& o/p to the discriminator is

$$\frac{d\, n_{dis}(t)}{dt} = \left\{w_c + \frac{1}{A_c}\frac{d n_s(t)}{dt}\right\}\sin\left(w_c t + \frac{n_s(t)}{A_c}\right)$$

The signal is again passed through envelope detector & LPF so o/p of LPF is :

$$n_o(t) = \frac{1}{A_c}\frac{d n_s(t)}{dt}$$

But there is no mathematical representation of $n_s(t)$ so we use spectrum method i.e psdf approach.

$$\boxed{\begin{array}{c} n_s(t) \\ S_{ns}(f) \end{array} \longrightarrow \begin{array}{|c|} \hline \text{Differentiator} \\ H(f) \\ \hline \end{array} \longrightarrow \begin{array}{c} n_o(t) \\ S_{no}(f) \end{array}}$$

The Psdf of the signal at the o/p of the differentiator is

$$S_{no}(f) = S_{ns}(f)\,|H(f)|^2$$

Transfer function of differentiator

$$H(f) = j\cdot 2\pi f$$

so, $S_{no}(f) = N_0\,|j 2\pi f|^2$

$$= 4\pi^2 N_0 f^2$$

This shows that the noise psd at the o/p of the discriminator is proportional to $f^2$

$\therefore$ noise power within message Bw is;

$$P_{NO} = \frac{1}{A_c^2} \int_{-B}^{B} S_{no}(f)\,df = \frac{1}{A_c^2} \times 4\pi^2 N_0 \int_{-B}^{B} f^2\,df$$

$$= \frac{8\pi^2 N_0 B^3}{3 A_c^2}$$

So, $SNR_0 = \dfrac{P_{so}}{P_{NO}} = \dfrac{4\pi^2 A_c^2 k_f^2 \overline{u_m^2(t)}}{\dfrac{8\pi^2 N_0 B^3}{3 A_c^2}}$

$$= 4\pi^2 A_c^2 k_f^2 \overline{u_m^2(t)} \times \frac{3 A_c^2}{8\pi^2 N_0 B^3}$$

$$= \frac{3 A_c^4 k_f^2 \overline{u_m^2(t)}}{2 N_0 B^3}$$

∴ Detection gain $(Y) = \dfrac{SNR_0}{SNR_i}$

$$= \frac{3 A_c^4 k_f^2 \overline{u_m^2(t)}}{2 N_0 B^3} \Bigg/ \frac{A_c^2}{4 B N_0}$$

$$= \frac{3 A_c^4 k_f^2 \overline{u_m^2(t)}}{2 N_0 B^3} \times \frac{4 B N_0}{A_c^2}$$

$$\boxed{Y = \frac{6 A_c^2 k_f^2 \overline{u_m^2(t)}}{B^2}} \qquad /\!/$$

## FM threshold effect & its Remedy:

The detection gain $(Y)$ in FM is proportional to $\beta^2$. So for given SNR, the rise in $\beta$ will increase $Y$ or $SNR_0$. But with increase in $\beta$ will considerably increase the system BW at the Carson's rule rate;

$$B_{FM} = 2(\beta+1) fm \qquad —①$$

Increase in BW will subsequently increase i/p noise power and decrease $SNR_i$.

As assumed earlier the carrier amplitude $A_c$ is greater than noise. So this assumption will no longer remain valid. If we increase $\beta$ indiscriminately i.e. at some point of increase of $\beta$, $SNR_i$ will be so low that the signal as in case of large noise AM signal will be mutilated by the noise causing reception impossible.

In other words, there exists threshold effect at less than the threshold level because of which fm system will no more detect message signal.

The threshold level for fm is 10 dB.

Before reaching the threshold level the FM system

Produces noise clicks. As SNRi is further decreased then individual clicks are produced which are converted to crackling sound, with further reduced in SNRi & finally threshold level is reached the receiver breaksie, received signal is completely mutilated by the noise.

FM system owing threshold effect also exhibit locking property. If the SNRi is below the threshold level the receiver locks to the noise.

Remedy:

(i) Using FMFB demodulator: This demodulator has negative feedback & reduces noise threshold. Reduces THL by 5-7dB

(ii) Using PLL Demodulator: It reduces threshold level by 2 to 3 dB.

(iii) Use of Preemphasis & De-emphasis network: It improves system performance by 6dB.

Comparision of AM & FM: There are 3 criteria for comparision:

(i) Bandwidth efficiency: SSBSC has best performance because in this case channel bandwidth is equal to message

bandwidth. Therefore SSB-SB is widely used in applications where the BW is major constraint. such as microwave link, satellite communication, point-to-point communication.

The application which requires near to DC transmission then VSB-SC is used such as TB broadcasting.

(ii) Power efficiency: DSB-AM is least efficient & FM has the highest efficiency due to its high level of noise immunity.

Therefore, in power critical applications such as space vehicle communications, FM broadcasting. FM is preferred over other modulation techniques.

(iii) System complexity: The system complexity plays vital role when number of receiver is tremendous. Since DSB-AM is least complicated it is used for commercial radio broadcasting. In terms of system complexity, SSBSC is slightly more complex than DSB-SC but since DSB-SC is less effective in terms of Bandwidth it is not used mostly in practice.

Error Probability!

Q) Derive the expression for error probability in binary communication. 70ch[7]

Q) Derive the exp^n for error probability of binary PAM: 72kar[5] and extend it to M-ary: 69ch[6+2]

Q) Error Probability in coherent ASK! 72ch[4], 70Asd[4], 69ch[6], 74As[6].

Q) Error probability in binary Ask & extend it to M-Ary: #71ch[6], 73sh, 69ch[6+2],

A) Error probability of coherent PSK 72h[4], 76ch[6]

Q) show Ask requires double the average signal power than PSK for same error probability 72lh[3]

Q) Error probability for Many System: 78Bh[6] 71sh[5]

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# Error Probability for binary PAM system & extending it to M-ary system!

Performance of digital communication system is evaluated in terms of error probability. The error probability depends on input SNR (SNRi) and complexity of the decoder.

Let us consider a base-band data communication, where no carrier modulation is employed. The most frequently used is the PAM. So we start with analysis of PAM.
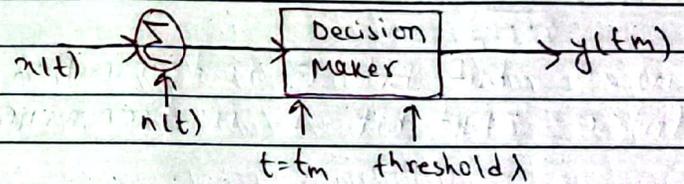


The signal output at the decision maker is,

$$y(t_m) = A_m + N_o(t_m) \qquad (1)$$

Where, $A_m = +A$ if $B_m = 1$ (Bit 1 is transmitted)
$\qquad = -A$ if $B_m = 0$ (Bit 0 is transmitted)

In absence of noise the decision making device

compares $y(t)$ against the threshold level $\lambda$ (which is $0v$)

* If $y(t_m) > 0$ then it is decided that bit 1 is transmitted.
* If $y(t_m) < 0$ then decision is favour of 0 bit.

The presence of noise & its strength may introduce errors in the decision making. The error is introduced when the decision is in the favour of 1 & actually 0 is transmitted & vice-versa.

It means, error is introduced when

$\quad y(t_m) > 0 \quad$ but bit 0 is transmitted
$\quad y(t_m) < 0 \quad$ but bit 1 is transmitted.

Now let us assume noise is present in system. Consider bit sequence 0 & 1 are equiprobable and statistically independent. Thus total probability will be sum of each error probability.

Here, $P(b_m = 0) = P(b_m = 1) = \dfrac{1}{2}$

and, $P[y(t_m) > 0 / b_m = 0]$ is the probability of $y(t_m) > 0$ when binary 0 is transmitted.

similarly, $P[y(t_m) < 0 / b_m = 1]$ is the probability of $y(t_m) < 0$ when binary 1 is transmitted.

Thus, Total probability is;

$P_e = P[y(t_m) > 0 \wedge b_m = 0] + P[y(t_m) < 0 \wedge b_m = 1]$

$\quad = P[y(t_m) > 0 / b_m = 0] * P(b_m = 0) + P[y(t_m) < 0 / b_m = 1]$

$\quad\quad * P(b_m = 1)$

$P_e = \dfrac{1}{2} \left\{ P[y(t_m) > 0 / b_m = 0] + P[y(t_m) < 0 / b_m = 1] \right\}$

In terms of noise, the above expression can be written as,

$P_e = \dfrac{1}{2} \left\{ P[n_0(t_m) > A] + P[n_0(t_m) < A] \right\}$

$P_e = \dfrac{1}{2} \left\{ P[ |n_0(t_m)| > A ] \right\}$

As input noise $n(t)$ is zero

Mean Gaussian process with variance No. So, output noise $n_o(t)$ is also Gaussian process with variance No.

