Simple Harmonic Motion

→ S.H.M is a type of periodic motion in which the acceleration of oscillating particle at any instant is directly proportional to the displacement of the particle from mean position at that instant. And, the motion is always directed towards the mean position.

→ I.e. Acceleration is directly proportional to the displacement of the particle.

→ Acceleration $(a) \propto$ displacement (y)

or $a \propto y$

$\Rightarrow \boxed{a = -ky}$

Where, 'k' is a proportionality constant and '-ve' sign indicates that the motion is directed towards mean or equilibrium position.

(i) Displacement

$y = a \sin \omega t$

(ii) Amplitude, $(y_{max} = a)$

(iii) Velocity $(v) = \omega\sqrt{a^2 - y^2}$

Cond^n. (I) At mean position $(y = 0)$

$v = a\omega = v_{max}$

Cond^n (II) At Extreme or End point. $(y = a)$

$v = 0 = V_{min}$

(iv) **Acceleration (a):-**

$$A = -\omega^2 y$$

Magnitude only, $A = \omega^2 y$

Condⁿ. (I) At mean position $(y = 0)$

$A = 0 = A_{min}$

Condⁿ (II) At extreme point $(y = a)$

$A = \omega^2 a = A_{max}$

(v) **Time Period (T) :-**

$$T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

(vi) **Frequency (f):-**

$$f = \frac{1}{2\pi} \sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$

$\rightarrow P = 1/f$

(vii) **Phase Angle (ϕ)**

(4) Equation of SHM :-

→ If 'y' be the displacement of the particle from mean position at any instant of time 't'. Then, the restoring force is given by,

$$f = -ky \quad \text{----- (i)}$$

Where, 'k' is a force constant.

From Newton's 2nd law of motion. The force is given by,

$$f = mA$$

or $$-ky = m \cdot \frac{d^2y}{dt^2}$$

or $$m\frac{d^2y}{dt^2} + ky = 0$$

or $$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

or, $$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0} \text{ --- (ii)} \quad \text{where, } \omega^2 = \frac{k}{m}$$

This is the eq^n of SHM.

Now,

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

or, $$\frac{dv}{dt} = -\omega^2 y$$

or $$\frac{dv}{dy} \cdot \frac{dy}{dt} = -\omega^2 y$$

or, $$v \cdot \frac{dv}{dy} = -\omega^2 y \quad \left[\because v = \frac{dy}{dt}\right]$$

or $$v \cdot dv = -\omega^2 y \cdot dy$$

Integrating both sides, we get,

$$\int v \cdot dv = -\omega^2 \int y \, dy$$

or, $\frac{v^2}{2} = -\frac{\omega^2 y^2}{2} + c$ ------ (iii)

Where, 'c' is an integration constant. Its value is determined by using boundary conditions,

At $y = a$ (amplitude), $v = 0$.

Now, from eqn (iii),

$$0 = -\frac{\omega^2 a^2}{2} + c$$

or, $c = \frac{\omega^2 a^2}{2}$

putting the value of 'c' in eqn (iii);

or, $\frac{v^2}{2} = -\frac{\omega^2 y^2}{2} + \frac{\omega^2 a^2}{2}$

or, $v^2 = \omega^2 (a^2 - y^2)$

$$\Rightarrow \boxed{v = \omega \sqrt{a^2 - y^2}}$$

Again, $\frac{dy}{dt} = \omega \sqrt{a^2 - y^2}$

or, $\frac{dy}{\sqrt{a^2 - y^2}} = \omega dt$

integrating both sides; we get,

$$\int \frac{dy}{\sqrt{a^2 - y^2}} = \omega \int dt$$

Let, $y = a \sin\theta$

$dy = a \cos\theta \cdot d\theta$

or $\int \frac{a\cos\theta \cdot d\theta}{\sqrt{a^2 - a^2\sin^2\theta}} = \omega t + \phi$

or, $\theta = \omega t + \phi$ {'ϕ' is a Integration constant.}

Hence, $\boxed{y = a\sin(\omega t + \phi)}$

This is the solⁿ of eqⁿ of SHM.

Free Oscillation :-

→ The oscillation of a body in which the body oscillates with its own natural frequency at constant amplitude when it left free is called free oscillation. The free oscillation is due to restoring force. It is an ideal case, The ~~loaded mass spring~~ motion of loaded mass-spring system can be considered as free oscillation.

Consider a body of mass 'm' executes SHM. Its velocity and displacement at a time 't' are 'v' and 'y' respectively.

the K.E. of the body is,

$K.E. = \frac{1}{2} mv^2$ & the P.E. of the body is,

$P.E. = \frac{1}{2} ky^2$

The total energy of the body is,

Total Energy $(E) = K.E + P.E.$

or, $E = \frac{1}{2} mv^2 + \frac{1}{2} ky^2$

since, for free oscillation, the Total energy is constant.

i.e. $\frac{1}{2}mv^2 + \frac{1}{2}ky^2 = \text{constant}$

differentiating both sides w.r. to t,

or, $\frac{1}{2}m\frac{dv^2}{dv}\cdot\frac{dv}{dt} + \frac{1}{2}k\cdot\frac{dy^2}{dy}\cdot\frac{dy}{dt} = 0$

or, $\frac{1}{\cancel{2}}m\cdot\cancel{2}v\cdot\frac{dv}{dt} + \frac{1}{\cancel{2}}k\cdot\cancel{2}y\cdot\frac{dy}{dt} = 0$

or, $mv\cdot\frac{dv}{dt} + ky\cdot\frac{dy}{dt} = 0$

or, $mv\cdot\frac{dv}{dt} + ky\cdot v = 0$

or, $m\cdot\frac{dv}{dt} + k\cdot y = 0$ [Taking 'v' common & dividing both sides by 'v']

or, $\boxed{\frac{d^2y}{dt^2} + \frac{k}{m}y = 0}$ --- (i) [$\because \frac{dv}{dt} = \frac{d^2y}{dt^2}$ & dividing by 'm']

→ This is the eqn of free oscillation.

This eqn is similar to the eqn of SHM.

$\frac{d^2y}{dt^2} + \omega^2 y = 0$ --- (ii)

Compairing eqn (i) & (ii); we get,

$\omega^2 = \frac{k}{m}$

$\Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{k/m}}$

→ Soln. ⇒ Similar as previous.

Damped Oscillation:-

→ The oscillation in which amplitude of oscillating particle gradually decreases and finally becomes zero by the effect of dissipative force like viscous force, frictional force etc is called damped oscillation.

If 'v' be the velocity of oscillating particle then the damping force is,

$$F_d \propto v$$

$$\text{or} \quad F_d = -bv \quad \text{---} \text{(i)}$$

Where, 'b' be the proportionality constant called damping constant. '-ve' sign indicates that damping force and velocity are in opposite direction.

The restoring force is given by,

$$F_r = -ky$$

From Newton's 2nd law of Motion,

$$\Sigma F = ma$$

$$\text{or} \quad F_d + F_r = ma$$

$$\text{or,} \quad -bv - ky = ma$$

$$\text{or} \quad ma + bv + ky = 0$$

$$\text{or} \quad m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$$

$$\text{or} \quad \frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = 0 \quad \text{---} \text{(ii)}$$

This is the eqn of damped oscillation which is similar to the differential eqn;

$$\boxed{\frac{d^2y}{dt^2} + 2\delta\frac{dy}{dt} + \omega_0^2 y = 0} \quad \text{----} \text{(iii)}$$

Comparing eqn (ii) & (iii), we get

$$\delta = \frac{b}{2m} \quad \& \quad \omega_0^2 = k/m$$

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The sol^n of eq^n (iii) is,

$$y = a e^{-\delta t} \sin(\omega t - \phi)$$

where,

$$\omega = \sqrt{\omega_0^2 - \delta^2}$$

so, the sol^n of eq^n (i) is,

$$y = a e^{-\frac{bt}{2m}} \sin(\omega t - \phi) \quad \left[\because \delta = \frac{b}{2m}\right]$$

or $y = A \sin(\omega t - \phi)$

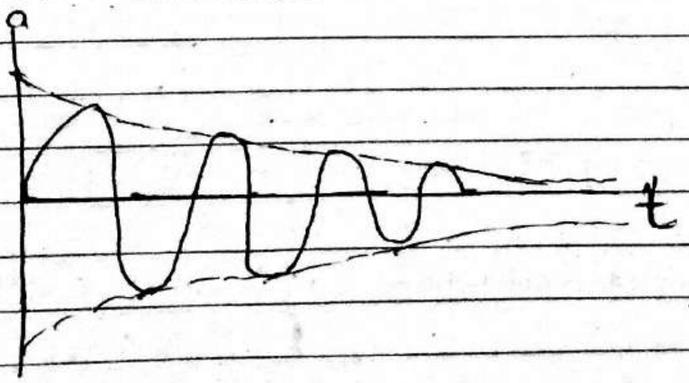
Where, $A = \left(a e^{-\frac{bt}{2m}}\right)$ is the amplitude of damped oscillation.

Again, $\omega = \sqrt{\omega_0^2 - \delta^2}$

or $\omega = \sqrt{K/m - \frac{b^2}{4m^2}}$

$$\boxed{\text{or, } f = \frac{1}{2\pi}\sqrt{K/m - \frac{b^2}{4m^2}}}$$

$(2\pi f) = \sqrt{K/m - \frac{b^2}{4m^2}}$



Forced oscillation :-

→ If the energy loss by oscillating body due to damping is equal to the energy supplied by external force, then the body oscillates with constant amplitude. This type of oscillation is called forced oscillation. Here, The damping force is F_d, restoring force is F_r and applied force F_a.
Damping force is given by, '$-bv$', restoring force is given by 'ky' and applied force is defined by ($F\cos\omega t$).
from Newton 2^{nd} law of motion,

$$\Sigma F = ma$$

or $F_d + F_r + F_a = ma$

or, $-bv - ky + F\cos\omega t = ma$, where, ω is the frequency of External force.

or $ma + bv + ky = F\cos\omega t$

or, $m\dfrac{d^2y}{dt^2} + b\dfrac{dy}{dt} + ky = F\cos\omega t$

or $\dfrac{d^2y}{dt^2} + \dfrac{b}{m}\dfrac{dy}{dt} + \dfrac{k}{m}y = \dfrac{F}{m}\cos\omega t$ --- (i)

This is the eqn of forced oscillation. which is similar to the differential eqn ;

$$\boxed{\frac{d^2y}{dt^2} + 2\delta\frac{dy}{dt} + \omega_0^2 y = f\cos\omega t} \quad \text{--- (ii)}$$

Comparing eq^n (i) & (ii) ; we get

$\delta = \dfrac{b}{2m}$, $\omega_0^2 = k/m$. $f = \dfrac{F}{m}$

The sol^n of eq^n (ii) is,

$$y = y_m \sin(\omega t - \phi)$$

Where, $y_m = \dfrac{f}{\sqrt{(\omega_0^2 - \omega_0^2)^2 + (2\delta\omega)^2}}$

and $\phi = \cancel{\tan^{-1}\left(\frac{2\delta\omega}{\omega^2-\omega_0^2}\right)} = \tan^{-1}\left(\frac{\omega^2 - \omega_0^2}{2\delta\omega}\right)$

For resonance, y_m is maximum,
i.e. $\omega = \omega_0$

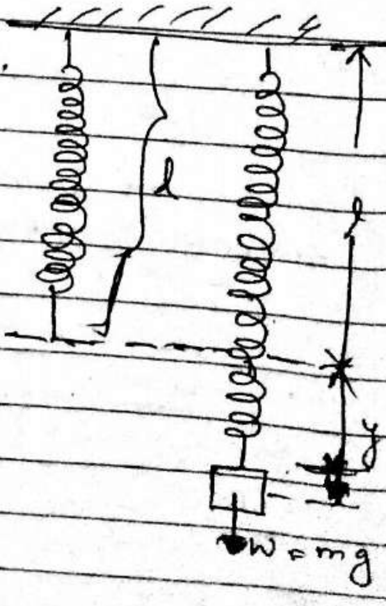
or, $$(y_m)_{max} = \frac{f}{2\delta\omega}$$

$$= \frac{F/m}{2 \cdot \frac{b}{2m} \cdot \omega_0} \quad [\because \omega = \omega_0]$$

$$\boxed{(y_m)_{max} = \frac{F}{b}\sqrt{\frac{m}{k}}}$$

(*) Vertical Mass-spring system:-

→ Consider a mass-less spring of length 'l' and attached to a mass of 'm' vertically downward. When mass is not attached on it, it maintains equilibrium position having no change in its original length. After attaching mass to it, then ~~[illegible]~~ stretched by ~~length~~ 'y' having total length as '$l+y$'. Then restoring force, $F_r = [-k \times length(l)]$ --



$\Rightarrow F_1 = -kl$

$\Rightarrow F_2 = -k(l+y)$

$\therefore F = F_2 - F_1 = -ky$

$\Rightarrow$ From Newton's 2nd law of motion, $F = ma$

or, $ma = -ky$

or, $a = -\frac{k}{m} y$ -- (i)

$\Rightarrow a \propto y$ (same as SHM)

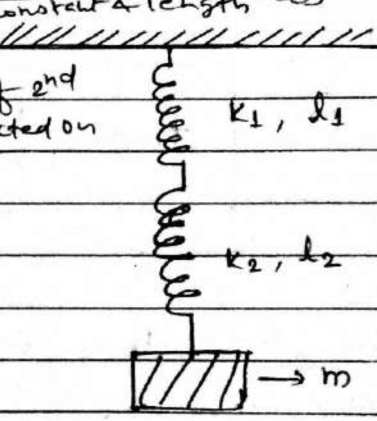
$\rightarrow a = -\omega^2 y$ -- (ii)

$\omega^2 = k/m$

$$\boxed{f = \frac{1}{2\pi}\sqrt{k/m}}$$

(#) (a) When two springs are connected in series :-

→ let us consider, two springs are connected in series having spring constant & length as k_1, l_1 and k_2, l_2 respectively and load of mass 'm' is hanging at the bottom end of 2nd spring. The first spring is connected on a rigid supports. Then,



→ Here,

$F = -k_1 l_1$

or, $l_1 = \frac{-F}{k_1}$

and,

$F = -k_2 l_2$

or, $l_2 = \frac{-F}{k_2}$

The total extension of the combination of spring is,

$l = l_1 + l_2$

or, $l = \frac{-F}{k_1} - \frac{F}{k_2}$

or, $l = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

or, $l = -F\cdot\left(\frac{k_1+k_2}{k_1\cdot k_2}\right)$

or, $F = -\left(\frac{k_1\cdot k_2}{k_1+k_2}\right)\cdot l$

or, $ma = -\left(\frac{k_1 . k_2}{k_1+k_2}\right) l$

or, $a = -\frac{1}{m}\left(\frac{k_1 . k_2}{k_1+k_2}\right) . l$ ---- (1)

or, $a \propto l$

i.e. The motion of above combination of spring is SHM.

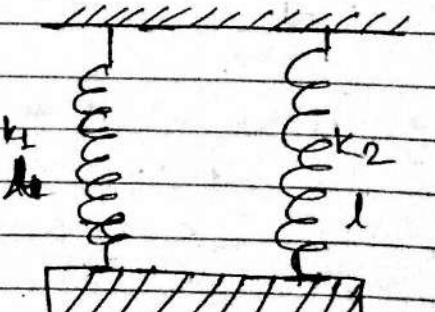
Compairing eqn (1) with,

$a = -\omega^2 l$

we get, $\omega^2 = \frac{k_1 . k_2}{m(k_1+k_2)}$

or, $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k_1 . k_2}{m(k_1+k_2)}}}$

(b) When two springs are connected in parallel :-



⇒ Let us consider two springs are connected parallel having rigid support at one end. The length of both springs are same i.e. 'l'. and their spring constants are 'k_1' & 'k_2' respectively for 1st & 2nd spring. Both of the spring are connected with a body of mass 'm' at other end towards bottom direction.

Then, the force given by the spring is given by, Here, $F_1 = -k_1 l$

$F_2 = -k_2 l$

The restoring force is

$F = F_1 + F_2$

or, $F = -k_1 l - k_2 l$

or, $F = -l(k_1 + k_2)$

$\therefore$ from newton's 2nd law of motion,

or, $ma = -(k_1 + k_2) l$

or, $a = -\left(\frac{k_1 + k_2}{m}\right) l$

$\Rightarrow a \propto l$

i.e. The motion of above combination of spring is SHM.

Compairing eqn (i) with,

$a = -\omega^2 l$

we get, $\omega^2 = \left(\frac{k_1 + k_2}{m}\right)$

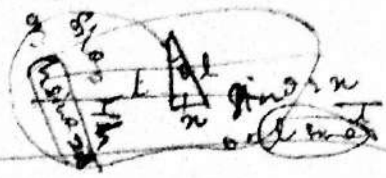
frequency.

or, $$\boxed{f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$

IMP.

(*) Compound Pendulum (Physical Pendulum) :-

$\Rightarrow$ Any body whatever the shape capable of oscillating in vertical plane about a horizontal axis passes through the body but not passes through the C.G. of the body is called compound pendulum.



→ Consider a compound pendulum of mass 'm' oscillates about a horizontal axis passes through the point of suspension 'S'. 'L' be the distance between point of suspension and C.G. ('G') of the body. When the body displaced from equilibrium position by small angle 'θ', the C.G. shifted from 'G' to 'G''.