Therefore,

$$P_e = \frac{1}{2} \int\limits_{A}^{\alpha} \frac{1}{2\pi N_o} \exp\left(\frac{-x^2}{2N_o}\right) dx$$

let, $z^2 = \dfrac{x^2}{2\sqrt{N_o}}$

$$x = z\sqrt{2N_o}$$

$$\therefore P_e = 2 \times \frac{1}{2} \int\limits_{A/\sqrt{2N_o}}^{\alpha} \frac{1}{\sqrt{\pi}} \exp\left(-z^2\right) dz$$

$$\longrightarrow \frac{1}{2} \int\limits_{A/\sqrt{2N_o}}^{\alpha} \frac{2}{\sqrt{\pi}} \exp\left(-z^2\right) dz$$

$$= \frac{1}{2} \times erfc\left(\frac{A}{\sqrt{2N_o}}\right)$$

$$\therefore P_e = \frac{1}{2} \times erfc(u)$$

Where, $erfc(u) = \dfrac{2}{\sqrt{\pi}} \int\limits_{u}^{\alpha} \exp(-z^2) dz$.

In many,

$$P_e = \frac{M-1}{M} erfc\left(\frac{A}{\sqrt{2N_o}}\right)$$

Remaining in copy - II

Error probability in coherent detection of ASK:

In binary ASK, the i/p signals can be represented as

$S_1(t) = 0$ ——(i)

$S_2(t) = A \cos \omega_c t$ ——(ii)

subtracting (i) from (ii)

$S_2(t) - S_1(t) = A \cos \omega_c t$ ——(iii)

The output signal for each case is

$$S_{o1}(KT_b) = \int_0^{T_b} S_1(t)[S_2(t) - S_1(t)] \, dt = 0 \quad \because S_1(t) = 0$$

$$S_{o2}(KT_b) = \int_0^{T_b} S_2(t)[S_2(t) - S_1(t)] \, dt$$

$$= \int_0^{T_b} (A \cos \omega_c t) \cdot (A \cos \omega_c t) \, dt$$

$$= \int_0^{T_b} A^2 \cos^2 \omega_c t \, dt$$

$$\therefore S_{o2}(KT_b) = \frac{A^2}{2} T_b \quad\quad ⓐ$$

The max value of SNR, $Y$ will be,

$$Y_{max}^2 = \int_{-\alpha}^{\alpha} \frac{[S_2(t) - S_1(t)]^2}{N_0/2} \, dt$$

$$= \frac{2}{N_0} \int_0^{T_b} A^2 \cos^2 \omega_c t \, dt$$

$$= \frac{2}{N_0} \frac{A^2 T_b}{2}$$

$$Y_{max}^2 = \frac{A^2 T_b}{N_0} \quad\quad ⓑ$$

$\therefore$ The error probability will be

$$P_{emin} = erfc\left(\frac{Y_{max}}{2}\right)$$

or, $\boxed{P_e = erfc\left(\sqrt{\frac{A^2 T_b}{4 N_0}}\right)}$ ——(iv)

Since $S1$ & $S2$ are equiprobable the average signal power will be

$$S_{av} = \frac{A^2}{4} \qquad \text{(v)}$$

So, in terms of average signal power,

$$P_e = erfc\left(\sqrt{\frac{S_{av}T_b}{N_0}}\right) = erfc\sqrt{\frac{E_{av}}{N_0}} \qquad \text{(vi)}$$

and the average signal energy per bit Information is,

$$E_{av} = S_{av} \cdot T_b \qquad \text{(vii)}$$

Coherent detection of PSK:

The demodulation used in PSK is always coherent.

The input signal can be expressed as:

$$S_1(t) = -A \cos w_c t \qquad \text{(viii)}$$
$$S_2(t) = A \cos w_c t \qquad \text{(ix)}$$

and their respective output signal will be,

$$S_{01}(KT_b) = -A^2 T_b \qquad \text{(x)}$$

$$S_{02}(KT_b) = A^2 T_b \qquad \text{(xi)}$$

The maximum SNR will be

$$Y^2_{max} = \int_{-\alpha}^{\alpha} \frac{[S_2(t) - S_1(t)]^2}{N_0/2} dt$$

$$= \int \frac{(2A \cos w_c t)^2}{N_0/2} dt$$

$$= \frac{4}{N_0} \int 2A^2 \cos^2 w_c t \, dt$$

$$Y^2_{max} = \frac{4}{N_0} A^2 T_b \qquad \text{(a)}$$

$$\text{or, } Y/y = 2A\sqrt{\frac{T_b}{N_0}} \qquad \text{(b)}$$

and, average signal power is, $S_{av} = \frac{A^2}{2} \qquad \text{(c)}$

Now, error probability in terms of average energy per bit is;

$$P_{emin} = erfc\left(\frac{Y_{max}}{2}\right)$$

$$= erfc \frac{2A}{2} \sqrt{\frac{T_b}{N_0}}$$

$$P_e = erfc \sqrt{\frac{A^2 T_b}{N_0}} \quad \text{—(xii)}$$

Since $S_{av} = \frac{A^2}{2}$

$$P_e = erfc \sqrt{\frac{2 S_{av} T_b}{N_0}} \quad \text{—(xiii)}$$

Similarly, $E_{av} = S_{av} \cdot T_b$.

$$P_e = erfc \sqrt{\frac{2 E_{av}}{N_0}} \quad \text{—(xiv)}$$

Now, comparing (vi) & (xiv); we can
see that _____ ASK
requires double the average signal
power than PSK for same error
probability.

Error probability of M-ary system:

Let us consider the case of M=4.



Threshold level
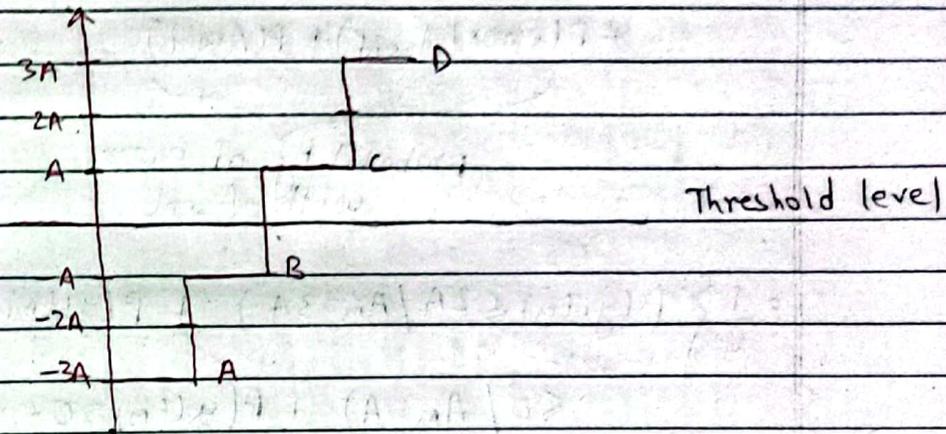
Decoding algorithm is,

$y(t_m) \leq -2A \longrightarrow$ Symbol 'A'
$-2A < y(t_m) \leq 0 \longrightarrow$ Symbol 'B'
$0 < y(t_m) \leq 2A \longrightarrow$ Symbol 'C'
$y(t_m) > 2A \longrightarrow$ Symbol 'D'

Here, symbol A, B, C and D are equiprobable

So, $P(A \text{ sent}) = P(B \text{ sent}) = P(C \text{ sent}) = P(D \text{ sent}) = \frac{1}{4}$

Now, Probability of error is

$P_e = P(error/D \, sent) * P(D \, sent) + P(error/C \, sent)$

$* P(C \, sent) + P(error/B \, sent) * P(B \, sent)$

$+ P(error/A \, sent) * P(A \, sent)$

↳ probability of error
with A sent.

$= \frac{1}{4} \Big\{ P(y(tm) \leq 2A / A_m = 3A) + P(y(tm) > 2A \, or$

$\leq 0 / A_m = A) + P(y(tm) > 0 \, or < -2A/$

$A_m = -A + P(y(tm) > -2A / A_m = -3A) \Big\}$

Since we know that,

$y(tm) = A_m + n_0(tm)$ so the

expression can be written in terms
of noise as,

$P_0 = \frac{1}{4} \Big\{ P(n_0(tm) < -A) + P(n_0(tm) > A \, or$

$P(n_0(tm) < -A) + P(n_0(tm) > A \, or \, n_0(tm) < -A +$

$n_0(tm) < -A) + P(n_0(tm) > A \, or \, n_0(tm) < -A +$

$P(n_0(tm) > A) \Big\}$

$= \frac{1}{4} \Big\{ P(|n_0(tm)| > A) + P(|n_0(tm)| > A)$

$+ P(|n_0(tm)| > A) \Big\}$

↳ from 1st to 4th term,

$= \frac{3}{4} \Big\{ P(|n_0(tm)| > A) \Big\} \quad \text{——①}$

Assuming $n_0(t)$ follow gaussian distribution

$P_e = \frac{3}{4} \times 2 \int_{\pi_A}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \left( \frac{-x^2}{2N_0} \right) dx$

$= \frac{3}{4} \times 2 \int_{A/\sqrt{2N_0}}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-\tau^2) d\tau$

$= \frac{3}{4} \, erfc \left( \frac{A}{\sqrt{2N_0}} \right)$

In general; $P_e = \frac{(m-1)}{m} erfc \left( \frac{A}{\sqrt{2N_0}} \right) \quad \text{——(ii)}$

Error control coding techniques (4H/7Marks)

- ✓ Define Hamming distance & Hamming code 7uch[2],
- 7uch[6] ✓ Explain operation of a ⅓ convolution encoder.
- • ✓ Define Hamming weight & Hamming distance with examples. 73sh[3]
- • ✓ Explain convolution coder with suitable example 72kar[6].
- • Define Hamming weight & hamming distance 71ch[2], 7oAsd[2], 7uAs[2], 72ch[2]
- • ✓ Importance of hamming distance and hamming weight in coding theory. 71sh[2]
- • ✓ Explain with example the syndrome decoding method in linear block coding. 71sh[5]
- • ✓ What is binary cyclic code. 7oAsd[1]
- Sn: • ✓ syndrome calculation in linear systematic block code: 6xh[4]
- • Why convolution coder are better suited than block coder. 73ch[2]

Hamming code: Hamming code is a set of error-correction codes that can be used to detect and correct the errors that can occur when the data is moved or stored from the sender to the receiver.

Hamming distance: Let us consider two code words having the same number of elements. The Hamming distance or simply distance between the two code words is defined as the number of locations in which their respective elements differ.
Eg: let us consider two code words given below

code word 1:  1  1  0  1  0  1  0  0
                  ↑        ↑  ↑       Hamming dist = 3
codeword 2:   0  1  0  1  1  1  1  0

The key significance of Hamming distance is that if two codewords have a hamming distance of d between them, then it would take d single bit errors to turn one of them into the other.

**Hamming weight:** Hamming weight of a code word $x$ is defined as the number of non-zero elements in the code word. Hamming weight of a code word is the distance between that code word and an all-zero code vector (A code having all elements equal to zero).

Eg: let $x = 11010100$.

As number of non zero elements in above code word is 4, the Hamming weight $W(x) = 4$.

It is important as it gives numeric values. It is mostly used in determining path lengths between nodes & error detection.

**Syndrome decoding for Block Codes:**

For the detector to detect or correct errors by sorting bit pattern of all possible valid code words more memory is required for large block lengths. To avoid this problem syndrome decoding is used. For this purpose, parity check matrix H is used.

For an $(n,k)$ linear block code, this parity check matrix H is of size $(n-k) \times n$.

H is given as;

$$H = [P^T : I_{n-k}]_{(n-k) \times n}$$

where,
$P^T \rightarrow$ transpose of $P$ matrix
$I_{n-k} \rightarrow$ identity matrix

Similarly, Transpose of H is given by

$$H^T = \begin{bmatrix} P \\ \cdots \\ I_{n-k} \end{bmatrix}_{n \times (n-k)} \qquad \text{—①}$$

where, $P \rightarrow$ coefficient matrix.

The transpose of parity check matrix $H^T$ exhibits a very important property i.e,

$$X H^T = (0, 0, \cdots 0) \qquad \text{—②}$$

This means that the product of any code word X and the transpose of the parity

check matrix will always be 0.

We shall use this property for the detection of errors in the received code words as under:

At the receiver, we have

If, $YH^T = (0, 0, \ldots, 0)$, then $Y = X$ ie no error

But if, $YH^T \neq (0, 0, \ldots, 0)$ then $Y \neq X$

     ie, error exists in the received code word.

The syndrome is defined as the non-zero output of the product $YH^T$. Thus, the non-zero syndrome represents the presence of errors in the received code word. The syndrome is represented by $S$ & is mathematically given as,

$$S = YH^T \qquad —(ii)$$

All zero elements of syndrome represent there is no error, & a non-zero value

of an element in syndrome represents the presence of error. But sometimes, even if all the syndrome elements have zero value, the error exists. This is shown in figure below:

$$S = 0$$

either, $Y = X$, ie there is no error in the received signal

or, $Y \neq X$, but $Y$ is some other valid codeword (other than $X$)
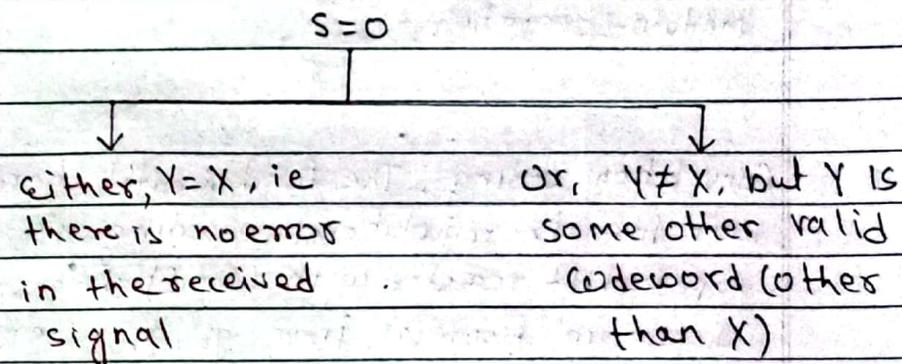
Binary cyclic code: It is the special case of linear block codes in which encoding and syndrome calculations can be easily implemented using simple shift registers.

A $(n, k)$ linear code $C$ is called cyclic code if cyclic shifts of the code are also code vectors of $C$. Thus in polynomial form if $C_1(x)$ is the code vector given by,

$$C_1(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_{n-1} x^{n-1} \qquad —(i)$$

Then, the shifted vector,

$$C_2(x) = C_{n-1} + C_0 x + C_1 x^2 + C_2 x^3 + \ldots + C_{n-2} x^{n-1} \qquad (ii)$$

is also called the code vector.
Here all addition & multiplication are modulo-2 operation.

**Convolution coding:** The fundamental hardware unit for the convolutional encoding is a taped shift register with $(L+1)$ stages, as shown in figure(a). Here, $g_0, g_1, \ldots$ etc are the tap gains which are nothing but binary digits 0s or 1s. A tap gain of 0 represents an open circuit whereas a tap gain of 1 represents a short circuit.
The message bits enter one by one into the tapped shift register, which are then combined by mod-2 addition to form the encoded bit $x$.
Therefore, we have

$$x = m_L g_L \oplus \ldots \ldots \oplus m_1 g_1 \oplus m_0 g_0 \qquad (i)$$

$$\text{or,} \quad x = \sum_{i=0}^{L} m_i g_i \quad (\text{mod-2 addition})$$

The name convolutional encoding comes from the fact that eqn (i) has a form of binary convolution which is analogous to the convolutional integral. The message bit $m_0$ in figure (a) represents the current input whereas the bits $m_1$ to $m_L$ represent the past input or state of the shift register. From eqn (i) it is clear that a message bit $x$ depends on the current message bit $m_0$ and the state of the shift register defined by the previous $L$ message bits.
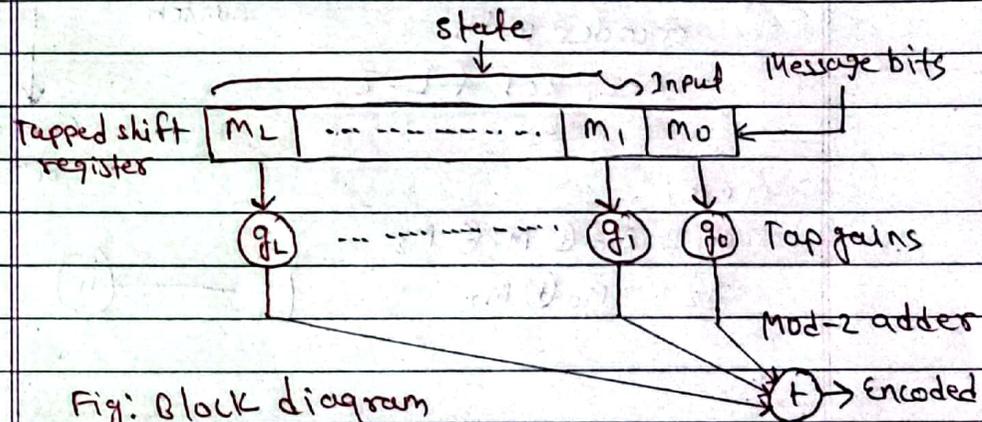