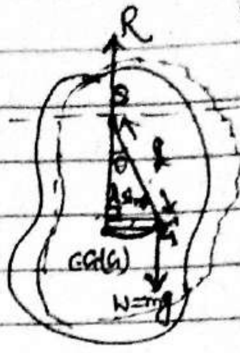


Fig:

$\frac{P}{l} = \sin\theta$

$(P = l\sin\theta)$

The restoring torque (τ) is produced due to the couple formed by weight of the body ($W = mg$) acting vertically downward at point 'G'' and normal reaction (R) acting vertically upward at point 'G'. The restoring torque is given by,

$$\tau = -mgl\sin\theta$$

for small angle 'θ', $\sin\theta \approx \theta$

or, $\tau = -mgl\theta$ --- (i)

If 'I' be the moment of Inertia of the body about the given axis and 'α' be the angular acceleration. then the torque is given by,

$$\tau = I\alpha \text{ --- (ii)}$$

equating eq^n (i) and (ii); we get,

$$I\alpha = -mgl\theta$$

or, $\alpha = -\left(\frac{mgl}{I}\right)\theta$ --- (iii)

or, $\boxed{\alpha \propto \theta}$ → SHM

Hence, the motion of compound pendulum is angular simple harmoni
compairing eqn (iii) with,

$$\alpha = -\omega^2 \theta \text{ , we get}$$

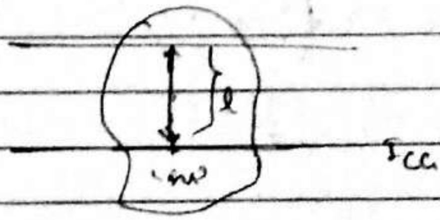
$$\omega^2 = \left(\frac{mgl}{I}\right)$$

or, $$\omega = \sqrt{\frac{mgl}{I}}$$

or, $$\frac{2\pi}{T} = \sqrt{\frac{mgl}{I}}$$

or, $$\boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$$

Using parallel axis theorem,



$$I = I_{CG} + ml^2$$

or, $$I = mk^2 + ml^2$$

Where,
'k' is the radius of gyration of the body.

Now,
$$T = 2\pi\sqrt{\frac{mk^2 + ml^2}{mgl}}$$

or, $$T = 2\pi\sqrt{\frac{\frac{k^2+l^2}{l}}{g}}$$

or, $$T = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

or, $$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

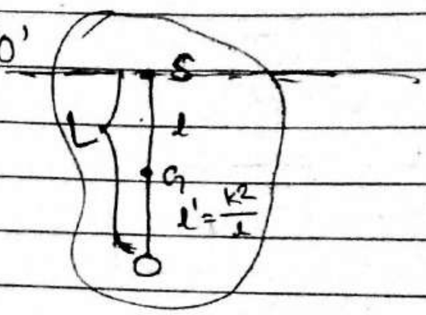
This is the time period of compound pendulum.

Where $L = \frac{K^2}{l} + l = l' + l$, is the equivalent simple pendulum length of the body.

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{l'+l}{g}}}$$

(#) Interchangeability of point of suspension and point of oscillation.

⇒ Produce the point 'G' to the point 'O' (point of oscillation) such that,



$$GO = \frac{K^2}{l} = l'$$

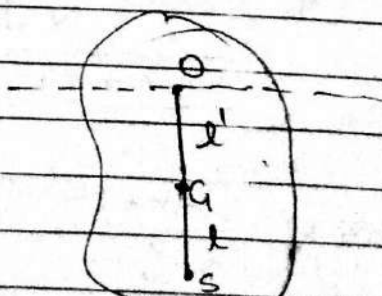
or, $K^2 = l \cdot l'$

The time period is,

$$T = 2\pi\sqrt{\frac{K^2/l + l}{g}}$$

or, $T = 2\pi\sqrt{\frac{l'+l}{g}}$ ----- (*)

Now, interchanging the point of suspension and point of oscillation. The time period is given by,



$$T' = 2\pi\sqrt{\frac{\frac{K^2}{l'} + l'}{g}}$$

or, $T' = 2\pi\sqrt{\frac{\frac{l \cdot l'}{l'} + l'}{g}}$

or, $T' = 2\pi\sqrt{\frac{l+l'}{g}}$ ------ (**)

from eqⁿ (*) & (**)

$$\boxed{T = T'}$$

Hence, point of suspension and point of oscillation of a compound pendulum can be interchanged.

(*) Minimum Time period :-

→ We know that, the time period of compound pendulum is,

$$T = 2\pi \sqrt{\frac{\frac{K^2}{l} + l}{g}}$$

$$\text{or, } T^2 = \frac{4\pi^2 \left(K^2/l + l\right)}{g}$$

differentiating both sides w.r.to 'l'.

$$2T \cdot \frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{K^2}{l^2} + 1\right) \quad \text{--- (i)}$$

$$\text{or, } \frac{dT}{dl} = \frac{4\pi^2}{2Tg}\left(-K^2/l^2 + 1\right)$$

For time period to be minimum,

$$\frac{dT}{dl} = 0$$

$$\text{or, } \frac{4\pi^2}{2Tg}\left(-\frac{K^2}{l^2} + 1\right) = 0$$

$$\text{or, } -\frac{K^2}{l^2} + 1 = 0$$

$$\text{or, } K^2 = l^2$$

$$\Rightarrow \boxed{K = l}$$

Again, differentiating eqⁿ (i), with respect to 'l',

$$2T \cdot \frac{d^2T}{dl^2} + \cancel{2\frac{dT}{dl}}^{\,0} = \frac{4\pi^2}{g}\left(\frac{2K^2}{l^3}\right)$$

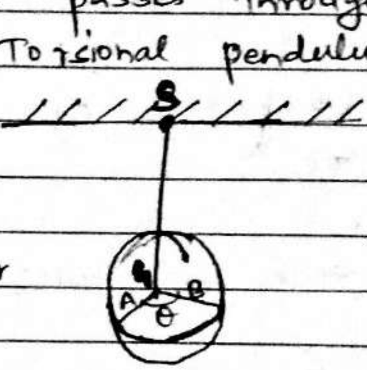
or, $$\frac{d^2T}{dl^2} = \frac{4\pi^2}{2Tg}\left(\frac{2K^2}{l^3}\right) > 0$$

$\frac{d^2T}{dl^2}$ is positive.

Hence The time period of compound pendulum is min^m at $\underline{\underline{K = l}}$.

Torsional Pendulum :-

→ Any body whatever the shape capable of oscillating in horizontal plane about a vertical axis passes through the C.G. of the body, is called Torsional pendulum.



⇒ Consider a disc of mass 'm' and radius 'r'. One end of a string is connected at center of gravity (CG) of the disc and other end is connected to the rigid support. The disc act as a torsional pendulum which oscillates in horizontal plane. The disc is twisted by an angle 'θ' and then release. The disc oscillates in horizontal plane. The restoring ~~force~~ torque is directly proportional to the angle twisted.

i.e. $\tau \propto \theta$

or $\tau = -C\theta$ --- (1) ; where, 'C' is torsional constant.

The '-ve' sign ~~shows~~ indicates that the restoring torque is opposite in direction to the angle twisted.

If 'I' be the moment of Inertia of the disc and 'α' be its angular acceleration. Then the torque is given by

$$\tau = I\alpha \quad --- (ii)$$

From eqⁿ (i) & (ii);

$$I\alpha = -C\theta$$

or, $\alpha = \frac{-C\theta}{I} \quad --- (iii)$

$\Rightarrow \boxed{\alpha \propto \theta}$

Hence, the motion of torsional pendulum is angular simple harmonic

Comparing eqⁿ (iii) with $\alpha = -\omega^2\theta$; we get.

$$\omega^2 = \frac{C}{I}$$

or $\omega = \sqrt{\frac{C}{I}}$

or $\frac{2\pi}{T} = \sqrt{\frac{C}{I}}$

or, $\boxed{T = 2\pi\sqrt{\frac{I}{C}}}$

This is the time period of torsional pendulum. for a given wire,

$$C = \frac{\pi\eta r^4}{2l}$$

where,

$\eta \rightarrow$ modulus of rigidity of wire

$l \rightarrow$ length of wire.

Now,
$$T^2 = 4\pi^2 \, I/c$$

or, $$T^2 = 4\pi^2 I * \frac{2l}{\cancel{\pi} \eta r^4}$$

or, $$\eta = \frac{\cancel{4\pi^2 I * 2l}}{T^2 \cancel{\pi} r^4}$$

$$\boxed{\eta = \frac{8\pi I l}{T^2 r^4}}$$

Q(i) Consider the solⁿ of differential eqⁿ in damped oscillation is represented by $y = ae^{-mt}\sin\omega t$, the eqⁿ for displacement of a point on a damped oscillator is given by $y = 5e^{-0.25t}\sin \pi/2\, t$ metre. Find the velocity of oscillation at $t = T/4$, where 'T' is period of oscillation.

→ Solⁿ: (i) Given,
$$y = ae^{-mt}\sin\omega t$$

given $$y = 5 \cdot e^{-0.25t}\sin \pi/2\, t \quad \text{--- (i)}$$

comparing this eqⁿ (i) with above eqn. we get,
$$y = ae^{-mt}\sin\omega t$$

$$\omega = \pi/2$$

or, $$\frac{2\pi}{T} = \pi/2$$

or, $$T = 4 \text{ sec}$$

Now, $$v = \left.\frac{dy}{dt}\right|_{t = \frac{T}{4} = 1 \text{ sec}}$$

$$\Rightarrow v = \left[5e^{-0.25t} \cdot (\pi/2) \cdot \cos(\pi/2)t + 5(-0.25)e^{-0.25t} \sin(\pi/2) \cdot t\right]_{t=1}$$

or, $v = -5 * 0.25 e^{-0.25*1} \cdot \sin(\pi/2) \cdot 1$

or, $v = -5 * 0.25 * e^{-0.25}$

$\therefore$ $v = -0.97$ m/s

Q① The amplitude of lightly damped oscillator decreases by 3% during each cycle. What fraction of energy of oscillator lost in each cycle.

Soln: Given,

$A = a - 3\%$ of a

or, $A = 0.97a$

Lost in energy $(\Delta E) = \frac{1}{2} m\omega^2 a^2 - \frac{1}{2} m\omega^2 A^2$

$= \frac{1}{2} m\omega^2 (a^2 - A^2)$

$= \frac{1}{2} m\omega^2 [a^2 - (0.97a)^2]$

$= \frac{1}{2} m\omega^2 a^2 [1 - 0.97^2]$

$= 0.059 * \left(\frac{1}{2} m\omega^2 a^2\right)$

Fraction of lost in energy in each cycle,

$= \frac{\Delta E}{\frac{1}{2} m\omega^2 a^2} = \frac{0.059 * \frac{1}{2} m\omega^2 a^2}{\frac{1}{2} m\omega^2 a^2}$

$= 0.059$

Q② Damped oscillator has mass 250 gm, spring constant 85 N/m and damping constant 70 gm/sec.

(i) How long does it take for the amplitude of the damped oscillator to drop to half of its initial value?

(ii) How long does it take for the mechanical energy to drop to half of its initial value?

⇒ Given, sol^n:

$m = 250gm = 0.25kg$

$k = 85 N/m$

$b = 70gm/s$

$= 7 \times 10^{-2} kg/s$

$y = A \sin(\omega t - \phi)$

$A = a e^{-bt/2m}$

(i) $t = ?$ For $A = a/2$

then, $a e^{-bt/2m} = a/2$

or, $e^{\frac{bt}{2m}} = 2$

or, $\frac{bt}{2m} = \ln(2)$

or, $t = \frac{2m \cdot \ln(2)}{b}$

or, $t = \frac{2 * 0.25 * \ln(2)}{7 * 10^{-2}}$

or, $t = 4.95$ sec

(ii) $t = ?$ for $E = \frac{E_0}{2}$

Now, $E = \frac{E_0}{2}$

or, $\cancel{E_0} e^{-bt/m} = \frac{\cancel{E_0}}{2}$

or, $e^{bt/m} = 2$

or, $\frac{bt}{m} = \ln(2)$

or, $t = \frac{m * \ln(2)}{b}$

or, $t = \frac{0.25 * \ln(2)}{7 * 10^{-2}}$

$\therefore t = 2.475$ sec

Note:-

$E \propto A^2$

$E_0 \propto a^2$

$\frac{E}{E_0} = \frac{A^2}{a^2}$

or, $\frac{E}{E_0} = \frac{\cancel{a^2} \cdot e^{-\frac{bt}{2m} \times 2}}{\cancel{a^2}}$

or, $\frac{E}{E_0} = e^{-\frac{bt}{m}}$

or, $E = E_0 e^{-bt/m}$

Q.④ A particle is moving with SHM in a straight line. If it has a speed 'v_1' when the displacement is 'x_1' and speed 'v_2' when the displacement is 'x_2'. Then show that the amplitude of the motion is $\boxed{a = \sqrt{\dfrac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}}}$

⇒ Soln; In SHM.

$$v_1 = \omega\sqrt{a^2 - x_1^2}$$

or, $v_1^2 = \omega^2(a^2 - x_1^2)$ --- (i)

and, $v_2^2 = \omega^2(a^2 - x_2^2)$ --- (ii)

from eqn (ii) ÷ (i)

$$\frac{v_2^2}{v_1^2} = \frac{\cancel{\omega^2}(a^2 - x_2^2)}{\cancel{\omega^2}(a^2 - x_1^2)}$$

or, $a^2 v_2^2 - x_1^2 \cdot v_2^2 - v_1^2 \cdot a^2 + v_1^2 x_2^2 = 0$

or, $a^2(v_2^2 - v_1^2) = v_2^2 \cdot x_1^2 - v_1^2 \cdot x_2^2$

or, $\boxed{a = \sqrt{\dfrac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}}}$ proved.

Q⑤ A uniform circular disc of radius 'R' oscillates in a vertical plane about the horizontal axis, find the distance of rotation from the center(C) for which the time period is minm. Find the value of Time period.

⇒ Given, radius of circuler disc = R

Here,

$$\cancel{m}K^2 = \frac{1}{2}\cancel{m}R^2$$

or, $$K = \frac{R}{\sqrt{2}}$$

Hence, the time period is min^m if 'l' = $R/\sqrt{2}$.

Again,

$$T = 2\pi\sqrt{\frac{\frac{K^2}{l} + l}{g}} \quad (\because K = l)$$

$$= 2\pi\sqrt{\frac{2l}{g}}$$

$$= 2\pi\sqrt{\frac{2 \times R/\sqrt{2}}{g}}$$

$$= 2\pi\sqrt{\frac{\sqrt{2}R}{g}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{1.414R}{g}}}$$

Q⑥ A wire has torsional constant 2 N-m/rad. A disc of radius 5 cm and mass 100gm is suspended to at its center. What is the frequency?

⇒ Given,

$C = 2\ Nm/rad$

$R = 5\ cm = 5 \times 10^{-2}\ m$

$M = 100\ gm = 0.1\ kg$

$f = ?$

$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

$$\because \cancel{f} = 2\pi\sqrt{\frac{I}{C}} \qquad = 2\pi\sqrt{\frac{\frac{1}{2}MR^2}{C}}$$

$$= 2\pi\sqrt{\frac{1 * 0.1 * (5\times10^{-2})^2}{2 * 2}}$$

$$= 0.05$$

Now,

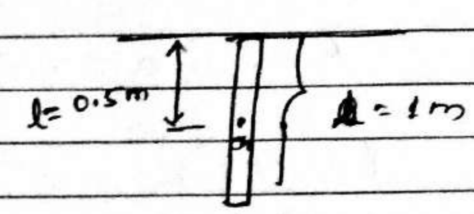
$$f = 1/T = 20 \text{ rad/sec.}$$

Q.⑥ A thin straight uniform rod of length 1 m and mass 100 gm hangs from a ~~point~~ pivot at one end. What is its period for small oscillation? What is the length of a simple pendulum that will have same period?

⇒ Soln: given,

$$m = 100\ gm = 0.1\ kg$$

$$l = 0.5\ m$$



$$T = ?$$

length of simple pendulum equivalent to rod is,

$$L = \frac{K^2}{l} + l$$

Now,

$$\cancel{m}K^2 = \frac{\cancel{m} * 1^2}{3} \quad \left[\because I = \frac{M * (length)^2}{3}\right]$$

or, $K^2 = 1/3$

Hence,

$$L = \frac{1}{3 * 0.5} + 0.5 = 1.167\ m \text{ Ans.}$$

Again,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$= 2\pi\sqrt{\frac{1.167}{10}}$$

$$= 2.146\ m \text{ Ans.}$$

Q⑧ A damped harmonic oscillator consist of a block ($m = 2kg$), a spring ($k = 10\ N/m$) and a damping force $F = -bv$. Initially it oscillates with amplitude 25cm, because of damping, the amplitude falls to $3/4$ of its initial value after the completion of 4 oscillations. What is the value of b? How much energy has lost during 4 oscillations?

⇒ Sol^n,

$m = 2kg$

$k = 10\ N/m$

$a = 25\ cm = 0.25\ m$

$A = \frac{3}{4}a = \frac{3}{4} \times 0.25 = 0.1875\ m$

$t = 4T$

$b = ?$

$\Delta E = ?$

Now, we know that,

$$T = 2\pi\sqrt{m/k} = 2\pi \times \sqrt{2/10} = 2.81\ sec$$

Again

$$A = a e^{-\frac{bt}{2m}}$$

or, $\frac{3}{4}\cancel{a} = \cancel{a}\, e^{-bt/2m}$

or, $\frac{4}{3} = e^{\frac{b \times 4 \times 2.81}{2 \times 2}}$ $[\because t = 4T]$

or, $\ln(4/3) = 2.81 \times b$

$\therefore b = 0.102\ kg/s$

Also,

Also,

$$\Delta E = \frac{1}{2} m\omega^2 a^2 - \frac{1}{2} m\omega^2 A^2$$

$$= \frac{1}{2} m\omega^2 (a^2 - A^2)$$

$$= \frac{1}{2} * m * \left(\frac{2\pi}{T}\right)^2 \cdot (a^2 - A^2)$$

$$= \frac{1}{2} * 2 * \left(\frac{2\pi}{2.81}\right)^2 * (0.25^2 - 0.1875^2)$$

$$= 0.1367 \text{ Joule}$$

Q.(9) A mass of 2kg is suspended from a spring constⁿ 18 N/m. If the undamped frequency is $\frac{2}{\sqrt{3}}$ times the damped frequency what will be the damping factor (i.e. damping constⁿ)?