Fig: Block diagram of a general convolution encoder

An example of practical convolution encoder with $n=2, K=1$ & $L=2$ is shown in figure below:

state

Input

message bits
input rate
$r_b$

| $m_2$ | $m_1$ | $m_0$ |
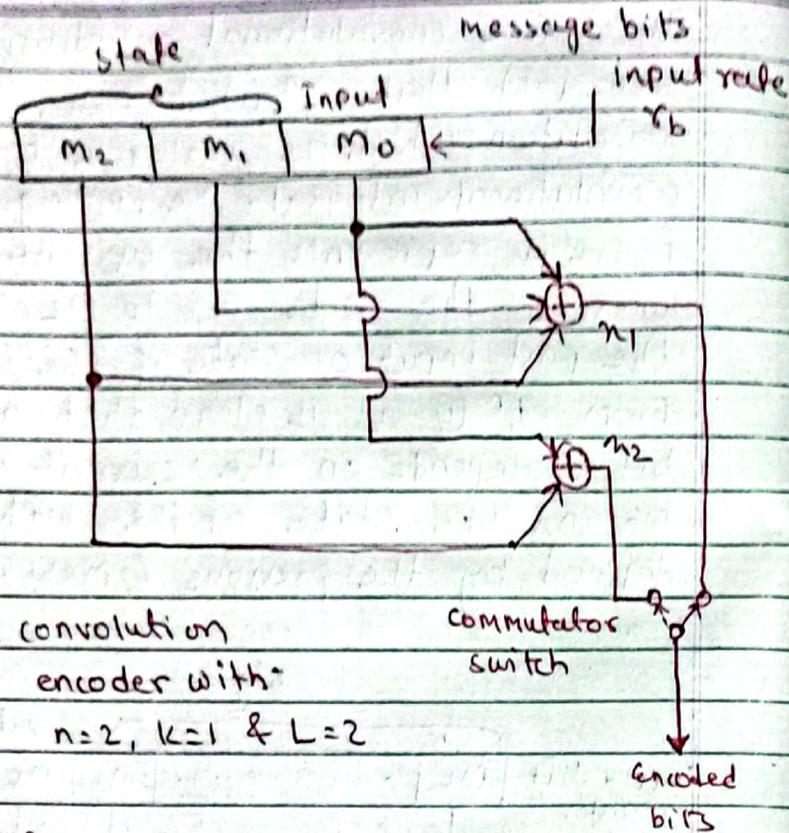
Fig: convolution
encoder with
$n=2, k=1 \& L=2$

commutator
switch

Encoded
bits

Here,
$n_1 = m_0 \oplus m_1 \oplus m_2$
$n_2 = m_0 \oplus m_2$ } —— (ii)

ie position of parity or check bits
is not defined

(ii) They are suitable for detecting &
preventing burst errors.

(iii) The hardware component is simpler
and encoding process is easy.

Q) why convolutional coder are better suited
than Block coder.
= Because of following reasons!

(i) They are preferred in non-systematic
form which do not have efficient structure

# Chapter - 1
## Numericals:

78 Bhadra! Q 1!

given:

Source, $S = \{A, B, C, D\}$

$P(A) = \dfrac{1}{2} = 0.5$  |  $P(C) = \dfrac{1}{8} = 0.125$

$P(B) = \dfrac{1}{4} = 0.25$  |  $P(D) = \dfrac{1}{8} = 0.125$

| Encode using : BCD | Evaluate |
|---|---|
| Shannon - Fano | coding efficiency. |
| Huffman | |

1) Using BCD coding: BCD is fixed length coding method that usually uses 4 to 8 bits depending on requirement.

| Symbol $n_i$ | Probabilities $P(n_i)$ | BCD code code | $n_i$ fixed length |
|---|---|---|---|
| A | 0.5 | 1100 | 4 |
| B | 0.25 | 1101 | 4 |
| C | 0.125 | 1110 | 4 |
| D | 0.125 | 1111 | 4 |

Entropy $(H_n) = 0.5 \log_2\left(\dfrac{1}{0.5}\right) + 0.25 \log_2\left(\dfrac{1}{0.25}\right) + 2 \times 0.125 \log_2\left(\dfrac{1}{0.125}\right)$

$H(x) = 0.5 + 0.5 + 2 \times 0.375$

$= 1.75 \text{ bits/symbol}$

$L_{BCD} = 4 \times (1) = 4 \text{ bits/symbol}$

$\eta = \dfrac{H(x)}{L_{BCD}} \times 100\% = 43.75\%$

2) Shannon Fano coding Huffman:

$P(CD) = 0.125 + 0.125 = 0.25$
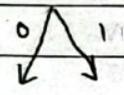


P(C)   P(D)

$P(BCD) = 0.25 + 0.25 = 0.5$



P(B)   P(CD)

$P(ABCD) = 0.5 + 0.5 = 1$



P(A)   P(BCD)
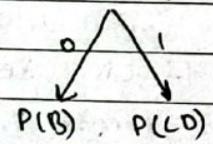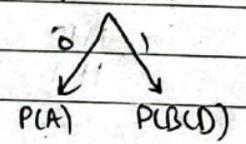
Now, making Huffman tree!

$P(ABCD)$



P(A)   P(BCD)



P(B)   P(CD)



P(C)   P(D)
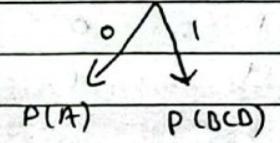
$P(A) = 0$
$P(B) = 10$
$P(C) = 110$
$P(D) = 111$
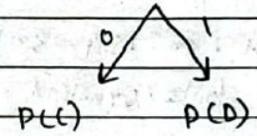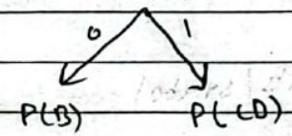
$H(x) = 1.75 \text{ bits/symbol}$

$L_H = 1 \times 0.5 + 2 \times 0.25 + 3 \times 0.125 + 3 \times 0.125$

$= 0.5 + 0.5 + 0.375 + 0.375$

$= 1.75 \text{ bits/symbol}.$

Now, $\eta_H = \dfrac{1.75}{1.75} \times 100\% = 100\%.$

3) Shannon Fano!

| $n_i$ | $P(x_i)$ | step 1 | step 2 | step 3 | code |
|---|---|---|---|---|---|
| A | 0.5 | 0 | | | 0 |
| B | 0.25 | 1 | 0 | | 10 |
| C | 0.125 | 1 | 1 | 0 | 110 |
| D | 0.125 | 1 | 1 | 1 | 111 |

$H(x) = 1.75$ bits/symbol)

$L_{SF} = 1 \times 0.5 + 2 \times 0.25 + 3 \times 0.125 + 3 \times 0.125$
$= 1.75$ bits/symbol).

efficiency, $\eta = \dfrac{1.75}{1.75} \times 100\% = 100\%.$

∴ Variable length coding like Shannon-fano & Huffman coding have greater efficiency then compared to Fixed length coding like BCD coding.

**76th Q no. 1**

$P = \{0.36, 0.18, 0.18, 0.12, 0.09, 0.07\}$

Calculate coding efficiency using shannon fano & huffman coding.