⇒ Given

$$m = 2 \text{ kg}$$
$$K = 18 \text{ N/m}$$
$$\frac{2}{\sqrt{3}} f = f_0$$

⇒ Here,

$$\frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \frac{2}{\sqrt{3}} \cdot \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \left(\because f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}},\ f = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right)$$

or, Squaring both sides we get,

$$\frac{k}{m} - \frac{b^2}{4m^2} = \frac{4}{3} \times \frac{k}{m}$$

or, $\frac{k}{m} - \frac{4}{3}\frac{k}{m} = \frac{b^2}{4m^2}$

or, −

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}}$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

or, $\frac{2}{\sqrt{3}} \times \frac{1}{2\pi} \sqrt{k/m - b^2/4m^2} = \frac{1}{2\pi} \sqrt{k/m}$

or, $\frac{4}{3} \times \left(k/m - \frac{b^2}{4m^2} \right) = k/m$

or, $\frac{4}{3} k/m - k/m = \frac{4}{3} \times \frac{b^2}{4m^2}$

$= \frac{b^2}{3m^2}$

or, $\frac{1}{3} k/m$

or, $18 \times 2 = b^2$

or, $b = \sqrt{36} = 6$ kg/s

Q. (10)

A mass of 0.01 kg suspended from a spring oscillates with a time period of 1 sec. When it is immersed in oil and allowed to oscillate, the time period increases by 0.2 sec. Calculate the damping constn.

=> Soln,

Given,

$m = 0.01$ kg

$T_0 = 1$ sec

$T = 1 + 0.2 = 1.2$ sec.

$\omega = \sqrt{k/m - \frac{b^2}{4m^2}}$

We know that,

$\omega = \sqrt{k/m - \frac{b^2}{4m^2}}$

or, $\frac{2\pi}{T} = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}}$

on squaring,

$$\frac{4\pi^2}{T^2} = \left(\frac{2\pi}{T_0}\right)^2 - \frac{b^2}{4m^2}$$

or, $$\frac{4\pi^2}{T^2} = \frac{4\pi^2}{T_0^2} - \frac{b^2}{4m^2}$$

or, $$b^2 = 4m^2\left(\frac{4\pi^2}{T_0^2} - \frac{4\pi^2}{T^2}\right)$$

or, $$b^2 = 4\times(0.01)^2 \cdot \left(\frac{4\cdot\pi^2}{(1)^2} - \frac{4\pi^2}{(1.2)^2}\right)$$

$\therefore$ $b = 0.069$ kg/s.

Q. (11) A simple pendulum of length 20 cm and mass 5 gm is suspended in a race car travelling with 70 m/s around a circle of radius 50 m. If the pendulum undergoes in a small oscillation in a radial direction, what is the frequency of oscillation?

$\Rightarrow$ Soln, Given,

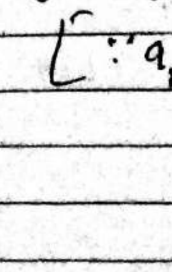
$l = 20$ cm $= 0.2$ m

$m = 5$ gm $= 5\times10^{-3}$ kg

$v = 70$ m/s

$r = 50$ m

$f = \frac{1}{2\pi}\sqrt{g/l}$



$\Rightarrow g' = \sqrt{g^2 + (a_r)^2}$

or, $g' = \sqrt{10^2 + \left(\frac{v^2}{r}\right)^2}$ $\left[\because a_r = \frac{v^2}{r}\right]$

or, $g' = \sqrt{100 + (70^2/50)^2}$

or, $g' = 92.50$ m/s^2

Now,

$f = \frac{1}{2\pi}\sqrt{g'/l} = 3.53$ Hz

Q. A mass of 2kg hung on a spring of force const^n $K = 2\times10^4$ N/m. The system subjected to vibrating force described by a force $F = 3\cos\omega t$. A static force of 3N causes a deflection of 0.15 mm. Find the amplitude of vibration for $\omega = 50$ rad/s & $b = 10$ kg/s.

⇒ Sol^n, Given,

$$y_m = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}}$$

$\because f = F/m$

$\omega_0^2 = K/m$

$\delta = b/2m$

$\Rightarrow \because f = \frac{F}{m}$

$\therefore f = \frac{3}{2}$

$\Rightarrow \omega_0^2 = \frac{2\times10^4}{2} = 10^4$

$\Rightarrow \delta = \frac{b}{2m} = \frac{10}{2*2} = 2.5$

$\Rightarrow \omega = 50$ rad/s

Now,

$$y_m = \frac{f}{\sqrt{(\omega_0^2 - \omega^2) + (2\delta\omega)^2}}$$

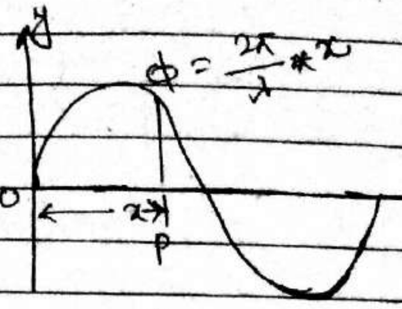
$$= \frac{3/2}{\sqrt{[(10^4) - (50)^2]^2 + (2*2.5*50)^2}}$$

$= 2\times10^{-4}$ m

Wave Motion :-

Progressive Wave :-

→ The wave in which the disturbance continuously transferred in a certain direction is called progressive ~~motion~~. wave.



Consider a progressive wave propagated along positive x-axis. The particles of the medium vibrate simple harmonically so the displacement at point 'O' is,

$$y = a \sin \omega t$$

where,

$a \Rightarrow$ amplitude of vibration

$\omega \Rightarrow$ Angular frequency.

Let, the wave is at point 'P' at a distance 'x' from 'O'. The displacement of particle at 'P' is,

$$y = a \sin(\omega t - \phi)$$

where,

$\phi \rightarrow$ phase difference betn points 'O' and 'P'.

We know that,

For path difference 'λ' the phase difference is '2π'.

" " " 'x', the phase difference is, $\frac{2\pi}{\lambda} * x$.

$$\Rightarrow \phi = \frac{2\pi}{\lambda} * x$$

Now,

$$y = a \sin\left(\omega t - \frac{2\pi}{\lambda} * x\right)$$

or $y = a \sin(\omega t - kx)$ - - - (i) ; where, $k = \frac{2\pi}{\lambda}$ is called wave number.

or, $y = a \sin\left(\frac{2\pi}{T} \times t - \frac{2\pi}{\lambda} x\right)$

or, $y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ ---- (ii)

or, $y = a \sin 2\pi\left(t \times f - \frac{x}{\lambda}\right)$ $\left[\because f = \frac{1}{T}\right]$

or, $y = a \sin 2\pi\left(t \cdot \frac{v}{\lambda} - \frac{x}{\lambda}\right)$ $[\because v = f \times \lambda]$

or, $y = a \sin \frac{2\pi}{\lambda}\left(vt - x\right)$ ---- (iii)

Eq^n (i), (ii) & (iii) are different forms of eq^n of progressive wave.

Wave velocity, particle velocity and particle acceleration:-

We know that, the eq^n of progressive wave is,

$$y = a \sin(\omega t - kx) \text{ ---- (i)}$$

since,

the wave travels in positive x-axis. So, the wave velocity is the rate of change of distance along x-axis.

or, wave velocity $(u) = \frac{dx}{dt}$

Here,

$(\omega t - kx) = \text{constant}$.

differentiating both sides w.r.t. t,

or,

$$\omega - k \cdot \frac{dx}{dt} = 0$$

or, $\frac{dx}{dt} = \frac{\omega}{k}$

or, $\boxed{u = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f \times \lambda}$

Again,
since, the particle vibrates along y-axis. so, the particle velocity is the rate of change of distance along y-axis.

i.e. particle velocity $(v) = \frac{dy}{dt}$

or, $v = \omega a \cos(\omega t - kx)$ --- (ii)

diff eqn (i) w.r.to x,

or, $\frac{dy}{dx} = -ka\cos(\omega t - kx)$ ---- (iii)

dividing eqn (ii) by (iii);

$$\frac{v}{\frac{dy}{dx}} = \frac{\omega a \cos(\omega t - kx)}{-ka\cos(\omega t - kx)}$$

or, $v = \frac{-\omega}{k} \times \frac{dy}{dx}$

or $\boxed{v = -u \times \frac{dy}{dx}}$

$\Rightarrow$ $\boxed{\text{(particle velocity at a point)} = -\text{(wave velocity)} \times \text{(slope of displacement curve at that point)}}$

Now, particle acceleration is given by,

$$A = \frac{d^2y}{dt^2} = -\omega^2 a \sin(\omega t - kx)$$

or, $A = -\omega^2 y$ --- (iv)

taking 2nd order diff. in eqn (i) w.r.to x;

$$\frac{d^2y}{dx^2} = -k^2[a\sin(\omega t - kx)]$$

or $\frac{d^2y}{dx^2} = -k^2y$ ---- (v)

dividing eqn (iv) by (v),

$$\frac{A}{\frac{d^2y}{dx^2}} = \frac{-\omega^2 y}{-k^2 y}$$

or, $A = \frac{\omega^2}{k^2} \times \frac{d^2y}{dx^2}$

or, $\boxed{A = u^2 \cdot \frac{d^2y}{dx^2}}$

⇒ (particle acceln at a point) = (wave velocity)^2 × (curvature of displacement curve at that point)

(4) Energy of progressive wave :

⇒ We know that, the eqn of progressive wave is given by,

$$y = a \sin(\omega t - kx) \quad --\ (I)$$

where,

$a \Rightarrow$ amplitude
$\omega \Rightarrow$ angular frequency $\left(\omega = \frac{2\pi}{T}\right)$
$k \Rightarrow$ wave number $\left(k = \frac{2\pi}{\lambda}\right)$

Here,

$$v = \frac{dy}{dt} = \omega a \cos(\omega t - kx)$$

& $A = \frac{d^2y}{dt^2} = -\omega^2 a \sin(\omega t - kx) = -\omega^2 y$

Now,
the K.E. of the wave is,

$$K.E. = \frac{1}{2} m v^2$$

or $K.E. = \frac{1}{2} m \omega^2 a^2 \cos^2(\omega t - kx)$

The K.E. per unit volume is given by,

$$E_K = \frac{K.E.}{V} = \frac{1}{2} \frac{m}{V} \omega^2 a^2 \cos^2(\omega t - kx)$$

or, $E_K = \frac{1}{2} \rho \omega^2 a^2 \cos^2(\omega t - kx)$ ---- (ii)

Where,
$\rho \to$ density of med^m. $(\rho = \frac{m}{V})$

Again,
According to Newton's 2nd law of motion.

$$F = mA$$

or, $F = -m\omega^2 y$

The small work done for small displacement. dy is,

$$dW = -F \times dy$$

or, $dW = m\omega^2 y \, dy$

The total work done for displacement 'y' is,

$$W = \int_0^y dW$$

or, $W = m\omega^2 \int_0^y y \, dy$

or $W = m\omega^2 \frac{y^2}{2}$

This work done is equal to the potential energy,

$$P.E. = \frac{1}{2} m\omega^2 y^2$$

The P.E per unit volume is,

$$E_p = \frac{P.E}{V} = \frac{1}{2}\frac{m}{V}\omega^2 y^2$$

or, $E_p = \frac{1}{2}\rho\omega^2 a^2 \sin^2(\omega t - kx)$ -- (iii) $[\because y = a\sin(\omega t - kx)]$

The total energy of progressive wave per unit volume is,

$$E = E_p + E_k$$

or, $E = \frac{1}{2}\rho\omega^2 a^2 \sin^2(\omega t - kx) + \frac{1}{2}\rho\omega^2 a^2 \cos^2(\omega t - kx)$

or, $E = \frac{1}{2}\rho\omega^2 a^2[\sin^2(\omega t - kx) + \cos^2(\omega t - kx)]$

or, $E = \frac{1}{2}\rho\omega^2 a^2$

or, $E = \frac{1}{2}\rho(2\pi f)^2 \cdot a^2$

or, $\boxed{E = 2\pi^2\rho \cdot f^2 \cdot a^2}$

This is the reqd expression.

Again, the Intensity of progressive wave is,

$$I = \frac{\text{Energy}}{A \times t}$$

or $I = \frac{\text{Energy} \times l}{V \times t}$

or, $I = \left(\frac{\text{Energy}}{V}\right) \times \frac{l}{t}$

or, $I = E \times v$

$\Rightarrow \boxed{I = 2\pi^2\rho \cdot f^2 a^2 v}$

Interference of wave:-

⇒ The process of superposition of two waves of same frequency and wavelength propagating in a medium in same direction is called interference of wave.

Consider two waves of same frequency 'f' and same wavelength 'λ' are propagating in a medium in same direction. The displacements of these waves are,

$$y_1 = a_1 \sin(\omega t - kx)$$

& $$y_2 = a_2 \sin(\omega t - kx)$$

The resultant displacement is given by,

$$y = y_1 + y_2$$

or $$y = a_1 \sin(\omega t - kx) + a_2 \sin(\omega t - kx)$$

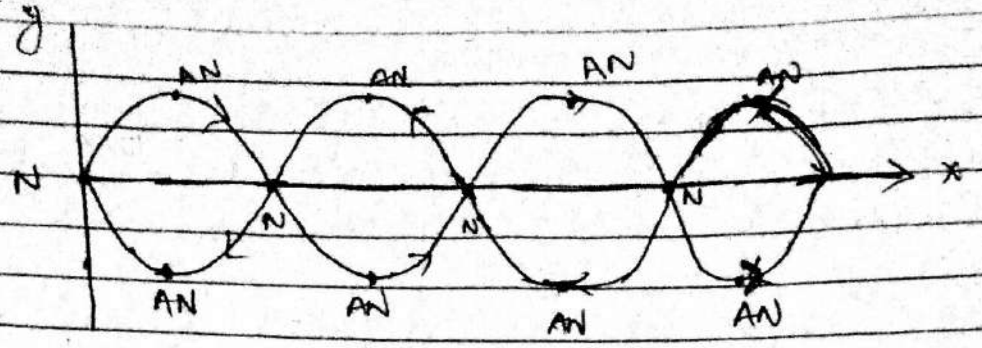
or $$y = (a_1 + a_2) \sin(\omega t - kx)$$

$$\Rightarrow \boxed{y = A \sin(\omega t - kx)}$$

Stationary wave:

⇒ When two progressive waves of same amplitude, frequency and wavelength propagating in a medium with same velocity but in opposite direction superimpose to each other, they give rise to a another wave called stationary wave.

- Generally, the stationary wave is formed by the superposition of incident and reflected waves.



Consider two wave of same amplitude (a), frequency (f) and wavelength (λ) are propagating in a medium with same velocity but in opposite direction. Their displacement are given by,

$$y_1 = a \sin(\omega t - kx) \quad \text{--- (i)}$$

$$y_2 = a \sin(\omega t + kx) \quad \text{--- (ii)}$$

When they superimpose to each other, the resultant displacement is given by,

$$y = y_1 + y_2$$

or, $y = a \sin(\omega t + kx) + a \sin(\omega t - kx)$

or, $y = a\left[\sin(\omega t + kx) + \sin(\omega t - kx)\right]$

or, $y = a\left[2 \cdot \sin\left(\frac{\omega t + kx + \omega t - kx}{2}\right) \cdot \cos\left(\frac{\omega t + kx - \omega t + kx}{2}\right)\right]$

or, $y = 2a \sin\omega t \cdot \cos kx$

or, $\boxed{y = 2a \cos kx \cdot \sin\omega t}$

or, $\boxed{y = A \sin\omega t}$

where, $A = 2a\cos kx$ is the amplitude of stationary wave.

For Antinode:

'A' is max^m.

Hence, $\cos kx = \pm 1$

or, $\cos kx = \cos n\pi \quad [n = 0, 1, 2, 3, ---]$

or, $\frac{2\pi}{\lambda} \times x = n\pi$

or, $x = \frac{n\lambda}{2}$

for, $n=0, \; x_0 = 0$

$n=1, \; x_1 = \lambda/2$

$n=2, \; x_2 = \frac{2\lambda}{2}$ and so, on

For Node :-

$\cos kx = 0$

or $\cos kx = \cos \left(\frac{2n+1}{2}\right)\pi \quad [n = 0, 1, 2, 3, ---]$

or, $\frac{2\cancel{\pi}}{\lambda} x = \left(\frac{2n+1}{2}\right)\cancel{\pi}$

or, $x = (2n+1)\frac{\lambda}{4}$

for,

$n=0, \; x_0 = \lambda/4$

$n=1, \; x_1 = \frac{3\lambda}{4}$

$n=2, \; x_2 = \frac{5\lambda}{4}$ and so on.

Q.①. A simple harmonic wave in a gas in the +ve x-axis. Its amplitude is 2cm, velocity 45 m/s and frequency is 75 per sec. Find out the displacement of the particle of the medium at a distance of 135 cm from the origin in the direction of wave at time t = 3 sec.

⇒ Soln

$y = a \sin(\omega t - kx)$ [∵ it is in +ve x-axis]

Given,

$a = 2\ cm = 0.02\ m$

$f = 75$ per sec.

$x = 135\ cm = 1.35\ m$

$t = 3\ sec$

$v = 45\ m/s$

$\left[\because v = \frac{\omega}{k}\right]$

Here,

$\omega = 2\pi f = 2\pi * 75 = 471.23\ rad/s$

& $k = \frac{\omega}{v} = \frac{471.23}{45} = 10.47\ m^{-1}$

Now, $y = a \sin(\omega t - kx)$

$= 0.02 \sin(471.23 * 3 - 10.47 * 1.35)$

$= -0.0128\ m$

Q② The eqn of stationary wave is given by $y = 12 \cos\left(\frac{\pi}{5}\right) x \sin \omega t$ where 'x' and 'y' are in cm and 't' is in second. Find frequency, wavelength, velocity and amplitude of progressive wave.