1) Shannon fano.

| $n_i$ | $P_{xi}$ | step 1 | step 2 | step 3 | step 4 | code |
|---|---|---|---|---|---|---|
| $n_1$ | 0.36 | 0 | 0 | | | 00 |
| $n_2$ | 0.18 | 0 | 1 | | | 01 |
| $n_3$ | 0.18 | 1 | 0 | | | 10 |
| $n_4$ | 0.12 | 1 | 1 | 0 | | 110 |
| $n_5$ | 0.09 | 1 | 1 | 1 | 0 | 1110 |
| $n_6$ | 0.07 | 1 | 1 | 1 | 1 | 111 |

$H(x) = 0.36 \log_2\left(\dfrac{1}{0.36}\right) + 0.18 \log_2\left(\dfrac{1}{0.18}\right) + 0.12 \log_2\left(\dfrac{1}{0.12}\right)$
$+ 0.09 \log_2\left(\dfrac{1}{0.09}\right) + 0.07 \log_2\left(\dfrac{1}{0.07}\right)$

$= 0.530 + 0.445 + 0.445 + 0.307 + 0.313 + 0.268$
$= 2.368$ bits/symbol.

$L_{SF} = 2 \times 0.36 + 2 \times 0.18 + 2 \times 0.18 + 3 \times 0.12 + 4 \times 0.09 +$
$4 \times 0.07 = 0.72 + 0.36 + 0.36 + 0.36 + 0.36 + 0.28$
$= 2.44$ bits/symbol

$$\eta_{SE} = \frac{2.368}{2.44} \times 100\% = 0.97 \times 100\% = 97\%$$

## Huffman coding:

$$P(x_5 x_6) = 0.09 + 0.07 = 0.16$$
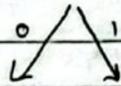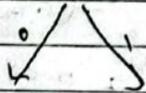
$P(x_5)$  $P(x_6)$
0.09    0.07

$$P(x_4 x_5 x_6) = 0.16 + 0.12 = 0.28$$
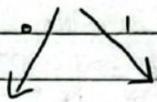
$P(x_5 x_6)$   $P(x_4)$
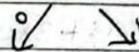0.16        0.12

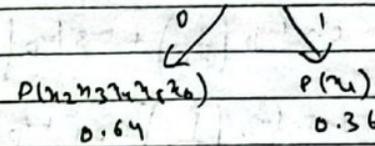$$P(x_2 x_3) = 0.18 + 0.18 = 0.36$$

$P(x_2)$   $P(x_3)$
0.18     0.18

$$P(x_2 x_3 x_4 x_5 x_6) = 0.36 + 0.28 = 0.64$$
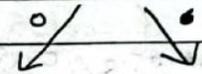
$P(x_2 x_3)$   $P(x_4 x_5 x_6)$
0.36         0.28
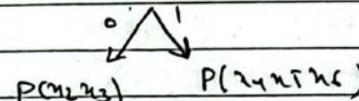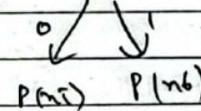
---

$$P(x_1 x_2 x_3 x_4 x_5 x_6) = 0.64 + 0.36 = 1$$

$P(x_2 x_3 x_4 x_5 x_6)$    $P(x_1)$
0.64                    0.36

### creating Huffman tree:

$P(x_1 x_2 x_3 x_4 x_5 x_6)$

$P(x_2 x_3 x_4 x_5 x_6)$   $P(x_1)$

$P(x_2 x_3)$   $P(x_4 x_5 x_6)$

$P(x_2)$   $P(x_3)$   $P(x_5 x_6)$   $P(x_4)$

$P(x_5)$   $P(x_6)$

$P(x_1) = 1$
$P(x_2) = 000$
$P(x_3) = 001$
$P(x_4) = 011$
$P(x_5) = 0100$
$P(x_6) = 0101$

$$L_H = 1 \times 0.36 + 3 \times 0.18 + 3 \times 0.18 + 3 \times 0.12 + 4 \times 0.09$$
$$+ 4 \times 0.07$$

$$= 0.36 + 0.54 + 0.54 + 0.36 + 0.36 + 0.28$$
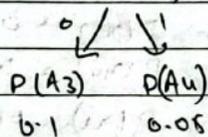
$$= 2.44 \text{ bits / symbol}$$

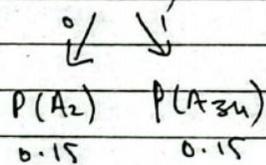$$\therefore \eta_H = \frac{2.368}{2.44} \times 100\% = 97\%$$

| | |
|---|---|
| AO | 0.4 |
| A1 | 0.3 |
| A2 | 0.15 |
| A3 | 0.1 |
| Au | 0.05 |

$$H(x) = 0.4 \log_2\left(\frac{1}{0.4}\right) + 0.3 \log_2\left(\frac{1}{0.3}\right) +$$
$$0.15 \log_2\left(\frac{1}{0.15}\right) + 0.1 \log_2\left(\frac{1}{0.1}\right)$$
$$+ 0.05 \log_2\left(\frac{1}{0.05}\right) \quad = 0.529 + 0.521$$
$$+ 0.41 + 0.332$$
$$+ 0.216 = 2.006$$
bits/symbol

**1) Huffman**

$P(A3Au) = 0.1 + 0.05 = 0.15$

0 / 1
P(A3)    P(Au)
0.1       0.05

$P(A234) = 0.15 + 0.15 = 0.3$

0 / 1
P(A2)    P(A34)
0.15      0.15

$P(A1234) = 0.3 + 0.3 = 0.6$

0 / 1
P(A1)    P(A234)
0.3       0.3

---

$P(A01234) = 0.6 + 0.4 = 1$

0 / 1
P(A1234)    P(Ao)
0.6          0.4

**Huffman tree:**

P(A01234)

0 / 1
P(A1234)    P(Ao)

0 / 1
P(A1)    P(A234)

0 / 1
P(A2)    P(A34)

0 / 1
P(A3)    P(Au)

P(Ao)   Ao = 1
P(    a1 = 00
A2 = 010
A3 = 0110
Au = 0111

$$L_H = 1 \times 0.4 + 2 \times 0.3 + 3 \times 0.15 + 4 \times 0.1 + 4 \times 0.05$$
$$\sim 0.4 + 0.6 + 0.45 + 0.4 + 0.2 = 2.05$$

## 2) Shannon fano

| $A_i$ | $P(A_i)$ | step1 | step2 | step3 | step4 | code |
|-------|----------|-------|-------|-------|-------|------|
| A0 | 0.4 | 0 | | | | 0 |
| A1 | 0.3 | 1 | 0 | | | 10 |
| A2 | 0.15 | 1 | 1 | 0 | | 110 |
| A3 | 0.1 | 1 | 1 | 1 | 0 | 1110 |
| A4 | 0.05 | 1 | 1 | 1 | 1 | 1111 |

$$\eta = L_H = L_{SF} = \frac{2.00}{2.05} \times 100\% = 97.56\%$$

If, symbol rate = 1000 symbols per second.

output bit = H(a) × symbol rate

= 2.00 × 1000

= 2000 bps

---

72ch:

| $x_i$ | $P(x_i)$ | | arranging in decreasing probability |
|-------|----------|---|-------------------------------------|
| | | | $x_i$ let ($P(x_i)$) |
| A | 0.3 ✓ | | $E \to A = 0.4$ |
| B | 0.1 ✓ | | $A \to B = 0.3$ |
| C | 0.02 ✓ | | $D \to C = 0.15$ |
| D | 0.15 ✓ | | $B \to D = 0.1$ |
| E | 0.4 ✓ | | $F \to E = 0.03$ |
| F | 0.03 | | $C \to F = 0.02$ |

Rate $R_s = 14.4$ K baud. = 14400 baud.

using BCD coding:

| $x_i$ | $P(x)$ | code | $x_i$ |
|-------|--------|------|-------|
| E → A | 0.4 | 1010 | 4 |
| A → B | 0.3 | 1011 | 4 |
| D → C | 0.15 | 1100 | 4 |
| B → D | 0.1 | 1101 | 4 |
| F → E | 0.03 | 1110 | 4 |
| C → F | 0.02 | 1111 | 4 |