⇒ Given

$y = 12 \cos\left(\frac{\pi}{5}\right) \cdot x \cdot \sin \omega t$

comparing this eqn. with,

$y = 2a \cos kx \cdot \sin \omega t$; we get,

(i) $a = 6\,cm$

$K = \pi/5$

& $\omega = 20\pi$

(ii) $f = \frac{\omega}{2\pi}$

$= \frac{20\pi}{2\pi}$

$= 10$ rad/s

$= 10\ H_z$

Here,

(iii) $\frac{2\pi}{\lambda} = K = \pi/5$

or $\lambda = 2 \times 5 = 10$ cm

Again, (iv) $v = f \times \lambda = 10 \times 10 = 100\ cm/s.$

Q(3) A certain transverse wave is describe by $y = 0.65cm \cos 2\pi\left(\frac{x}{28cm} - \frac{t}{0.0365s}\right)$ determining amplitude, wavelength, frequency, speed and direction of propagation.

⇒ Soln, Given eqn,

$$y = 0.65\,cm\ \cos 2\pi\left(\frac{x}{28\,cm} - \frac{t}{0.0365\,sec}\right)$$

comparing this eqn with,

~~$y = 0.65\,cm\ \cos 2\pi\left(\frac{x}{28}\right)$~~

$$y = a.\cos 2\pi\left(x/\lambda - \frac{t}{T}\right)$$; we get,

then,

$a = 0.65$ cm

$\lambda = 28$ cm

$T = 0.0365$ sec

$$v = \frac{\lambda}{T} = \frac{28}{0.0365} = 767.123 \text{ cm/s}$$

& $$f = \frac{1}{T} = \frac{1}{0.0365} = 27.397 \text{ Hz}$$

Again,

$$y = 0.65 \cos 2\pi \left\{ \left(\frac{t}{0.0365} - \frac{x}{28} \right) \right\}$$

$$= 0.65 \cos 2\pi \left(\frac{t}{0.0365} - x/28 \right)$$

Hence, the wave propagates along positive x-axis.

Q(4) Find the intensity of a progressive wave in a fluid of density 1.3 kg/m3. The displacement of the wave is given by $y = 10^{-5} \cos(7000t - 20x)$ where x & y are in meter and 't' is in seconds.

⇒ Sol^n Given eq^n.

$$y = 10^{-5} \cos(7000t - 20x)$$

comparing this eq^n with.

$$y = a \cos(\omega t - kx)$$

So,

$a = 10^{-5}$ m

$\omega = 7000$ rad/s

$K = 20$

$\because$ $\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{7000}{2\times\pi} = 1114.08$

$K = \frac{\omega}{v} \Rightarrow v = \omega/k = \frac{7000}{20} = 350$

Now,

$$\text{Intensity } (I) = 2\pi^2 \rho \cdot f^2 a^2 v$$ or [illegible]

$$= 2 * \pi^2 * 1.3 * (10^{-5})^2 * 350 * (114.08)^2$$

$$= 1.115 \text{ W/m}^2 \quad (\text{watt/m}^2)$$

Q(5) A source of sound has a frequency of 512 Hz and amplitude 0.25 cm. What is the intensity of sound if velocity of sound in air is 340 m/s & density of air is 0.00129 g/cm³

⇒ Given

frequency of sound $(f) = 512$ Hz

Amplitude $(a) = 0.25$ cm $= 2.5 \times 10^{-3}$ m

velocity of sound $(v) = 340$ m/s

density of air $(\rho) = 0.00129$ g/cm³

$$= \frac{0.00129 * 10^6}{10^3} = 1.29 \text{ kg/m}^3$$

Intensity of sound $(I) = ?$

∵ We know that,

$$I = 2\pi^2 \rho \cdot f^2 a^2 v$$

$$= 2\pi^2 * (1.29) * (512)^2 * (2.5 * 10^{-3})^2 * 340$$

$$= 14184.64 \text{ watt/m}^2$$

Q.6 A wave travelling along a string is described by $y = 0.00327 \sin (72.1x - 2.72t)$ in which numerical constants are in SI unit. Find amplitude, wavelength, frequency and ~~wavelength~~ velocity of wave. What is the displacement at $x = 22.5$ cm and $t = 18.9$ sec.

$\{y = a \sin (kx - \omega t)\}$

Q.7 A stationary wave is represented by $y = 0.2 \cos (2\pi x) \sin (\ldots t)$ where, x, y & t are in SI unit. Calculate the distance betn two successive antinodes.

Q.8 A sinusoidal wave of frequency 500 Hz has a speed of 350 m/s. How far two points that differ, Its displacement betn two points at a time are in 1 millisecond apart?

⇒ Soln, Given,

frequency $(f) = 500$ Hz

velocity $(v) = 350$ m/s

Here,

Time period $(T) = \dfrac{1}{f} = \dfrac{1}{500} = 2 \times 10^{-3}$ sec

wavelength $(\lambda) = \dfrac{v}{f} = \dfrac{350}{500} = 0.7$ m

Now, for time difference 'T' the path diff is 'λ'.

" " " (1×10^{-3}) sec, the path diff is,

$= \dfrac{\lambda}{T} \times 1 \times 10^{-3}$

$= \dfrac{0.7 \times 1 \times 10^{-3}}{2 \times 10^{-3}}$

$= 0.35$ M.

Q.6

Sol^n: Given eq^n is,

$$y = 0.00327 \sin(72.1x - 2.72t)$$

~~Comparing this eq^n~~

$$y = 0.00327 \sin[-(2.72t - 72.1x)]$$
$$= -0.00327 \sin(2.72t - 72.1x)$$

Now, compairing this eq^n with,

$$y = a \sin(\omega t - kx)$$

then,

(i) $a = 0.00327$ m

(iii) $\omega = 2.72$ rad/s

$k = 72.1$

(ii) ~~Wavelength (λ)~~

$\because \frac{2\pi}{\lambda} = k \Rightarrow \lambda = \frac{2\pi}{k}$

Now,

$\Rightarrow \frac{2\pi}{72.1} \Rightarrow 0.087$ m

Now, velocity (v) = $f * \lambda$

$$= \frac{\omega}{2\pi} * \lambda$$

$\left[\because \omega = 2\pi f \text{ or, } f = \frac{\omega}{2\pi}\right]$

$$= \frac{2.72}{2\pi} * 0.087$$

$$= 0.0376 \text{ m/s}$$

Now, for displacement at $x = 22.5$ cm $\Rightarrow 0.225$ m and $t = 18.9$ sec

~~$\Rightarrow y = 0.00327 \sin(72.1x - 2.72t)$~~

~~$= 0.00327 \sin(72.1 * 0.225 - 2.72 * 18.9)$~~

$\Rightarrow y = -0.00327 \sin(2.72t - 72.1x)$

$= -0.00327 \sin(2.72 * 18.9 - 72.1 * 0.225)$

~~$= 3.255 * 10^{-3}$ m~~ $\{-1.884 * 10^{-3}$ m

~~$y = 3.255 * 10^{-3}$ m~~

$\boxed{\Rightarrow y = 1.884 * 10^{-3} \text{ m}}$

Q. 7

Soln Given eqn is

$$y = 0.2 \cos(2\pi x) \cdot \sin(4\pi t)$$

Comparing this eqn with;

$$y = 2a \cos kx \cdot \sin \omega t$$

$\therefore$ $2a = 0.2$

$\Rightarrow a = \frac{0.2}{2} = 0.1\ m$

$\Rightarrow k = 2\pi$

$\Rightarrow \omega = 4\pi$

$\because k = \frac{2\pi}{\lambda}$

or, $\cancel{2\pi} = \frac{\cancel{2\pi}}{\lambda}$

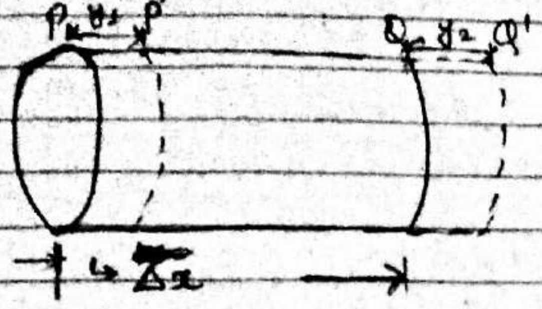
$\therefore \lambda = 1\ m$

$\therefore$ distance betn two successive antinodes $= \frac{\lambda}{2}$

$= \frac{1}{2} = 0.5\ m$

Unit - 3 Acoustics

(ii) Pressure Amplitude:-



Since the soundwave is a progressive wave, the displacement is given by

$$y = a \sin(\omega t - kx)$$

where,
- $a \to$ Amplitude
- $\omega \to$ Angular frequency
- $k \to$ wave number

When the sound wave travels in a medium the value of atmospheric pressure fluctuate above and below its normal value. The maximum change in pressure due to the propagation of sound wave in the medium is called pressure amplitude.

Imagine a cylinder of medium of cross-sectional area A and length 'Δx' along the direction of propagation of wave. Here its initial volume '$A\Delta x$.'

When the sound wave travels the end 'P' of the cylinder displaces by distance 'y_1' and the end 'Q' displaces by distance 'y_2'. Now, the volume of disturbed cylinder is,

$$A(\Delta x + y_2 - y_1) = A(\Delta x + \Delta y)$$

The change in volume is $(\Delta V) = A(\Delta x + \Delta y) - A\Delta x$

or, $\boxed{\Delta V = A\Delta y}$.

Here, If $\Delta y > 0$, the pressure decreases,
and, If $\Delta y < 0$, the pressure increases,

The fractional change in volume is,

$$\frac{dv}{v} = \lim_{\Delta V \to 0} \frac{\Delta V}{V} = \lim_{\Delta x \to 0} \frac{\cancel{A}\,\Delta y}{\cancel{A}\,\Delta x} = \frac{dy}{dx} \qquad [\because \Delta V = A\Delta y \ \& \ V = A\Delta x]$$

or, $$\frac{dv}{v} = -ka\cos(\omega t - kx) \qquad \left[\because \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}\right]$$

We know that, the bulk modulus of elasticity is,

$$B = \frac{-p}{\frac{dv}{v}}$$; Where, 'p' is change in pressure.

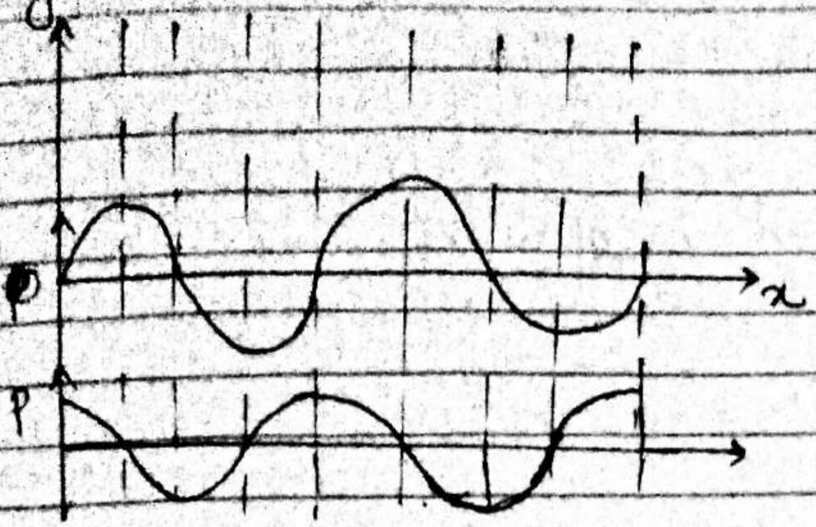
or, $$p = -B \cdot \frac{dv}{v}$$

or, $$p = -B\left[-ka\cos(\omega t - kx)\right]$$

or, $$p = Bak\cos(\omega t - kx)$$

or, $$\boxed{p = P_{max}\cos(\omega t - kx)};$$

This is the pressure eqn of sound wave.
Where,
$P_{max} = Bak$, is known as pressure amplitude.



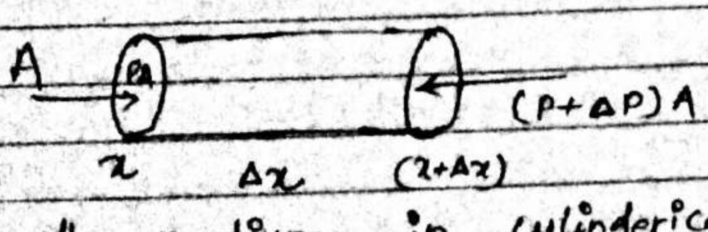
(ii) Velocity of sound in material medium:-

⇒ Consider a sound wave travel in material medium (fluid) in forward direction. The displacement eqn is given by,

$$y = a\sin(\omega t - kx) \quad \text{---} \; (i)$$

~~And~~,
Also pressure wave is given by,

$$P = Bak\cos(\omega t - kx) \quad \text{---} \; (ii)$$



→ Consider the medium in cylinderical region of cross-sectional area 'A' and the length 'Δx'. The medium is between 'x' and 'x+Δx'. The force at 'x' is 'PA' and the force at (x+Δx) is (P+ΔP)A.

the net force act on the cylinder is,

$$\Delta F = (P+\Delta P)A - PA$$

or $$\Delta F = \Delta P A \quad \text{---} \; (iii)$$

The mass of medium in the cylinder is '$\rho A \Delta x$', where 'ρ' is density of the medm.

Now the acceleration is given by,

$$a_c = -\frac{\Delta F}{m}$$

or, $$a_c = -\frac{\Delta P A}{\rho A \Delta x}$$

or $$a_c = -\frac{1}{\rho} \cdot \frac{\Delta P}{\Delta x}$$

or $$a_c = -\frac{1}{\rho} \cdot \frac{dP}{dx}$$

or, $$a_c = -\frac{1}{\rho} B a k^2 \sin(\omega t - kx) \cdots \text{(iv)}$$

Again the acceleration is given by,

$$a_c = \frac{d^2y}{dt^2} = -a\omega^2 \sin(\omega t - kx) \cdots \text{(v)}$$

Equating eqn (iv) and (v); we get,

$$\Rightarrow -\frac{1}{\rho} B a k^2 \sin(\omega t - kx) = -a\omega^2 \sin(\omega t - kx)$$

or, $$\frac{Bk^2}{\rho} = \omega^2$$

or, $$\frac{B}{\rho} = \frac{\omega^2}{k^2}$$

or, $$v^2 = \frac{B}{\rho}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{B}{\rho}}}$$

This is the expression of velocity of sound in medium (fluid).

In solid the velocity of sound is given by,

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

Where, $Y \longrightarrow$ Young's Modulus of Elasticity.

(#) Newton's formula for velocity of sound in air :-

⇒ Newton assume that, when the sound wave travel in a medium, then the compressions and rarefactions are formed. During the formation of compressions and rarefactions the temperature of the medium remains constant. i.e. The process is isothermal process.

We know that, in isothermal process,

$PV =$ Constant

differentiating both sides we get,

$$PdV + VdP = 0$$

or, $PdV = -VdP$

or, $P = \frac{-VdP}{dV}$

or, $P = \frac{-dP}{\frac{dV}{V}}$

or, $P = B$ (where, $B = -\frac{dP}{\frac{dV}{V}}$ is bulk modulus of elasticity)

Also, we know that the velocity of sound in the gas is,

$$V = \sqrt{\frac{B}{\rho}} \quad \text{where, } \rho \rightarrow \text{density of medium.}$$

or $$v = \sqrt{\frac{P}{\rho}}$$

At NTP, $P = 1.01 \times 10^5$ Pa

$\rho = 1.293$ kg/m³

Now, we get

$v = 280$ m/s

But, the experimental value of velocity of sound in air at NTP is 332 m/s. So, this difference in value of velocity of sound in air is not due to experimental error. Hence, assumption of Newton is wrong.

Laplace correction:-

Laplace said that the assumption made by Newton is wrong i.e. when the sound wave travels in a gas compression and rarefactions are formed and there is no temp^r constant but the heat is constant. The process is adiabatic process.

We know that,

In adiabatic process,

$PV^{\gamma} = \text{const}^n$. where $\gamma \rightarrow$ The ratio of molar heat capacities of a gas at const^n pressure and volume.

differentiating both sides; we get

$$P \cdot \gamma V^{\gamma-1} \cdot dV + V^{\gamma} \cdot dP = 0$$

or, $p = -\dfrac{dp}{\gamma \cdot \frac{dV}{V}}$

or, $p = \dfrac{B}{\gamma}$ $\left[\text{where, } B = -\dfrac{dp}{\frac{dV}{V}} \right)$

or, $\gamma p = B$

Now, the velocity of sound is given by,

$v = \sqrt{B/\rho}$

or $\boxed{v = \sqrt{\dfrac{\gamma p}{\rho}}}$

At NTP,

$\gamma = 1.4$ [∵ Gas is diatomic]

$p = 1.01 \times 10^5$ Pa

$\rho = 1.293$ kg/m³

We get,

$v = 330.69$ m/s

Which is close to the experimental value, so the formula derive by laplace ($v = \sqrt{\gamma p/\rho}$) is correct for velocity of sound in gas.

* Beat :-

When two sound waves having same amplitude, but slightly different frequency propagating in same direction superimpose to each other, then the amplitude (Intensity) of sound changes periodically, this phenomenon is called Beat.

The Number of beat per second is called beat frequency.

Consider two sound waves having same amplitude but slightly different frequencies f_1 and f_2 are propagating along the same direction in a medium. Their displacements at $x=0$, are,

$$y_1 = a \sin \omega_1 t$$

and,
$$y_2 = a \sin \omega_2 t$$

When they superimpose to each other, the resultant displacement is given by,

$$y = y_1 + y_2$$

or, $y = a \sin \omega_1 t + a \sin \omega_2 t$

or, $y = a (\sin \omega_1 t + \sin \omega_2 t)$

or, $y = 2a \sin\left(\frac{\omega_1 t + \omega_2 t}{2}\right) \cdot \cos\left(\frac{\omega_1 t - \omega_2 t}{2}\right)$

or, $\boxed{y = A \sin\left(\frac{\omega_1 + \omega_2}{2}\right)t}$ → is the eqn of resultant wave

Where, $\boxed{A = 2a \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t}$ → is the amplitude of resultant wave.