$H(x) = 2.057$ bits/symbol.   $L_{BCD} = 4$

For Huffman

$P(EF) = 0.03 + 0.02 = 0.05$

```
      0 ↙   ↘ 1
   P(E)    P(F)
   0.03    0.02
```

$P(DEF) = 0.1 + 0.05 = 0.15$

```
      0 ↙   ↘ 1
   P(D)    P(EF)
   0.1     0.05
```

$P(CDEF) = 0.15 + 0.15 = 0.3$

```
      0 ↙   ↘ 1
   P(C)    P(DEF)
   0.15     0.15
```

$P(BCDEF) = 0.3 + 0.3 = 0.6$

```
      0 ↙   ↘ 1
   P(B)    P(CDEF)
   0.3      0.3
```

$P(ABCDEF)$

```
      0 ↙   ↘ 1
  P(BCDEF)  P(A)
    0.6     0.4
```

---

Huffman tree:

```
        P(ABCDEF)
      0 ↙      ↘ 1
  P(BCDEF)     P(A)
   0 ↙ ↘ 1
 P(B)  P(CDEF)
      0 ↙ ↘ 1
    P(C)  P(DEF)
        0 ↙ ↘ 1
      P(D)  P(EF)
          0 ↙ ↘ 1
        P(E)  P(F)
```
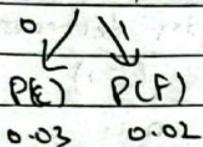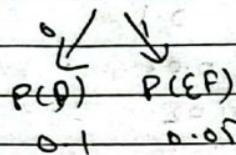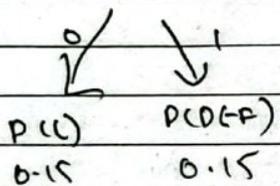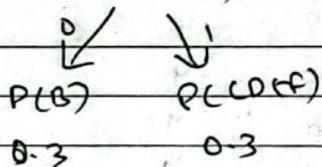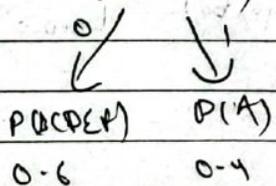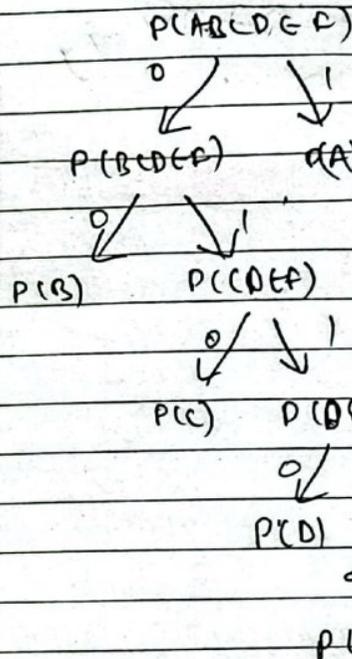
$P(A) = 1$
$P(B) = 00$
$P(C) = 010$
$P(D) = 0110$
$P(E) = 01110$
$P(F) = 01111$

$L_H = 1 \times 0.4 + 2 \times 0.3 + 3 \times 0.15 + 4 \times 0.1 + 5 \times 0.03$
$+ 5 \times 0.02 = 2.1 \text{ bits/symbol}$

(i) $R_s = 14.4 \text{ kbaud}$
$= 14400 \text{ baud}$

o/p bit rate / information rate $= H \times R_s$
$= 2.057 \times 14400$
$= 29620 \text{ bps}$

Date : ........................
Page: ........................

Chapter 2
Sampling

Date : ........................
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$$I_{080} = \frac{2.057}{4} \times 1000 + 51.4257.$$

$$I_H = \frac{2.057}{2.1} \times 1000 = 97.976 \, ./$$

73ch 2) = soln

$$g(t) = 10 \cos(20\pi t) \cos(200\pi t)$$
$$f_s = 250 \text{ samples/sec.}$$

① determine the spectrum of resulting sampled signal

② specify the cut-off frequency of the ideal reconstructed filter so as to recover $g(t)$ from its sampled version

③ determine the Nyquist rate for $g(t)$.

= soln

$$g(t) = 10 \cos(20\pi t) \cdot \cos(200\pi t)$$
$$= 5 [2 \cos(20\pi t) \cdot \cos(200\pi t)]$$
$$= 5 [\cos(220\pi t) + \cos(180\pi t)]$$

comparing with standard eq 2

| $w_1 = 220\pi$ | $w_2 = 180\pi$ |
|---|---|
| $2\pi f_1 = 220\pi$ | $2\pi f_2 = 180\pi$ |
| $f_1 = 110 \, Hz$ | $f_2 = 90 \, Hz$ |

since $f_1 > f_2$
$$f_m = f_1 = 110 \, Hz$$

and, sampling frequency $(f_s) = 210 \, Hz.$

1) spectrum of sampled



$g(w)$

$-f_s = \frac{240}{Hz}$   $-f_m$   $110\,Hz$   $0$   $f_m$   $110\,Hz$   $f_s = 210Hz$   $2f_s$ → $r$
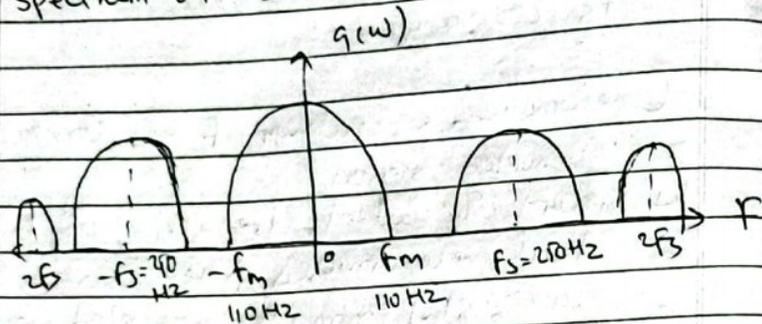
(i) cut off frequency of the ideal reconstruction filter so as to recover $g(t)$;

$$F_s = 2Fs$$
$$= 2 \times 110 = 220 \, Hz.$$

(ii) Nyquist Rate $= \dfrac{1}{2f_m} = \dfrac{1}{2 \times 110}$

$$= 4.545 \times 10^{-3} sec$$
$$= 4.545 \, ms.$$

---

2 ch 2)

$\frac{72 \, ch}{2}$

2) Nyquist rate=?
Nyquist interval=?.

$x(t) = 6\cos 50\pi t + 20 \sin 300 \pi t - 10 \cos 100 \pi t$   ch.

is sampled & quantized using 512 lod

$= s\eta n$

$\omega_1 = 50\pi$        $\omega_2 = 300\pi$       $\omega_3 = 100\pi$

$f_1 = 25 \, Hz$      $f_2 = 150 \, Hz$      $f_3 = 50Hz$

Also signal is sampled & quantized using 512 levels.

So, $q = 2^V$

$$512 = 2^V$$
$$2^9 = 2^V$$
$$\therefore V = 9 \, bits.$$

$\therefore f_2 > f_3 > f_1$

$\therefore f_m = f_2 = 150 \, Hz.$

Nyquist rate$(f_s) = 2 f_m$

$$= 2 \times 150 = 300 \, Hz.$$

Nyquist interval $(is) = \dfrac{1}{f_s} = \dfrac{1}{300} = 0.037 \, sec$

$f_{max} = 3.4 KHz = 3.4 \times 10^3 Hz$ per signal.

(i) Total Bandwidth, $B \geq (f_{max} \times 24)$ $(N \times f_m)$

$= 3.4 KHz \times 24$

$B = 81.6 KHz$

(ii) Signaling rate of system is given as,

$r = 1.5 \times 10^6 bits/sec$

since there are 24 channels, the bit rate of individual channel is,

$r(one\ channel) = \dfrac{1.5 \times 10^6}{24} = 62500\ bits/sec$

Further, since sample is encoded using 8 bits, the samples per second will be,

$Sample/second = \dfrac{r(one\ channel)\ bits/sec}{bits/sample}$

$sample/sec \rightarrow$ sampling frequency.

$f_s = 62500\ b/sec / 8\ b/sample$

---

$f_s = 7812.5 Hz.$

3 b)

encoder $\rightarrow$ 7 bit binary encoder.

bit rate $= 50 \times 10^6 bits/sec.$

$B_{max} = ?$

$= 50 \times 10^{12}$

Let $BW = fm\ Hz.$

$\therefore f_s \geq 2fm$

Number of bits, $v = 7 bits.$

we know that signalling rate

$r \geq v \cdot f_s$

$r \geq 7 \cdot 2fm$

substituting for r,

$50 \times 10^6 \geq 7.2\ fm$

$fm \leq 3.57 MHz,$

$\therefore BW = 3.57 MHz.$

**zuch**

3) = sol<sup>m</sup>.

given,

BW = 4.2 MHz

so, fm = 4.2 MHz

given quantization levels, q = 512

① 
$$q = 2^v$$
$$512 = 2^v$$
$$2^9 = 2^v$$
$$\therefore v = 9$$

∴ number of bits/code word length = 9 bits

⑪ Transmission BW ≥ V fm

$$BW \geq 9 \times 4.2 \, MHz$$

$$BW \geq 37.8 \, MHz.$$

⑪ Final bit rate.