(a) for max^m Amplitude:-

$$\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t = \pm 1$$

or, $\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t = \cos n\pi$, where, $n = 0, 1, 2, 3 \ldots$

or, $\frac{2\pi(f_1 - f_2)}{2} \times t = n\pi$

or, $\boxed{t = \frac{n}{f_1 - f_2}}$

Here,

$n=0, \quad t_0 = 0.$

$n=1, \quad t_1 = \frac{1}{f_1 - f_2}$

$n=2, \quad t_2 = \frac{2}{f_1 - f_2}$ & so on.

Now,

Beat period $(T) = t_2 - t_1$

$$\text{or, } T = \frac{2}{f_1 - f_2} - \frac{1}{f_1 - f_2}$$

$$\text{or, } T = \frac{1}{f_1 - f_2}$$

Again,

The beat frequency is,

$$f = \frac{1}{T} = f_1 - f_2$$

$$\Rightarrow \boxed{f = |f_1 - f_2|}$$

(b) For minimum amplitude;

$$\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t = 0$$

$$\text{or, } \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t = \cos(2n+1)\pi/2 \text{ ; Where, } n = 0, 1, 2, 3, ---$$

$$\text{or, } \cancel{2}\pi(f_1 - f_2) \times t = (2n+1)\pi/\cancel{2}$$

$$\text{or, } \cancel{\frac{2(f_1 - f_2)}{2n+1} = t} \Rightarrow \boxed{t = \frac{2n+1}{2(f_1 - f_2)}}$$

Here,

$n=0, \; t_0 = \frac{1}{2(f_1-f_2)}$

$n=1, \; t_1 = \frac{3}{2(f_1-f_2)}$

$n=2, \; t_2 = \frac{5}{2(f_1-f_2)}$ and so on.

Now,

~~Beat~~ period $(T) = T_2 - T_1$

or $T = \frac{5}{2(f_1-f_2)} - \frac{3}{2(f_1-f_2)}$

$= \frac{\cancel{2}}{\cancel{2}(f_1-f_2)} = \frac{1}{f_1-f_2}$

$\therefore \boxed{T = \frac{1}{f_1-f_2}}$

Again,

The Beat frequency is,

$f = \frac{1}{T} = |f_1-f_2|$

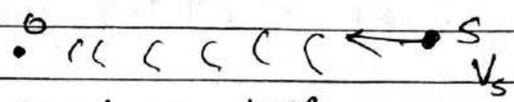
$\therefore \boxed{f = |f_1-f_2|}$ //

Doppler's Effect:-

⇒ The apparent change in frequency (pitch) of the sound due to the relative motion between source of sound and observer is called doppler's Effect.

(a) When the source is in motion and the observer is at rest

(i) The source moving towards the ~~same~~ stationary observer.



Consider a sound producing source is moving towards the stationary observer with velocity V_s. Let, f, λ and ~~velocity~~ are frequency, wavelength and speed of sound wave.

When the sources moves towards the observer the distance between ~~source~~ observer and source decreased by V_s in each second so the sound wave is compressed.

Now, the apparent wavelength is,

$$\lambda' = \frac{V - V_s}{f}$$

Again, the apparent frequency is,

$$f' = \frac{V}{\lambda'}$$

or, $$\boxed{f' = \left(\frac{V}{V - V_s}\right) * f}$$

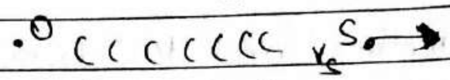
or, $\cancel{f'} =$

since, $\left(\frac{V}{V - V_s}\right) > 1$, So, $f' > f$.

Hence, the apparent frequency of sound increases when the source moves towards the stationary observer.

~~When the source is~~

(ii) The source moving away from the stationary observer.



Consider a sound producing source is moving away from the stationary observer with velocity V_s. Let, f, λ and v are frequency, wavelength and speed of sound wave.

When the source moves away from the observer the distance between observer and source increases by V_s in each second so the sound wave expand.

Now, the apperent wavelength is,

$$\lambda' = \frac{V + V_s}{f}$$

Again, the apperent frequency is,

$$f' = \frac{f}{\lambda'}$$

or, $$f' = \frac{\frac{V}{V+V_s}}{f}$$

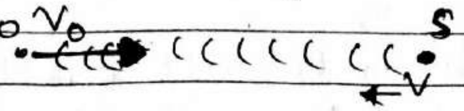
or, $$\boxed{f' = \left(\frac{V}{V+V_s}\right) \times f}$$

since, $\left(\frac{V}{V+V_s}\right) < 1$, $\therefore f' < f$,

Hence the apperent frequency of sound decreases when the source moves away from the stationary observer.

(b) When the source is at rest and observer is in motion.

(i) The observer is moving towards the stationary source.



Consider a source of sound produces the sound wave of frequency (f), Wavelength (λ) and velocity (v) which is at rest. The observer is moving towards the source with velocity V_o.

When the observer moves towards the source the relative velocity of sound increases which is given by

$$V_r = V + V_o$$

Now, the apparent frequency is given by,

$$f' = \frac{V_r}{\lambda}$$

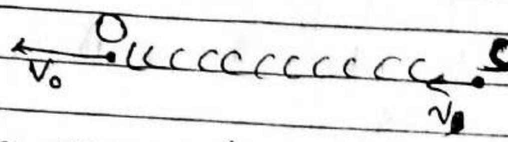
or $$f' = \frac{V + V_o}{V/f} \qquad \left[\because V = f \times \lambda \text{ or, } \lambda = V/f\right]$$

$$\therefore \boxed{f' = \left(\frac{V + V_o}{V}\right) f}$$

since, $\frac{V + V_o}{V} > 1, \quad \therefore f' > f$

Hence, the apparent frequency of sound increases when the observer moves towards the stationary source.

(ii) The observer moves away from the stationary source.



Consider a source of sound produces the sound wave of frequency (f), wavelength (λ), and velocity (V) which is at rest. The observer is moving away from the source with velocity v_o.

When the observer moves away from the source, the relative velocity of sound ~~is~~ decreases which is given by,

$$V_r = V - V_o$$

Now the apparent frequency is given by,

$$f' = \frac{V_r}{\lambda}$$

$$\text{or, } f' = \frac{V - V_o}{V/f}$$

$$\text{or, } \boxed{f' = \left(\frac{V - V_o}{V}\right) \times f}$$

since,

$$\frac{V - V_o}{V} < 1 \; ; \; f' < f,$$

Hence, The apparent frequency of sound is decreased when the observer moves away from the stationary source.

(C) When both source and observer are in motion:-

When both source and observer ~~both~~ are in motion then the relative velocity of sound as well as the wavelength of sound both changes.

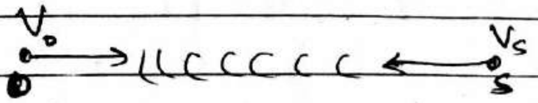
The apparent frequency is ;

$$f' = \frac{V_r}{\lambda'} \quad \text{--- (i)}$$

Where, $V_r \rightarrow$ relative velocity of sound w.r.to observer.

$\lambda' \Rightarrow$ apparent wavelength of sound.

(i) Both source and observer moving towards each other:-



Here,

$$V_r = V + V_o$$

and $$\lambda' = \frac{V - V_s}{f}$$

Now,

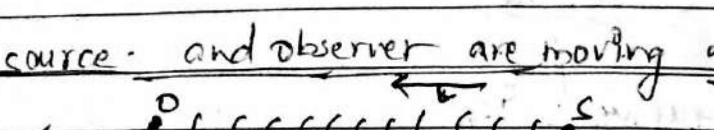
$$f' = \frac{V_r}{\lambda'}$$

or, $$f' = \frac{V + V_o}{\frac{V - V_s}{f}}$$

or $$\boxed{f' = \left(\frac{V + V_o}{V - V_s}\right) \times f}$$

since, $\left(\frac{V + V_o}{V - V_s}\right) > 1$, $\therefore f' > f$. Hence the apparent frequency of sound increases when both source and observer are moving towards each other.

(ii) When both source and observer are moving away from each other:



The relative velocity of sound with respect to observer

$$V_r = V - V_0$$

The apparent wavelength of soundwave

$$\lambda' = \frac{V + V_s}{f}$$

Now, apparent frequency is,

$$f' = \frac{V_r}{\lambda'}$$

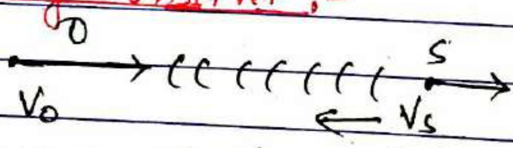
$$\text{or, } f' = \frac{V - V_0}{\left(\frac{V + V_s}{f}\right)}$$

$$\text{or } f' = \left(\frac{V - V_0}{V + V_s}\right) * f$$

Since, $\left(\frac{V - V_0}{V + V_s}\right) < 1$, $\therefore f' < f$.

i.e. The apparent frequency decreases.

(iii) Source leading observer :-



~~The apparent wavelength of soundwave~~

The relative velocity of sound with respect to observer.

$$V_r = V + V_0$$

The apparent wavelength of sound wave,

$$\lambda' = \frac{V + V_s}{f}$$

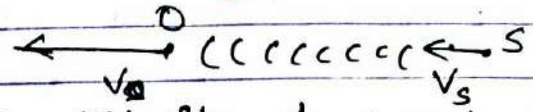
Now apparent frequency is,

$$f' = \frac{V_r}{\lambda'}$$

$$\text{or, } f' = \frac{v+v_0}{\left(\frac{v+v_s}{f}\right)}$$

$$\text{or, } \boxed{f' = \left(\frac{v+v_0}{v+v_s}\right) * f}$$

(iv) Observer leading source :-



The relative velocity of sound with respect to observer

$$v_r = v - v_0$$

The apparent wavelength of sound wave,

$$\lambda' = \frac{v-v_s}{f}$$

Now, Apparent frequency is,

$$f' = \frac{v_r}{\lambda'}$$

$$\text{or } f' = \frac{v-v_0}{\left(\frac{v-v_s}{f}\right)}$$

$$\text{or, } \boxed{f' = \left(\frac{v-v_0}{v-v_s}\right) * f}$$

$$\boxed{f' = \left(\frac{v \pm v_0}{v \mp v_s}\right) * f}$$

④ Ultra sound:-

⇒ The sound wave having frequency greater than 20 kHz (20000 Hz) is called Ultrasonic or Ultrasound.

Imp
⑥ Production of Ultrasound:- Two types of method of productions:- They are as follows:-

(a) Piezo electric or Electrostriction Method:-

→ If mechanical pressure is applied to the ~~the~~ opposite faces of certain crystal (quartz), the other pair of opposite faces develope opposite electric charges resulting a potential difference, this effect is called piezoelectric effect.

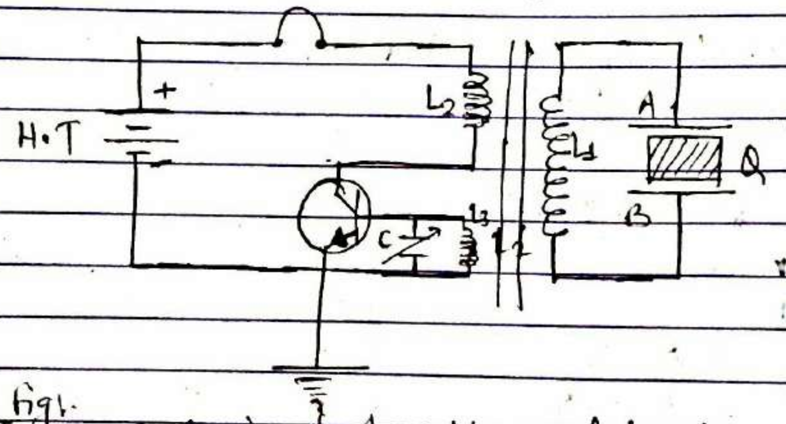


fig:- production of Ultrasound by piezoelectric Method.

Conversely,
If a periodic potential is applied to the opposite faces of the crystal, the periodic changes in dimension of crystal would take place. This periodic change in length of the crystal produces ultrasound. For the generation of ultrasound one can supply the periodic voltage of frequency equal to that of ultrasound which is to be produced.

The experimental circuit diagram for the production of ultrasound by ~~the~~ piezoelectric method is shown in above fig. 'Q' is a thin crystal of quertz which is placed between two metal plates 'A' and 'B' which are connected to the primary of the transformer which is coupled inductively to the oscillatory circuit. The variable capacitor 'C' is so adjusted that the frequency of oscillatory circuit is equal to the natural frequency of vibration of crystal, then the crystal vibrates with maximum amplitude and the ultrasound is produced.

Frequency of vibration

$$f = \frac{n}{2l}\sqrt{\frac{Y}{\rho}}$$

Where, $n = 1, 2, 3, \ldots\ldots$ (no. of modes of vibration)
l = length of crystal
Y = Young's Modulus of Elasticity
ρ = density of crystal.

(b) Magnetostriction Method :-

→ When a ferromagnetic rod is subjected to the alternating magnetic field, it expands and contracts alternatively with twice the frequency of applied magnetic field, this is called magnetostriction effect and these longitudinal expansion and contraction produces ultrasound.

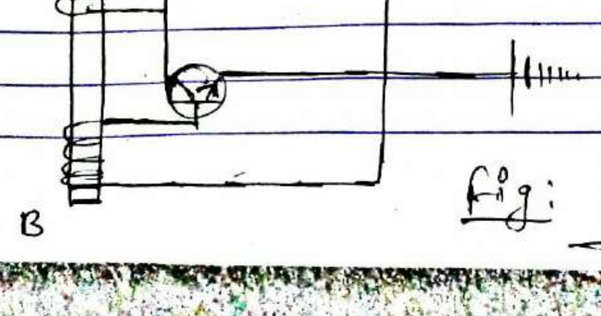


fig: production of ultrasound by Magnetostriction method.

The experimental arrangement for the production of ultrasound by magnetostriction method is shown in above fig.

A ferromagnetic rod 'AB' is magnetised and demagnetised periodically by applying the periodic potential. The variable capacitor 'C' is so adjusted that the frequency of oscillatory circuit is equal to the natural frequency of vibration of the rod due to which the rod vibrates with maximum amplitude and the ultrasound is produced.

Frequency of vibration,

$$f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

where,

$l \rightarrow$ length of rod.
$Y \rightarrow$ Young's Modulus of elasticity
$\rho \rightarrow$ density of rod.

imp

Application of Ultrasound:-

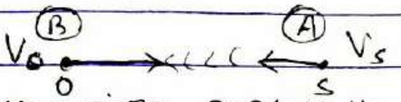
Ultrasound or ultrasonic can be used for various applications. Some of them are:-

(i) Used to measure the depth of sea by sending the ultrasonic waves and how much time will take to return after hitting the bottom surface of sea.
(ii) Used in Medicinal works, such as to recognize the embryo of unborn child or the internal diseases of human.
(iii) Scientific research on study of Cavitation.
(iv) Used for cleaning & automization.
(v) Used to find distance and direction of submarines.
(vi) Used in blood surgery.

Numericals:-

Q.1 Two airplane A and B are approaching each other with a speed of 360 km/hr. The frequency of the whistle emitted by A is 1000 Hz. Calculate the apparent pitch of the whistle as heard by the passengers of airplane B. velocity of sound in air = 350 m/s.

Ans: Given,



velocity of source = velocity observer, given 360 km/hr

$$V_O = V_s = 360\ km/hr$$

$$= \frac{360 \times 1000}{60 \times 60} = 100\,m/s$$

frequency $(f) = 1000$ Hz

Velocity of sound in air $(V) = 350\,m/s$

apparent frequency $(f') = ?$

Here,

$$f' = \left(\frac{V + V_O}{V - V_S}\right) \times f$$

$$= \left(\frac{350 + 100}{350 - 100}\right) \times 1000$$

$$= 1800\ Hz.$$

Q.2 The pitch of the whistle of an engine appear to drop to 2/3th of the original value when it passes a stationary observer. Assuming the velocity of sound in air to be 350 m/s, Calculate the speed of the engine.

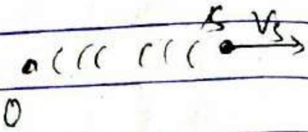
Ans.

given, frequency$(f) = f$ (say)

Apparent frequency $(f') = 2/3$ of f $= \frac{2f}{3}$

velocity of sound in air $(v) = 350$ m/s

Velocity of source $(V_s) = ?$



Here,

$$f' = \left(\frac{v}{v+V_s}\right) * f$$

$$\text{or, } \frac{2f}{3} = \left(\frac{350}{350+V_s}\right) * f$$

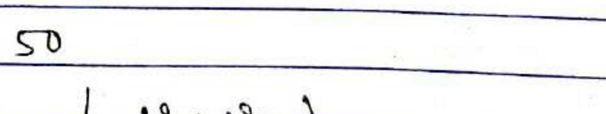
$$\text{or } 2(350+V_s) = 3 * 350$$

$$\text{or, } V_s = \frac{3*350 - 700}{2}$$

$$\therefore V_s = 175 \text{ m/s.}$$

Q.③ An observer travelling with a constᵗ velocity of 20 m/s passes close a stationary source of sound and notice that there is change of frequency of 50Hz as he passes the source. What is the frequency of source? (Speed of sound in air = 340 m/s)

Ans. Given,



$V_0 = 20$ m/s

$V = 340$ m/s

Here,

$$f' - f'' = 50$$

$$\text{or, } \left(\frac{v+v_0}{v}\right) * f - \left(\frac{v - v_0}{v}\right) * f = 50$$

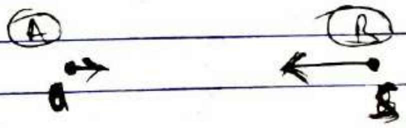
or, $\left(\frac{340+20}{340}\right) * f - \left(\frac{340-20}{360}\right) f = 50$

or, $f\left(\frac{360}{340} - \frac{320}{360}\right) = 50$ $\Rightarrow$ $\boxed{f = \frac{50}{0.1699} \Rightarrow 294.3\ Hz}$

or, $f * 0.1699 = 50$

Q(4) Two Aeroplane passes each other in opposite directions and one of them is blowing whistle of frequency 540 Hz. Calculate the frequency of notes heard in other aeroplane (i) before and (ii) After they have passed each other. velocity of either aeroplane is 540 km/hr and velocity of sound is 350 m/s.

⇒ Ans



(i) 1st case

Given
frequency $(f) = 540\ Hz$
velocity of sound $(V) = 350\ m/s$
velocity of plane (A) = velocity of plane B = 540 km/hr
$= \frac{540 \times 1000}{60 \times 60}$
$= 150\ m/s$

S → O
[A] [B]

(i) Before,

$$f' = \left(\frac{V + V_0}{V - V_s}\right) * f$$

$$= \left(\frac{350+150}{350-150}\right) * 540 = 1350\ Hz$$

(ii) After they passed each other,

(S) (O)
[A] [B]

$$f' = \left(\frac{V - V_0}{V + V_s}\right) * f$$

$$= \left(\frac{350-150}{350+150}\right) * 540 = 216\ Hz$$

Q5 The maximum pressure variation that the ear can tolerate in loud sounds is about 28 N/m^2. Normal atmospheric pressure is about 10^5 N/m^2. Find the corresponding maximum displacement for a sound wave in air having a frequency 1000 Hz. Given density of air = 1.293 kg/m^3 and velocity of sound in air 343 m/s.

$\Rightarrow$ Soln:

Pressure Amplitude $(P_{max}) = 28\ N/m^2$

Atm. pressure $(P_a) = 10^5\ N/m^2$

frequency $(f) = 1000\ Hz$

density of air $(\rho) = 1.293\ kg/m^3$

velocity of sound $(v) = 343\ m/s$

We know that,

$$P_{max} = B a k$$

or $$P_{max} = v^2 \cdot \rho \cdot a \cdot \frac{2\pi}{\lambda} \qquad \left[\because v = \sqrt{\frac{B}{\rho}} \text{ and } k = \frac{2\pi}{\lambda}\right]$$

or $$P_{max} = v^2 \cdot \rho \cdot a \frac{2\pi}{v/f} \qquad [\because v = f * \lambda]$$

or $$P_{max} = 2v \cdot \rho \cdot a \pi f$$

or $$a = \frac{P_{max}}{v \cdot \rho \cdot 2\pi f}$$

or $$a = \frac{28}{343 * 1.293 * 2\pi * 1000}$$

$$\therefore a = 1 \times 10^{-5}\ m$$

Q.6 A tuning fork 'A' of frequency 384 Hz gives 6 beats per second when sounded with another tuning fork B. On loading B with some wax, the number of beats per second becomes 4. What is the frequency of 'B'?

Ans:→ Given,

frequency of tuning fork A = 384 Hz

$384\,Hz \begin{cases} +\ 390\,Hz \\ -\ 378\,Hz \end{cases}$

→ possible frequencies of B are,

$\rightarrow 384 + 6 = 390\,Hz$

$\Rightarrow 384 - 6 = 378\,Hz$

1st cond^n

$384 \begin{cases} +4\ \ 388\,Hz \\ -4\ \ 380\,Hz \end{cases}$

→ After loading, possible frequencies of 'B' are,

$\Rightarrow 384 + 4 \Rightarrow 388\,Hz$

$\Rightarrow 384 - 4 \Rightarrow 380\,Hz.$

As, we know that,

Frequency of tuning fork decreases when some wax is loaded on it. So the frequency of 'B' is 390 Hz.

Q.7 A quartz crystal of thickness 0.001 m is vibrating at resonance. Calculate the fundamental frequency of the vibration. Given, Y = 7.9 × … N/m² and density = $2.65 \times 10^3\ kg/m^3$

⇒ Given Soln

thickness of quartz crystal (l) = 0.001 m

Young's Modulus of Elasticity (Y) = $7.9 \times 10^{10}\ N/m^2$

density of crystal (ρ) = $2.65 \times 10^3\ kg/m^3$

Fundamental frequency (f) = ?

∵ We know that,

$$f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

$$= \frac{1}{2 \times 0.001} \times \sqrt{\frac{7.9 \times 10^{11}}{2.65 \times 10^{3}}}$$

$= 8632977.58$ Hz

$= 8.633$ MHz.

Q. 8 Calculate the velocity of sound in gas in which two waves of lengths 1m and 1.01m produces 10 beats in 3 seconds.

⇒ Soln.

Given,

wavelength of 1st wave $(\lambda_1) = 1m$

" " 2nd " $(\lambda_2) = 1.01$.

beats in 3 seconds $= 10$ beats.

∴ $= \frac{10}{3} = 3.33$ beats

At first, ∵ We know that,

$V = f_1 \lambda_1$

or, $f_1 = \frac{V}{\lambda_1}$

$= \frac{V}{1m}$

And, $V = f_2\lambda_2 \Rightarrow f_2 = \frac{V}{\lambda_2}$

Now, from questions;-

$|f_2 - f_1| = 10/3$

or, $\left(\frac{V}{\lambda_2} - \frac{V}{\lambda_1}\right) = 10/3$

or, $v\left\{\left[\frac{1}{1.01} - \frac{1}{1}\right]\right\} = 3.33$

or, $v \times \{0.99 - 1\} = 3.33$

or, $v = \frac{3.33}{0.01} = 336.67\ m/s$

$\therefore v = 336.67\ m/s.$

Hence velocity of sound in air 336.67 m/s.

(9) A tuning fork produces 6 beats per second when sounded with a tuning fork of frequency 256 Hz. The same tuning fork when sounded with another tuning fork of frequency 252 Hz produced 2 beats per second. Find the frequency of sound.

⇒ Soln,

Given,

Frequency of tuning fork $(f_1) = 256\ Hz$

1st condn. beats produced = 6 beats/second.

2nd condn :- beats produced = 2 beats/second

frequency of another tuning fork $(f_2) = 252\ Hz$

$\because$ 1st condn :-

$f_1 \xrightarrow{+6} 256 + 6 = 262\ Hz$

$f_1 \xrightarrow{-6} 256 - 6 = 250\ Hz$

Similarly, 2nd condn,

$f_2 \xrightarrow{+2} 252 + 2 = 254\ Hz$

$f_2 \xrightarrow{-2} 252 - 2 = 250\ Hz$

Same result.

⇒ Hence, the required frequency of tuning fork is 250 Hz.

Q(10) A ferromagnetic rod has a length of 40mm and the density of the material is 7250 kg/m³. Evaluate the natural frequency of rod if young's modulus of elasticity of the material is 11.5×10^{10} N/m²

⇒ Given

length of (l) = 40 mm = 40×10^{-3} m

density of the material (ρ) = 7250 kg/m³

young's Modulus of material (Y) = 11.5×10^{10} N/m²

natural frequency (f) = ?

∴ We know that,

$$f = \frac{1}{2l}\sqrt{Y/\rho}$$

$$= \frac{1}{2 \times 40 \times 10^{-3}}\sqrt{\frac{11.5 \times 10^{10}}{7250}}$$

$= 49784.01$ Hz

$= 49.78$ kHz.

Interference :-

Coherent sources:-

⇒ Two light ~~waves~~ sources which continuously emit light waves of same frequency, wavelength and amplitude and always have a constant phase difference are called coherent sources of light.

Interference:-

The phenomenon of non-uniform distribution of intensity of light due to the superposition of two light waves from coherent source is called interference.

→ (a) Constructive Interference:-

When two light waves from coherent source having same phase superimposed to each other, then the resultant amplitude is the sum of amplitudes of individual waves and the intensity of light becomes maximum, this type of interference is called constructive interference.

→ (b) Destructive Interference:-

When the two light waves from coherent source having phase difference of 'π' superimposed to each other, then the resultant amplitude is the difference of amplitudes of ~~waves~~ individual wave and the intensity of light is minimum this type of interference is called destructive interference.

Analytical Treatment of Interference:-

⇒ Consider two lights waves from coherent source.

$$y_1 = a\sin\omega t$$

$$y_2 = a\sin(\omega t+\delta)$$

Where, 'δ' be the phase difference between light waves.

When both of them superimpose to each other, then the resultant displacement is, (i)

$y = y_1 + y_2$

or, $y = a \sin \omega t + a \sin(\omega t + \delta)$

or, $y = a \sin \omega t + a \sin \omega t \cdot \cos \delta + a \cos \omega t \cdot \sin \delta$

or, $y = a \sin \omega t (1 + \cos \delta) + a \cos \omega t \cdot \sin \delta$

Let,

$a(1 + \cos \delta) = A \cos \phi$ --- (i)

and, $a \sin \delta = A \sin \phi$ --- (ii)

$\therefore y = A \sin \omega t \cos \phi + A \cos \omega t \cdot \sin \phi$

$\therefore \boxed{y = A \sin(\omega t + \phi)}$

This is the eq^n of resultant wave.

Where, $A \rightarrow$ Amplitude of resultant wave (ii)

Now, squaring and adding eq^n (i) and (ii);

$a^2(1 + \cos \delta)^2 + a^2 \sin^2 \delta = A^2(\sin^2 \phi + \cos^2 \phi)$

or, $a^2 + a^2 \cos^2 \delta + 2a^2 \cos \delta + a^2 \sin^2 \delta = A^2$

or, $a^2 + a^2(\cos^2 \delta + \sin^2 \delta) + 2a^2 \cos \delta = A^2$

or, $2a^2 + 2a^2 \cos \delta = A^2$

or, $2a^2(1 + \cos \delta) = A^2$

or, $2a^2 \cdot 2 \cos^2 \delta/2 = A^2$

or, $A^2 = 4a^2 \cos^2 \delta/2 = I$ (Intensity)

or, $\boxed{A = \pm 2a \cos \delta/2}$

(i) <u>constructive interference:-</u>

<u>For constructive interference,</u>

$$\cos \delta/2 = \pm 1$$

or, $$\cos \delta/2 = \cos n\pi$$

or, $$\boxed{\delta = 2n\pi}$$

or, $$\frac{\cancel{2}\cancel{\pi}}{\lambda} \times x = \cancel{2} n \cancel{\pi}$$ [$\delta \rightarrow$ phase difference, $x \rightarrow$ path difference]

~~or~~, where, $\lambda \rightarrow$ wavelength of light
$x \rightarrow$ path difference

$\Rightarrow$ $\boxed{x = n\lambda}$ where, $n = 0, 1, 2, 3, \ldots$

<u>Hence,</u> for constructive interference (Bright fringe), the path difference is integral multiple of wavelength.

(ii) <u>For destructive interference:-</u>

$$\cos \delta/2 = 0$$

or $$\cos \delta/2 = \cos (2n+1)\pi/2$$

or, $$\delta/\cancel{2} = (2n+1)\pi/\cancel{2}$$

or, $$\delta = (2n+1)\pi$$

or, $$\frac{2\cancel{\pi}}{\lambda} \times x = (2n+1)\cancel{\pi}$$

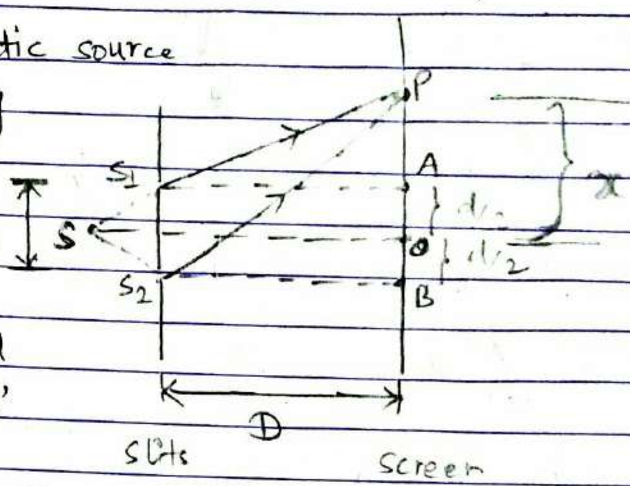
or, $$\boxed{x = \frac{(2n+1)\lambda}{2}}$$ where, $n = 0, 1, 2, 3, \ldots$

where, $x \rightarrow$ path difference,
$\lambda \rightarrow$ wavelength of light.

Hence, for destructive interference (Dark fringe), the path difference is odd integral multiple of ~~wavelength~~ half of wavelength.

Young's double slit experiment :-

⇒ Consider a monochromatic source of light 's' which continuously emits the light waves of wavelength 'λ'. 'S_1' and 'S_2' are two slits at equal distance from 's'. 'S_1' & 'S_2' act as a virtual coherent source. 'd' and 'D' are distances betn two slits and ~~slits~~ distance betn slits and screen respectively.



Take a point 'O' on the screen which is at equal distance from 'S_1' & 'S_2'. So the path difference between light waves from 'S_1' and 'S_2' to the point 'O' is zero. So, the point 'O' is central bright fringe. Dark and bright fringes are formed alternatively above and below the central bright fringe.

Take a point 'P' at a distance of 'x' from 'O'. The path difference between light wave 'S_1' and 'S_2' at 'P' is

path difference, $= S_2P - S_1P$

Now,

$$S_2P^2 - S_1P^2 = S_2B^2 + PB^2 - \{S_1A^2 + PA^2\}$$

or, $$(S_2P - S_1P)(S_2P + S_1P) = D^2 + (x + d/2)^2 - D^2 - (x - d/2)^2$$

or, $$(S_2P - S_1P) = \frac{2xd}{S_1P + S_2P}$$

In ΔS_2PB,

$S_2P^2 = S_2B^2 + PB^2$

In ΔS_1PA,

$S_1P^2 = S_1A^2 + PA^2$

Since, in Experiment the points 'O' and 'P' are very close to each other

so, $\boxed{S_2P + S_1P \approx 2D}$

Here,

$$S_2P - S_1P = \frac{2xd}{2D}$$

$\therefore$ Path difference $= \dfrac{xd}{D}$

(i) For Bright fringe:-

path difference $= n\lambda$, $n = 0, 1, 2, 3, \ldots$

or, $\dfrac{xd}{D} = n\lambda$

or, $\boxed{x = \dfrac{n\lambda D}{d}}$

for, $n=0$, $x_0 = 0$.

$n=1$, $x_1 = \dfrac{\lambda D}{d}$

n_2, $x_2 = \dfrac{2\lambda D}{d}$

.... and so, on.

Now, the fringe width $(\beta) = x_n - x_{n-1}$

$$= \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

(ii) for dark fringe:-

path diff $= (2n-1)\lambda/2$; $n = 1, 2, 3, \ldots$

or, $\dfrac{xd}{D} = (2n-1)\lambda/2$

or, $x = \frac{(2n-1)\lambda D}{2d}$

For, $n=1$, $x_1 = \frac{(2-1)\lambda D}{2d} = \frac{\lambda D}{2d}$

for $n=2$, $x_2 = \frac{(2.2-1)\lambda D}{2d} = \frac{3\lambda D}{2d}$

for $n=3$, $x_3 = \frac{(2.3-1)\lambda D}{2d} = \frac{5\lambda D}{2d}$

------ and so on,

Now, the fringe width $(\beta) = x_n - x_{n-1}$

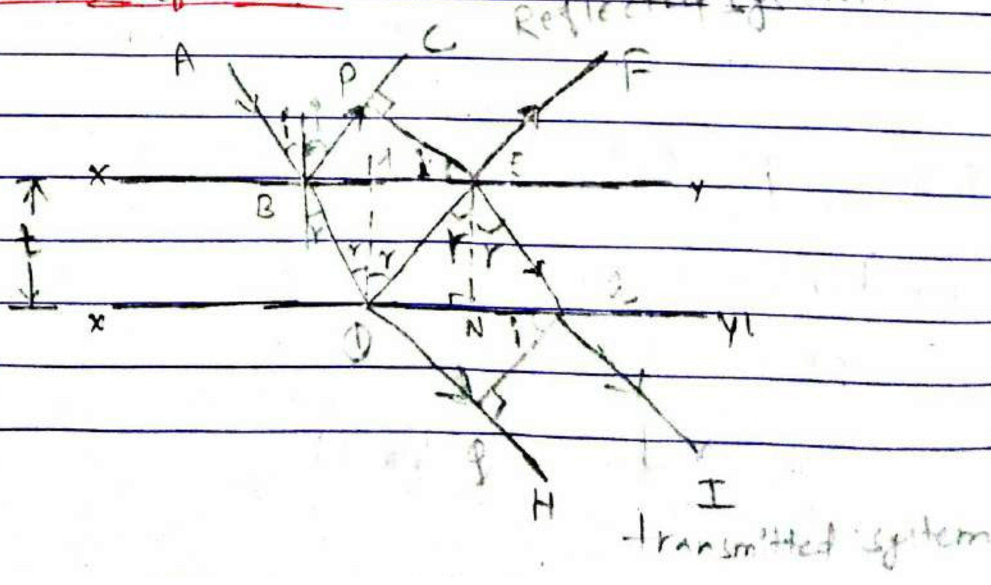
$$= \frac{(2n-1)\lambda D}{2d} - \frac{\{2(n-1)-1\}\lambda D}{2d}$$

$$= \frac{2n\lambda D - \lambda D - 2n\lambda D + 3\lambda D}{2d}$$

$$= \frac{\cancel{2}\lambda D}{\cancel{2}d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

(#) Interference in thin film:-



Consider a thin film of medium of thickness 't' and R.I. 'μ' mounted betn two parallel planes XY and X'Y'. A beam of light AB in incident on plane XY with angle of incidence 'i' at 'B'. The beam partially reflected along 'BC' and partially refracted along 'BD'. This process repeated at point D, E, G and so on.