It is equal to signaling rate.

$$r = V fs$$

---

$$r = 9 \times 2 fm$$
$$= 9 \times 2 \times 4.2 \times 10^6$$
$$= 75.6 \times 10^6 \, bits/sec.$$

⑭ O/P SQNR:

$$\left(\frac{S}{N}\right)_{dB} \leq (4.8 + 6V) \, dv.$$

$$V = 9$$

$$\left(\frac{S}{N}\right)_{dB} \leq 4.8 + 6 \times 9$$

$$\left(\frac{S}{N}\right)_{dB} \leq 58.8 \, dB.$$

---

**7884**

3 5)

BW = 6 MHz.

fm = 6 MHz

$$q = 256.$$
$$256 = 2^v$$
$$2^8 = 2^v$$
$$\therefore v = 8$$

① codeword length
   = 8 bits.

(ii) Practical BW :

$$BW \geq V \times f_m$$
$$BW \geq 8 \times 6 MHz$$
$$BW \geq 48 MHz$$

(III) final data rate.

$$r = V f_s$$
$$= V \, 2 f_m$$
$$= 8 \times 2 \times (6 MHz)$$
$$= 96 \times 10^6 \, bits/sec$$

(iv) Output SQNR :

$$\left(\frac{S}{N}\right)_{dB} \leq (4.8 + 6V) \, dB.$$
$$\leq 4.8 + 6 \times 8$$
$$\left(\frac{S}{N}\right)_{dB} \leq 52.8 \, dB.$$

3) · given, $f_m = 4 kHz$

$$q = 16$$

(i)
$$2^V = q$$
$$2^V = 16$$
$$2^v = 2^4$$
$$\therefore V = 4$$

∴ max bits per sample = 4 bits.

(ii)
$$f_s \geq 2 f_m$$
$$\geq 2 \times 4 kHz$$
$$\geq 8 kHz$$

∴ min sampling rate = 8 kHz.

(iii)
Bit rate, $r \geq V f_s$
$$\geq 4 \times 8 \, kHz$$
$$\geq 32 \, kHz$$
$$\geq 32 \times 10^3 \, bits/second$$

given

$f_m = 4kHz$

min dynamic range = $\pm 2.4V$

bit rate = $64kHz$

bit persample = ?

Quantization noise power = ?

$SQNR_{dB} = ?$

minimum $BW = ?$

bit persample $v = \dfrac{\log_{10} 64000}{\log_{10} 2}$

$= 16$ bits.

$X_{max} = 2.4V$

Quantization noise power, $P = \dfrac{A_m^2}{2}$

$= \dfrac{X_m^2}{2}$

$= \dfrac{(2.4)^2}{2}$

$= 2.88$

$\dfrac{S}{N} = \dfrac{3 P 2^{2v}}{X_{max}^2}$

$= \dfrac{3 \times 2.88 \times 2^{2 \times 16}}{(2.4)^2}$

$= 6442450944$

$\left(\dfrac{S}{N}\right)_{dB} = 10 \times \log_{10}\left(\dfrac{S}{N}\right)$

$= 10 \log_{10}(6442450944)$

$= 98.09 dB$

for min BW,

$f_m = 4kHz.$

$BW \geqslant v f_m$

since there are 10 audio signals multiplex

$BW \geqslant 10 \times v \times f_m$

$\geqslant 10 \times 16 \times 4$

$BW \geqslant 640 kHz$

given,

$$x(t) = 6 \cos(5000\pi t)$$

$$q = 2^V$$
$$128 = 2^V$$
$$2^7 = 2^V$$
$$V = 7 \text{ bit}$$

(a) $(SQNR) = 4.8 + 6V$
$$= 4.8 + 6 \times 7$$
$$= 46.8 \, dB.$$

(b) $3 \times 4^V = 3 \times 4^7 = 49152.$

we know that,

$$n = \frac{3}{8\pi^2} \left( \frac{f_s'}{f_n} \right)^3$$

$$5000\pi t = 2\pi f t$$
$$f = 2500 \, Hz$$

$$\therefore n = \frac{3}{8\pi^2} \left( \frac{f_s'}{f_n} \right)^3.$$

$$49152 = \frac{3}{8\pi^2} \left( \frac{f_s'}{2500} \right)^3.$$

$$f_s' = 272401.75 \, Hz.$$

$$\boxed{f_s' = 272.401 \, KHz}$$

(c) $SQNR = \frac{3}{8\pi^2} \left( \frac{2 f_n}{f_n} \right)^3$

$$= \frac{3}{8\pi^2} \times 2^3$$

$$= 0.30,$$

In dB; $10 \log_{10}(0.30) \approx -5.22 \, dB$

$\dfrac{6 \times h}{3}$

3) SNR

given,

$f_m = 4 \text{ kHz}$

$f_s = 2 \times 4 = 8 \text{ kHz}.$

$\pm A = \pm 1.1 V$

Bit rate $= 32 \text{ kbPS} = \vee f_s \text{ sample/sec.}$

$32 = 8n$

$\vee = 4 \text{ bits/sample.}$

$\therefore \text{SQNR} = 4.8 + 6 \vee$

$= 4.8 + 6 \times 4$

$= 28.8 \text{ dB}$

---

$\dfrac{32 \times 10^6}{4} = 30 \quad n$

given,

$f_m = 3 \text{ kHz}$

$f_s = 10 \text{ kHz}$

Nyquist rate $= 2 f_m = 2 \times 3 \text{ kHz}$

$= 6 \text{ kHz}$

Sampling interval $(T_s) = \dfrac{1}{f_s}$

$= \dfrac{1}{10000}$

$A_{max} = 1$

$\therefore$ step size $(\Delta) = A_m \cdot 2\pi f_m \cdot T_s$

$= \dfrac{A_m \cdot 2\pi f_m}{f_s}$

$= \dfrac{1 \times 2\pi \times 3 \times 10^3}{10 \times 10^3}$

$= \dfrac{6\pi}{10} = 1.88 V$

$$(SQNR) = \frac{3}{3\pi^2 f_m^2 f_M T_s^3}$$

$$f_m = f_M = 3 kHz$$

$$SQNR = \frac{3}{3\times\pi^2 \times 3^2 \times 3 \times 10^9 \times (10^{-4})^3}$$

$$= 3.75$$

$$= 5.74 \, dB$$

Min transmission BW $= f_m = 3 kHz$

---

72 kार
2)

$$m(t) = sinc(5\pi t)$$
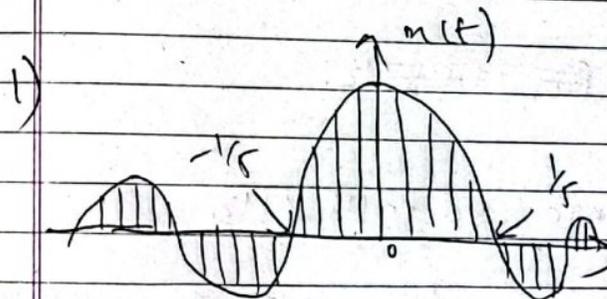$$f_s = 10 Hz$$

1) sketch the sampled of signal
2) sketch the spectrum of the sampled signal for the range $|f| < 30 Hz$.
3) Explain whether you can recover the signal $m(t)$ from the sampled signal.

given signal,
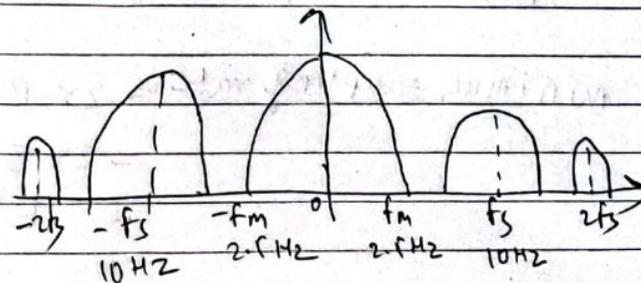$$m(t) = sinc(5\pi t) = sinc(2\times\pi\times 2.5t)$$

1)



11)

(III) given signal $x(t) = \text{sinc}(\pi t)$.

$f_m = 2.5 \, Hz$

$f_s = 10 \, Hz$

$f_s \geq 2 f_m$

$10 \geq 2 \times 2.5$

$10 \geq 5$

$\therefore x(t)$ can be recovered.