(i) In reflected system:

In reflected system, two reflected beams 'BC' and 'EF' produces the interference phenomenon. The path difference between 'BC' and 'EF'.

$$\text{path difference} = \mu(BD + DE) - BP$$

From fig:- in ΔBDM,

$$\cos r = \frac{DM}{BD}$$

$$\Rightarrow BD = \frac{t}{\cos r}$$

and, $$\tan r = \frac{BM}{DM}$$

$$\Rightarrow BM = t \cdot \tan r$$

Again, in ΔDME,

$$\cos r = \frac{DM}{DE}$$

$$\Rightarrow DE = \frac{t}{\cos r}$$

and, $$\tan r = \frac{ME}{DM}$$

$$\Rightarrow ME = t \cdot \tan r$$

Also, in ΔBPE,

$$\sin i = \frac{BP}{BE}$$

or, $BP = BE \cdot \sin i = (BM + ME)\sin i$

$\Rightarrow BP = 2t \cdot \tan r \cdot \sin i$

Now, the path difference becomes,

$$\text{path diff.} = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2t \tan r \cdot \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t\,\frac{\sin r}{\cos r} \cdot \sin i$$

$$= \frac{2\mu t}{\cos r} - \frac{2t \cdot \sin r \cdot \mu \sin r}{\cos r} \qquad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$

$$= \frac{2\mu t}{\cos r}\left(1 - \sin^2 r\right)$$

$$= \frac{2\mu t}{\cos r} \times \cos^2 r$$

$$= 2\mu t \cos r$$

$\Rightarrow$ Path difference $= 2\mu t \cdot \cos r$

Since, 'BC' is reflected from rarer medium but 'EF' is refracted from denser medium. So, there is an additional path diff. '$\lambda/2$'. The total path difference is,

$$\text{path difference} = 2\mu t \cos r + \lambda/2$$

⇒ For bright fringe,

path difference $= n\lambda$, $n = 0, 1, 2, 3, \ldots$

or, $2\mu t \cos r + \lambda/2 = n\lambda$

or, $2\mu t \cos r = n\lambda - \lambda/2$

$$\boxed{2\mu t \cos r = (2n-1)\lambda/2} \quad n = 1, 2, 3, \ldots$$

⇒ For dark fringe,

path difference $= (2n+1)\lambda/2$

or, $2\mu t \cos r + \cancel{\lambda/2} = 2n \cdot \lambda/2 + \cancel{\lambda/2}$

or, $\boxed{2\mu t \cos r = n\lambda}$ where, $n = 0, 1, 2, 3, \ldots$

(ii) Transmitted system :-

In transmitted system, the transmitted beams 'DH' & 'GI' produces the interference phenomenon. The path difference ~~from~~ between 'DH' & 'GI'.

path difference $= \mu(DE + EG) - QD$

From fig; In Δ DEN,

$$\cos r = \frac{EN}{DE}$$

or, $$DE = \frac{t}{\cos r}$$

and, $$\tan r = \frac{DN}{EN}$$

or, $DN = t \cdot \tan r$.

In Δ NEG,

$$\cos r = \frac{EN}{EG}$$

or, $$\cos r = \frac{t}{EG}$$

or $$EG = t/\cos r.$$

and $$\tan r = \frac{NG}{EN}$$

or, $$NG = t \cdot \tan r$$

Also, in Δ DQG,

$$\sin i = \frac{QD}{DG}$$

or, $$QD = DG \cdot \sin i$$
$$= (DN + NG) \sin i$$
$$= (t \cdot \tan r + t \cdot \tan r) \sin i$$
$$= 2t \tan r . \sin i$$

Now, the path difference becomes,

$$\text{path diff} = \mu (DE + EG) - QD$$

$$= \mu \left(\frac{t}{\cos r} + \frac{t}{\cos r} \right) - 2t \cdot \tan r . \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t \cdot \frac{\sin r . \sin i}{\cos r} \Big[\because$$

$$= \frac{2\mu t}{\cos r} - \frac{2t \sin r . \mu \sin r}{\cos r} \left[\because \mu = \frac{\sin i}{\sin r} \right]$$

$$= \frac{2\mu t}{\cos r}(1-\sin^2 r)$$

$$= \frac{2\mu t}{\cos r} \times \cos^2 r$$

$$= 2\mu t \cos r$$

Since, both beams 'DH' and 'GI' are transmitted from denser medium so, there is no any additional path difference. The total path difference is,

$$\text{path difference} = 2\mu t \cos r.$$

⇒ For bright fringe,

path difference $= n\lambda$ where, $n = 0, 1, 2, 3, \ldots$

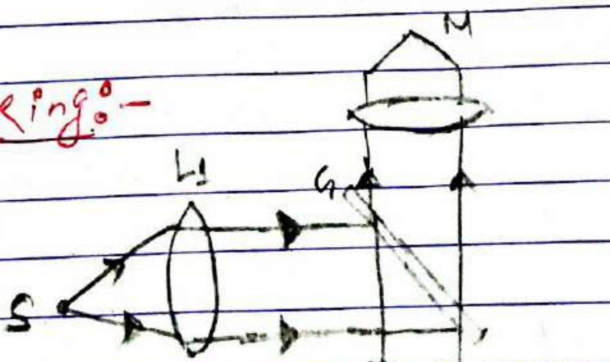
or $\boxed{2\mu t \cos r = n\lambda}$

⇒ For dark fringe

path difference $= (2n+1)\lambda/2$ [Where, $n = 0, 1, 2, 3, \ldots$

or $\boxed{2\mu t \cos r = \cancel{2n\cdot\frac{\lambda}{2}+\frac{\lambda}{2}}\ (2n+1)\lambda/2}$

(7) Newton's Ring:-



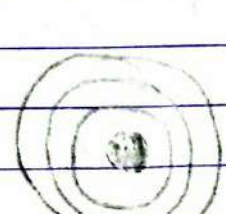
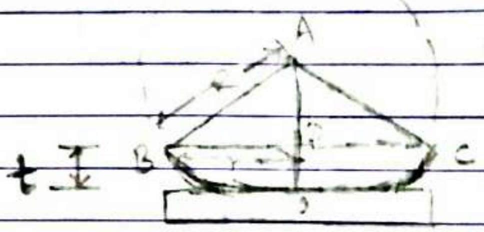


fig (b) Newton's ring

Fig. Newton's Ring experiment

⇒ A beam of light is made parallel by a convex lens 'L_1' and allowed to fall on a glass plate 'G' which is at an angle 45° with the incident beam. The light beam reflected from 'G' is fall on plano-convex lens (L) which is placed on the glass plate 'P'. The beam of light reflected from both plano-convex lens and glass plate which are observed by a microscope 'M'. Under the suitable arrangement Newton rings are seen concentric at a center and a ~~light~~ dark ring at the center. The Newton's rings are formed due to the interference in thin film of air between plano-convex lens and glass plate.

Theory:-



Let, 'R' be the radius of curvature of plano convex lens.

From fig:,

$AD = AO - OD$

or, $AD^2 = (AO - OD)^2$

or, $AD^2 = AO^2 - 2AO \cdot OD + OD^2$

or, $2AO \cdot OD - OD^2 = AO^2 - AD^2$

or, $2 \cdot R \cdot t - t^2 = AB^2 - AD^2$ [$\because AB = AO = R$ and $OD = t$ is thickness of air film at B].

or, $2Rt - t^2 = BD^2$

or, $r^2 = 2Rt - t^2$ [Where, $r = BD$ is radius of Newton's ring] since, 't' is very small, so, t^2 can be neglected,

$$r^2 = 2Rt$$

or, $$2t = \frac{r^2}{R} \quad \text{-----} \; (i)$$

We know that, for interference in thin film, the path difference is given by,

$\therefore$ path difference $= 2\mu t \cos r + \lambda/2$

For normal incidence, $i = 0$, $r = 0$,

$\Rightarrow \cos r = 1.$

For air, $\mu = 1$.

Now the path difference becomes;

path difference $= 2t + \lambda/2$

or, path difference $= \frac{r^2}{R} + \lambda/2$ ---- [from eq^n (i)].

(i) For dark ring:

path difference $= (2n+1)\lambda/2$; $n = 0, 1, 2, 3, \ldots$

or, $$\frac{r^2}{R} + \cancel{\lambda/2} = 2n \cdot \lambda/2 + \cancel{\lambda/2}$$

or, $$\frac{r^2}{R} = n\lambda$$

or, $$r^2 = n\lambda R$$

$$\Rightarrow \boxed{r_n = \sqrt{n\lambda R}}$$

(ii) For bright ring,

path difference $= n\lambda$ where $n = 1, 2, 3, \ldots$

or, $\frac{r^2}{R} + \lambda/2 = n\lambda$

or, $R\left(n\lambda - \lambda/2\right) = \frac{r^2}{1}$

or,

or, $(2n-1)\lambda/2 = r^2/R$

or, $r^2 = \frac{(2n-1)\lambda R}{2}$

$$\boxed{\therefore r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}}$$

This is the radius of n^{th} order Bright ring.

path diff = $2\mu t \cdot \cos$

$r_n = \sqrt{\frac{n\lambda R}{\mu}}$

$\mu = ?$

Determination of wavelength of light using Newton's ring:

⇒ We know that, the radius of n^{th} dark ring of newton's ring is,

$$r_n = \sqrt{n\lambda R}$$

where, $\lambda \rightarrow$ wavelength of light
$R \rightarrow$ Radius of curvature of planoconvex lens.

The diameter of n^{th} ring is,

$$D_n = 2\sqrt{n\lambda R}$$

or, $D_n^2 = 4 \cdot n\lambda R$

similarly, diameter of m^{th} ring is,

$$D_m^2 = 4m\lambda R$$

Now, $D_m^2 - D_n^2 = 4m\lambda R - 4n\lambda R$

or, $D_m^2 - D_n^2 = 4\lambda R(m-n)$

or, $\boxed{\lambda = \dfrac{D_m^2 - D_n^2}{4(m-n)R}}$

Using this relation we can determine the wavelength of light

Determination of refractive Index of liquid using Newton's ring:

⇒ When air is between planoconvex lens and glass plate

Here, $D_m^2 - D_n^2 = 4(m-n)\lambda R$ ------ (*)

When there is liquid between planoconvex and glass plate then the diameter of m^{th} and n^{th} dark fringes are

$$D_m'^2 = \frac{4m\lambda R}{\mu}$$

and, $$D_n'^2 = \frac{4n\lambda R}{\mu}$$

Now, $$D_m'^2 - D_n'^2 = \frac{4(m-n)\lambda R}{\mu} \quad \text{---} (**)$$

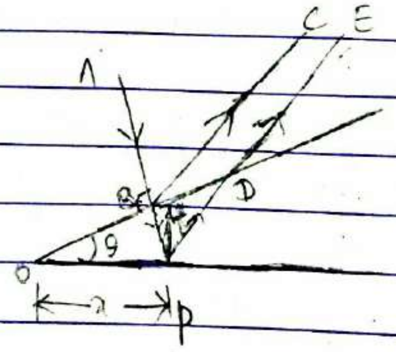
Dividing (*) by (**), we get,

or $$\frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2} = \frac{4(m-n)\lambda R}{\frac{4(m-n)\lambda R}{\mu}}$$

or $$\boxed{\mu = \frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2}}$$

Using this relation we can determine the refractive index of liquid.

(iii) Wedge shape Interference:-



⇒ The medium in which the thickness at one end is zero and gradually increases to a certain value at other end is called wedge.

Consider a wedge of the medium of refractive index 'μ' in which 'θ' be the angle made by the two surfaces of the wedge. A beam of light 'AB' is incident ~~at~~ normally on the upper surface of the wedge which partially reflected along 'BC' and partially refracted along 'BP' and this process is seen at point 'P', 'D' and so on. Two reflected beams 'BC' and 'DE' shows the interference phenomena. Let, 'x' be the distance between 'P' and 'O'. 't' be thickness of the medium at point 'P'.

for small angle,

$$\theta = \frac{t}{x}$$

or, $$t = \theta \cdot x$$

We know that, for interference in thin film, the path diff. is,

$$\text{path difference} = 2\mu t \cos r + \lambda/2$$

for normal incident, $i = r = 0°$,

$$\therefore \cos r = 1$$

$$\therefore \text{path difference} = 2\mu t + \lambda/2$$

or, $$\cancel{2\mu t + \lambda/2} = 2\mu\theta x + \lambda/2 \quad [\because t = \theta \cdot x]$$

(i) For Bright fringe,

$$\text{path diff} = n\lambda$$

or, $$2\mu\theta x + \lambda/2 = n\lambda$$

or, $$2\mu\theta x = (2n-1)\lambda/2$$

or, $$\boxed{x = \frac{(2n-1)\lambda}{4\mu\theta}}$$ where $n = 1, 2, 3, \ldots$

for, $n=1$, $x_1 = \frac{\lambda}{4\mu\theta}$

$n=2$, $x_2 = \frac{3\lambda}{4\mu\theta}$

$n=3$, $x_3 = \frac{5\lambda}{4\mu\theta}$ and so on,

Here, Fringe width $(\beta) = x_n - x_{n-1}$

$$\beta = \frac{(2n-1)\lambda}{4\mu\theta} - \frac{[2(n-1)-1)]\lambda}{4\mu\theta}$$

$$= \frac{\cancel{2n\lambda} - \cancel{\lambda} - \cancel{2n\lambda} + \cancel{\lambda} + 2\lambda + \cancel{\lambda}}{4\mu\theta}$$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

② for Dark fringe;

path diff. $= (2n+1)\lambda/2$

or $2\mu\theta x + \lambda/2 = \cancel{2n}\cdot\lambda/2 + \lambda/2$

or $2\mu\theta x = n\lambda$

or $x = \frac{n\lambda}{2\mu\theta}$ where $n = 0, 1, 2, 3, \ldots$

for $n=0$, $x_0 = 0$,

$n=1$, $x_1 = \frac{\lambda}{2\mu\theta}$

$n=2$, $x_2 = \frac{2\lambda}{2\mu\theta} = \frac{\lambda}{\mu\theta}$

$n=3$, $x_3 = \frac{3\lambda}{2\mu\theta}$ and so on.

For fringe width $(\beta) = x_n - x_{n-1}$

$$= \frac{n\lambda}{2\mu\theta} - \frac{(n-1)\lambda}{2\mu\theta}$$

$$= \frac{\cancel{n\lambda} - \cancel{n\lambda} + \lambda}{2\mu\theta}$$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

Numerical:

1. A thin soap film is illuminated by white light at an angle of incident $(i) = \sin^{-1}(4/5)$. In reflected light two dark consecutive over-lapping fringes are observed corresponding two wavelength 6.1×10^{-7} m and 6.0×10^{-7} m. 'μ' for soap solution is $4/3$ calculate the thickness of the film.

$\Rightarrow$ Given, Angle of incident $(i) = \sin^{-1}(4/5)$

or, $\sin i = \frac{4}{5}$

$\therefore \mu = \frac{\sin i}{\sin r}$

$\Rightarrow \frac{4}{3} = \frac{4/5}{\sin r} \Rightarrow \sin r = \frac{\cancel{4}1}{5} \times 3 \Rightarrow 3/5$

$\therefore \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = 4/5$

$\lambda_1 = 6.1 \times 10^{-7}$ m (wavelength of 1st light)

$\lambda_2 = 6. \times 10^{-7}$ m (wavelength of 2nd light).

Here, $2\mu t \cos r = n\lambda_1$ --- (i)

", $2\mu t \cos r = (n+1)\lambda_2$ --- (ii)

Now,

$n\lambda_1 = (n+1)\lambda_2$

or, $n \times 6.1\times10^{-7} = (n+1)\, 6\times10^{-7}$

or, $6.1n - 6n = 6$

or, $n = \frac{6}{0.1} = 60$

Now, from eqn ①

$2\mu t \cos r = 60 \cdot \lambda_1$

or, $2 \times \frac{4}{3} \times t \times \cos\frac{4}{5} = 60 \times 6.1\times10^{-7}$

or, $t = \frac{15 \times 60 \times 6.1 \times 10^{-7}}{2\times4\times4}$

$t = 1.715 \times 10^{-5}\ m$

Q ② Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.5. The fringes spacing is 1 mm and wavelength of light 5893 Å. Calculate the angle of wedge in second ~~~~ of an arc.

Given,

$\Rightarrow$ $\mu = 1.5$

$\lambda = 5893\,Å = 5893\times10^{-10}\ m$

Fringe spacing $(\beta) = 1\ mm = 1\times10^{-3}\ m$

$\because \beta = \frac{\lambda}{2\mu\theta}$

or, $\theta = \frac{\lambda}{2\cdot\mu\cdot\beta}$

$$\theta = \frac{5893 \times 10^{-10}}{2 \times 1.5 \times 1 \times 10^{-3}}$$

$$\theta = 1.938 \times 10^{-4} \text{ rad.}$$

1.964×10^{-4}

$$\therefore \theta = \left(1.938 \times 10^{-4} \times \frac{180}{\pi} \times 60 \times 60\right)$$

$\boxed{40.5''}$

$$\boxed{\theta = 39.98''}$$ Ans.

Q. (3) A double slit of 0.5 mm separation is illuminated by light of blue cardium of wavelength 4800 Å. How far behind the slit one go to obtain fringes that are 0.1 cm apart.

⇒ Given,

Seperation betn two slit $(d) = 0.5$ mm $= 0.5 \times 10^{-3}$ m

wavelength $(\lambda) = 4800$ Å $= 4800 \times 10^{-10}$ m

fringes width $(\beta) = 0.1$ cm $= 1 \times 10^{-3}$ m

distance betn slit & screen $(D) = ?$

∴ We know that,

$$\beta = \frac{\lambda D}{d}$$

or $$D = \frac{\beta d}{\lambda}$$

$$= \frac{1 \times 10^{-3} \times 0.5 \times 10^{-3}}{4800 \times 10^{-10}}$$

$$= 1.04 \text{ m}$$

Q(4) In Young's Experiment the fringe width is 0.6cm by using the light of wavelength 5000 Å. If the distance betn slit and screen is reduced to half, what should be the wavelength of light source to get the fringes 0.4cm wide?