———————X——————X——————

$\dfrac{69ch}{2)}$
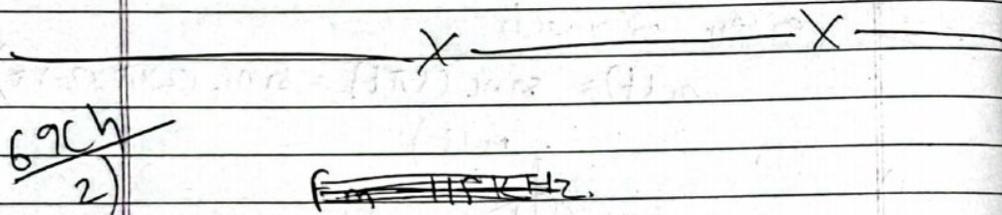
~~$f_m = 115 \, KHz$~~

Range: 80 to 115 KHz

$BW = 2f_m = 115 - 80 = 35 \, KHz$

minimum sampling rate $= 2 \times BW$

$= 2 \times 35$

$= 70 \, KHz$

nomally, range of min. sampling frequencies lies in between $4f_m$ to $8f_m$ for BP signals,

Range of min. sampling frequencies

$= 2 \times BW$ to $4 \times BW$

$= 2 \times 35 \, KHz$ to $4 \times 35 \, KHz$

$= 70 \, KHz$ to $140 \, KHz$ //

**30(4) Q)** The generator polynomial of a (7,4) cyclic code is $g(x) = 1 + x + x^3$. Find the code for the message signal in a vector 1011 in a non systematic & systematic form.

$=$ **Soln**

given, $n = 7$ and $k = 4$

$$g(x) = 1 + x + x^3$$

For message signal 1011,

$$M(x) = m_0 + m_1 x + m_2 x^2 + m_3 x^3$$
$$= 1 + 0 \cdot x + 1 x^2 + 1 \cdot x^3$$
$$= 1 + x^2 + x^3$$

∴ code vector will be;

$$C(x) = M(x) \cdot g(x)$$
$$= (1 + x^2 + x^3)(1 + x + x^3)$$
$$= 1 + x^2 + x^3 + x + x^3 + x^4 + x^3 + x^5 + x^6$$
$$= 1 + x + x^2 + x^3(1 \oplus 1 \oplus 1) + x^4 + x^5 + x^6$$
$$C(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6$$

∴ code word will be $C(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6$

$$= 1111111$$

which is in non-systematic form.

---

for systematic form

multiply $m(x)$ by $x^{n-k}$.

$$x^{n-k} m(x) = x^{7-4} m(x)$$
$$x^{n-k} m(x) = x^3(1 + x^2 + x^3) = x^3 + x^5 + x^6$$

Divide $x^{n-k} \{m(x)\}$ by $g(x)$

$$\frac{x^{n-k} \{m(x)\}}{g(x)} = \frac{x^3 + x^5 + x^6}{1 + x + x^3}$$

$$x^3 + x + 1 \,\big)\, x^6 + x^5 + x^3 \,\big(\, x^3$$
$$\underline{x^6 + x^4 + x^3}$$
$$x \Rightarrow r(x)$$

$$r(x) = [x^{n-k} m(x)] \oplus r(x)$$
$$= [0 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3 + 0 \cdot x^4 + 1 \cdot x^5 + 1 \cdot x^6] \oplus$$
$$[0 + 1 \cdot x]$$
$$= 0 + 1 \cdot x + 0 \cdot x^2 + 1 \cdot x^3 + 0 \cdot x^4 + 1 \cdot x^5 + 1 \cdot x^6$$
$$= 0101011$$

∴ code vector is : $x(x) = 0101011$
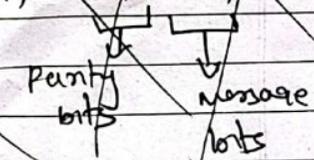
Parity bits    Message bits

Left column:

$$x^3+x+1 \,\big)\, \dfrac{x^6+x^5+x^3}{\quad} \,\big|\, x^3+x^2+x+1$$

$$x^6+x^4+x^3$$
$$\oplus \quad \oplus \quad \oplus$$

$$x^5+x^4$$
$$x^5+x^3+x^2$$
$$\oplus$$

$$x^4+x^3+x^2$$

$$x^4+x^2+x$$
$$\oplus \quad \oplus \quad \oplus$$

$$x^3+x$$
$$x^3+x+1$$
$$\oplus \quad \oplus$$

$$1 \rightarrow r(x)$$

$$x(x) = r(x) + x^3 M(x)$$
$$= 1 + x^3 + x^5 + x^6$$
$$= \text{ } 1001011$$

r , m

Right column:

Q) Construct a (7,4) cyclic code using generator polynomial $g(x) = x^3+x^2+1$ with data vector 1011

= solⁿ.

$$M(x) = 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x + 1 .$$
$$M(x) = x^3 + x + 1$$

$$g(x) = x^3 + x^2 + 1$$

$$x^{n-k} M(x) = x^3(x^3 + x + 1)$$
$$= x^6 + x^4 + x^3$$

$$\dfrac{x^3 M(x)}{g x} \Rightarrow x^3+x^2+1 \,\big)\, \dfrac{x^6+x^4+x^3}{\quad} \,\big|\, x^3+x^2$$

$$x^6 + x^5 + x^3$$

$$x^5 + x^4$$
$$x^5 + x^4 + x^2$$

$$x^2 \rightarrow r(x)$$

$$x^6 + x^4 + x^3 \oplus x^2$$
$$x^6 + x^4 + x^3 + x^2$$

$$1011100 \rightarrow \text{codeword}$$

message bits , parity bits

Q) For a code vector $x = (0111000)$ and the parity check matrix H given below. Prove that the given code is valid.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3 \times 7}$$

= soln

given code vector, $x = (0111000)$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

To be valid it must satisfy

$$x H^T = (0,0,\cdots 0)$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore x H^T = \begin{bmatrix} 0111000 \end{bmatrix}_{1 \times 7} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{7 \times 3}$$

$xH^T = (0 \times 1) \oplus (1 \times 1) \oplus (1 \times 1) \oplus (1 \times 0) \oplus (0 \times 1) \oplus (0 \times 0)$
$\oplus (0 \times 0), (0 \times 1) \oplus (1 \times 1) \oplus (1 \times 0) \oplus (1 \times 1) \oplus (0 \times 0) \oplus$
$(0 \times 1) \oplus (0 \times 0), (0 \times 1) \oplus (1 \times 0) \oplus (1 \times 1) \oplus (1 \times 1)$
$\oplus (0 \times 0) \oplus (0 \times 0) \oplus (0 \times 1)$

$= 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0,$
$0 \oplus 1 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0,$
$0 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0$

$xH^T = \begin{bmatrix} 0,0,0 \end{bmatrix}_{1 \times 3}$

$= (0,0,0)$

Hence this is valid codeword

---

27BH

$(6,3)$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$n = 6$
$k = 3$
message bits = $k = 3$.
parity bits = $n-k$
$= 3$

1) generator matrix!

$$g = [I_k | P]$$

$$\therefore I_k = I_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$1 \oplus 1 \to 0$
$0 \oplus 1 \to 1$
$1 \oplus 0 \to 1$

$0 \oplus 0 \to 0$

$$\therefore G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

2) All code words

Relation bet$^n$ parity bits & message bits is

$$[C_0, C_1, C_2] = [M_0, M_1, M_2] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$C_0 = (M_0 \times 1) \oplus (M_1 \times 1) \oplus (M_2 \times 1) = M_0 \oplus M_1 \oplus M_2$

$C_1 = (M_0 \times 1) \oplus (M_1 \times 1) \oplus (M_2 \times 0) = M_0 \oplus M_1$

$C_3 = (M_0 \times 1) \oplus (M_1 \times 0) \oplus (M_2 \times 1) = M_0 \oplus M_2$

| sNo | Message vector M₀ M₁ M₂ | | | Parity bits C₀ C₁ C₂ | | | Complete code vector M0 M1 M2 C0 C1 C2 | | | | | | wt |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2) | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 3 |
| 3) | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 3 |
| 4) | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 4 |
| 5) | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 4 |
| 6) | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 3 |
| 7) | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 4 |
| 8) | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 4 |

3) $d_{min} = $ min weight of any non zero code vector.

$d_{min} = 3$

No. of errors that can be detected is given by

$d_{min} \geq S + 1$

$3 \geq S + 1$

$\therefore S \leq 2$

Hence, at the most 2 errors can be detected.

Number of errors that can be corrected is given by

$d_{min} \geq 2t + 1$

$3 \geq 2t + 1$

$2 \geq 2t$

$t \leq 1$

This means at most one error can be corrected.

optional: correct upto t error & detect S > t errors per word:

$d_{min} \geq (t + s + 1)$