⇒ Soln Given

For 1st condn,

fringe width $(\beta_1) = 0.6cm = 6 \times 10^{-3} m$

wavelength $(\lambda_1) = 5000 Å = 5000 \times 10^{-10} m$

separation betn slit $(d_1) = d$ (say)

distn betn slit & screen $(D_1) = D$ (say)

2nd condn,

fringe width $(\beta_2) = 0.4cm = 4 \times 10^{-3} m$

wavelength $(\lambda_2) = ?$

separation betn slit $(d_2) = d$ (say)

distn betn slit & screen $(D_2) = D/2$

Now, $\because d_1 = d_2$

or, $\dfrac{\lambda_1 D_1}{\beta_1} = \dfrac{\lambda_2 D_2}{\beta_2}$

or, $\dfrac{5000 \times 10^{-10} \times \cancel{D}}{3\ \cancel{6 \times 10^{-3}}} = \dfrac{\lambda_2 \times \cancel{D}/2}{\cancel{4 \times 10^{-3}}\ 2}$

or, $\dfrac{5000 \times 10^{-10}}{3} = \lambda_2 \times 4$

$\therefore \lambda_2 = \dfrac{4 \times 5000 \times 10^{-10}}{3} = \boxed{6.67 \times 10^{-7}}$ m. //

Q(5) A soap film of refractive index 1.33 is illuminated by white light incident at an angle of 45°. The light refracted by it is examined by a spectrometer and a bright ~~fringe~~ band is found corresponding to a wavelength 6000 Å. Find the thickness of film.

⇒ $soln$.. Given

R.I. $(\mu) = 1.33$

Angle of incident $(i) = 45°$

$\because \mu = \frac{\sin i}{\sin r}$

or, $\sin r = \frac{\sin 45°}{\mu} = \frac{1/\sqrt{2}}{1.33} = \frac{1}{1.33\sqrt{2}} = 0.53$

$\therefore \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-0.58^2} = 0.848$

wavelength $(\lambda) = 6000 Å$

For path difference in refracted system (transmitted system);

path diff $= 2\mu t \cos r$

or, $n\lambda = 2\mu t \cos r$ [for bright fringe $= n\lambda$]

or, $n\lambda = 2 \times 1.33 \times 0.848 \times t$

or, $\frac{6000 \times 10^{-10}}{2 \times 1.33 \times 0.848} = t$ [for $n = 1$]

$$\boxed{\therefore t = 2.66 \times 10^{-7} \text{ m}}$$

Q.6 Newton's rings are ~~observed~~ observed in reflected light of wavelength 5900 Å. The diameter of 10th dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the air film.

⇒ $soln$, wavelength $(\lambda) = 5900 Å$

diameter of 10th dark ring $(D_{10}) = 0.5$ cm $= 0.5 \times 10^{-2}$ m

Radius of curvature $(R) = ?$

Thickness of air film $(t) = ?$

In Newton's ring Experiment,

for dark ring,

$r_n = \sqrt{n\lambda R}$

or, $D_n = 2\sqrt{n\lambda R}$

or, $(0.5 \times 10^{-2})^2 = 4 \times 10 \times 5900 \times 10^{-10} \times R$

or, $R = \dfrac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 5900 \times 10^{-10}}$

$\boxed{\therefore R = 1.059 \text{ m}}$

→ For Thickness of air film,

$2t = \dfrac{r^2}{R}$

or, $t = \dfrac{r^2}{2R}$

or, $t = \dfrac{(2.5 \times 10^{-3})^2}{2 \times 1.059}$

$\boxed{\therefore t = 2.95 \times 10^{-6} \text{ m}}$ //

Q.⑦ Newton's rings are formed by reflected light of wavelength 5895 A° with a liquid betn the plane and curve surfaces. If the diameter of the 5th ~~ring~~ bright ring is 3mm and radius of curvature of the curved surface is 100cm, calculate the R.I. of the liquid.

→ Soln

Wavelength $(\lambda) = 5895 \text{ A}^\circ = 5895 \times 10^{-10}$ m

Dia. of 5th Bright ring (D_5) = 3mm = 3×10^{-3} m

Radius of curvature (R) = 100 cm = 1 m

R.I. (μ) = ?

∴ We know that,

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2\mu}}$$

or $$D_n = 2\sqrt{\frac{(2n-1)\lambda R}{2\mu}}$$

or $$(D_n)^2 = \frac{\cancel{4}^{2} \times (2n-1)\lambda R}{\cancel{2}\mu}$$

or $$(D_5)^2 = \frac{2\cdot(2*5-1)\lambda R}{\mu}$$ [For 5th bright ring]

or $$\mu = \frac{2*9*5895*10^{-10}*1}{(3*10^{-3})^2}$$

$$\boxed{\mu = 1.179}$$ //Ans.

Q. 8 In Newton's ring's Expt, the diameter of the 10th ring changes from 1.40 cm to 1.27 cm when a liquid is placed (introduced) betn the lens and the plate, calculate the refractive index of the liquid.

⇒ Soln: Given,

Dia. of 10th ring in air medm $(D_{10a}) = 1.40\,cm = 1.4 \times 10^{-2}\,m$

Dia. of 10th ring in Liquid medm $(D_{10l}) = 1.27\,cm = 1.27 \times 10^{-2}\,m$

R.I. $(\mu) = ?$

For air medm;

$$2\sqrt{n\lambda R} = 1.40 \times 10^{-2} \quad \text{--- (i)}$$

$$2\sqrt{\frac{n\lambda R}{\mu}} = 1.27 \times 10^{-2} \quad \text{--- (ii)}$$

squaring (1st) & (2nd) eqn and dividing eqn (ii) by (i);

$$\frac{\cancel{4}\frac{n\lambda R}{\mu}}{\cancel{4}n\lambda R} = \frac{(1.27 \times 10^{-2})^2}{(1.40 \times 10^{-2})^2} \Rightarrow \mu = \frac{(1.4)^2}{(1.27)^2} = \boxed{1.215}$$ Ans.

Q.9 Determine the ratio of intensity at the centre of a bright fringe to the intensity found at a point one quarter of the distance betn two fringes from the centre.

⇒ Soln;

∵ We know that,

$$I = 4a^2 \cos^2 \frac{\phi}{2} \quad \text{---(1)}$$

At the centre,

$$\phi = 0,$$

$$\therefore I_c = 4a^2$$

Again,

At one quarter from the centre of bright fringe,

$$\phi = \frac{1}{4} \times 2\pi = \pi/2 \Rightarrow \frac{\phi}{2} = \pi/4$$

Rough,
I_c
$\leftarrow 2\pi \rightarrow$
$I_p = 1/4$
$\Rightarrow 1/4 \times 2\pi \Rightarrow \pi/2$

Now,

$$I_p = 4a^2 \cos^2 (\pi/4)$$

$$= 4a^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 2a^2$$

Again,

$$\frac{I_c}{I_p} = \frac{4\cancel{a^2}}{2\cancel{a^2}}$$

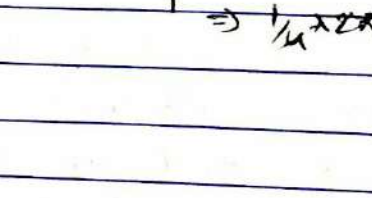
$$\therefore \boxed{I_c : I_p = 2:1} \quad \text{Ans.}$$

Q.10 A wedge shape film of air is produced by placing a fine wire of diameter D betn the ends of two flat glass plates of length 20cm. When the film is illuminated with light of wavelength 550nm, there are 12 dark fringes per centimeter. Find D?

⇒ Soln

Wavelength $(\lambda) = 550\,nm$

$$= 550 \times 10^{-9}\,m$$



Fringe width $(\beta) = \frac{1}{12}\,cm$

$$= \frac{1}{12} \times \frac{1}{100}\,m$$

$$= 8.33 \times 10^{-4}\,m$$

or,
$$\beta = \frac{\lambda}{2\mu\theta}$$

or, $\cancel{8.33\times10^{-4}\times2\times1\times\frac{20}{D} = 5500\times10^{-10}}$ $\left[\because \theta = D/20 \right]$ θ is very small, $\mu = 1$ (for air)

or, $\cancel{D = 6.602\times10^{-3}\,m}$

$D =$

or, $8.33\times10^{-4}\times2\times1 = \frac{5500\times10^{-10}\times20}{D}$

or, $D = \frac{5500\times10^{-10}\times20}{8.33\times10^{-4}\times2}$

$\therefore D = 6.602\times10^{-3}\,m$

Q.11 In double slit Expt, the distance betn slits is 5mm and the slits are 1m from the screen. Two interference patterns can be seen one due to light of wavelength 480nm and the other due to light of wavelengths 600nm. What is the separation on the screen betn the third order bright fringes of two interference patterns?

$\Rightarrow$ Given,
1st wavelength $(\lambda_1) = 480nm = 4800\times10^{-10}\,m$
2nd wavelength $(\lambda_2) = 600nm = 6000\times10^{-10}\,m$
slits separation $(d) = 5mm = 5\times10^{-3}\,m$ & distn betn slit & screen $(D) = 1m$

$\therefore$ For 3rd order bright fringe,

At λ_1,
$$x_3 = \frac{3\lambda_1 D}{d}$$

And At λ_2
$$x_3' = \frac{3\lambda_2 D}{d}$$

$$\Rightarrow x_3' - x_3 \Rightarrow \frac{3\lambda_2 D}{d} - \frac{3\lambda_1 D}{d}$$
$$\Rightarrow \frac{3\times600\times1}{5\times10^{-3}} - \frac{3\times480\times1}{5\times10^{-3}} \Rightarrow \frac{(1800-1440)\times10^{-9}}{5\times10^{-3}}$$
$$\Rightarrow 7.2\times10^{-5}\,m$$

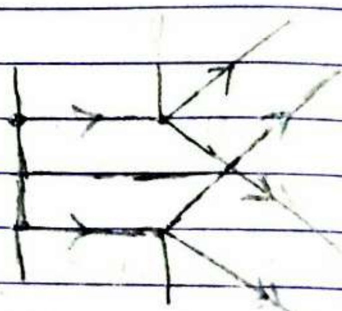
$$\Rightarrow \frac{\overset{72}{\cancel{360}} \times 10^{-9}}{\cancel{5} \times 10^{-3}}$$

$$\Rightarrow 7.2 \times 10^{-5}\ m$$

∴ The separation on the screen betn 3rd order bright fringes is $\underline{7.2 \times 10^{-5}\ m}$.

(4) Diffraction :-

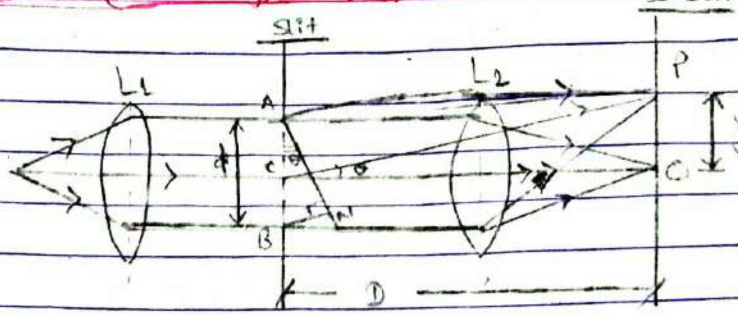
⇒ The phenomenon of spreading of light around the corners of a slits or obstacle placed on the path of the light is called diffraction. Both longitudinal and transverse waves can be diffracted. The main conditions of diffraction is the width of slit or the obstacle must be comparable of wavelength of wave.



(4) Fresnel and Fraunhoffer Diffraction :- [Difference betn]

Fresnel diffraction	Fraunhoffer diffraction
(1) The light suffering diffraction is not modified by lenses or mirrors.	(1) The light suffering diffraction is modified by the use of lenses or mirrors.
(2) In this case, either source or the screen or both are at finite distance from the obstacle or aperture.	(2) The sources of light and screen are effectively at infinite distances from the apertures or ~~holes~~ obstacle.
(3) The incident wavefront is either spherical or cylinderical.	(3) The incident and the diffracted wavefronts are plane.

⊕ Diffraction through single slit (Fraunhoffers diffraction):



⇒ Consider a light of wavelength 'λ' from a monochromatic source 'S' is made parallel by a convex lens 'L_1' and allowed to fall on a slit 'AB' of width 'd'.

The central bright ~~fringe~~ image of the source is formed at point 'O' on the screen at a distance of 'D' from the slit, which is known as central bright fringe. The dark and bright fringes are formed above and below the central bright fringe alternatively which are known as secondary minima and secondary maxima respectively.

Take a point 'P' on the screen at a distance of 'y' from 'O' to find the path difference betn the light waves from point 'A' & 'B' at 'P'. Draw a perpendicular 'AN' on the light wave 'BP'. The path difference between the light waves from point 'A' and 'B' at 'P' is given by,

path difference = BN = $d \sin\theta$ [where $\theta \rightarrow$ Angle of diffraction]

When the path difference is equal to one wavelength then we get Ist order secondary minima. For this, let us divide 'AB' into two equal halves, $\boxed{AC = BC = \frac{d}{2}}$. The light waves from every point of upper half and light waves from corresponding points of lower half have the path difference of $\lambda/2$. So, they interfere destructively and we get secondary minima.

Similarly, if the path difference is equal to '$n\lambda$' where, $n = 1, 2, 3, \ldots$ then we get secondary minima.

i.e. For secondary minima (Dark fringe)

$$\boxed{d \sin\theta_n = n\lambda}$$ where, $n = 1, 2, 3, \ldots$

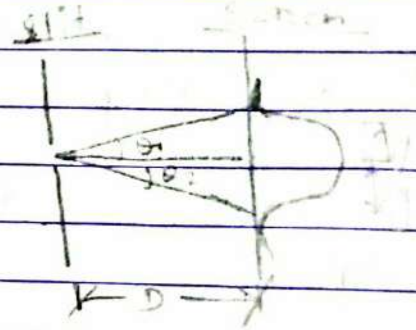
Also, If the path difference is equal to $(2n+1)\lambda/2$ then we get secondary maxima.

For secondary maxima (Bright fringe)

$\Rightarrow$ $\boxed{d \sin\theta_n = (2n+1)\lambda/2}$ $n = 1, 2, 3, \ldots$

(II) For width of Central Bright fringe:

$\Rightarrow$ The width of central bright fringe is the distance betw two 1st order secondary minima. We know that,



For 1st order secondary minima:

$$d \sin\theta_1 = \lambda$$

or $$\sin\theta_1 = \frac{\lambda}{d}$$

$\rightarrow$ For small angle θ_1, $\sin\theta_1 \approx \theta_1 = \lambda/d$ --- (i)

From figure:-

$$\theta = y/D \text{ --- (ii)}$$

Equating eqn (i) & (ii), we get,

$$\frac{y}{D} = \frac{\lambda}{d}$$

or $$y = \frac{\lambda D}{d}$$

Now, width of central bright is $(\beta) = 2y$

$\Rightarrow$ $$\boxed{\beta = \frac{2\lambda D}{d}}$$

(11) Intensity Distribution in single slit diffraction :-

fig (a)

fig (b)

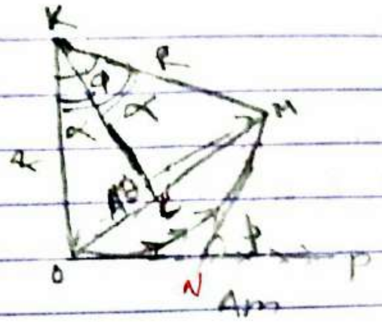


Fig (c)

Consider a parallel beam of light of wavelength 'λ' is incident on a slit of width 'd'. Let 'A_m' be the maximum amplitude at the central bright fringe and A_θ be the amplitude made by diffracted beam of light at an angle of diffraction 'θ'. 'A_m' is represented in fig (a) and 'A_θ' is represented in fig (b). The fig (c) shows that combination of fig (a) and fig (b). In fig (c), 'OP' gives the direction of initial vector (amplitude) an 'NM' gives the final vector. Let 'ϕ' be the phase difference between undiffracted and diffracted ray of light.

Now, $\sin\alpha = \frac{LM}{KM}$

or, $\sin\alpha = \frac{A_\theta/2}{R}$

or, $\sin\alpha = \frac{A_\theta}{2R}$

or, $A_\theta = 2R\sin\alpha$ ------ (i)

Again, in radian measure,

$$\phi = \frac{\widehat{OM}}{R} = \frac{A_m}{R}$$

or, $$2\alpha = \frac{A_m}{R} \qquad [\because \phi = 2\alpha]$$

or $$A_m = 2\alpha R \quad \text{---} \text{(ii)}$$

We know that,

intensity of undiffracted ray $(I_m) \propto A_m^2$
intensity of diffracted ray $(I_\theta) \propto A_\theta^2$

Hence, $$\frac{I_\theta}{I_m} = \left(\frac{A_\theta}{A_m}\right)^2$$

or $$\frac{I_\theta}{I_m} = \left(\frac{\cancel{2}\cancel{R}\sin\alpha}{\cancel{2}\alpha\cancel{R}}\right)^2$$

$$I_\theta = \left(\frac{\sin\alpha}{\alpha}\right)^2 \cdot I_m \quad \text{----} \text{(iii)}$$

Again, we know that,

$$\phi = \frac{2\pi}{\lambda} \times \text{path difference}$$

or $$\cancel{2}\alpha = \frac{\cancel{2}\pi}{\lambda} \times d\sin\theta$$

In single slit diffraction, path difference = $d\sin\theta$

$$\Rightarrow \alpha = \frac{\pi}{\lambda} \times d\sin\theta \quad \text{---} \text{(iv)}$$

Case - I For central maxima, $\theta = 0^\circ$

$$\boxed{\alpha = 0}$$

Here, from eq^n (iii),

$$I_\theta = \left(\frac{\sin 0}{0}\right)^2 \cdot I_m$$

$$\Rightarrow \boxed{I_\theta = I_m} \qquad \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

Hence, the intensity is maximum at central bright fringe.

Case - II :-

For secondary maxima,

we know that,

$$\alpha = \frac{\pi}{\lambda} * d\sin\theta$$

For secondary maxima,

$$d\sin\theta = (2n+1)\lambda/2$$

ⓐ For 1st order secondary maxima, $n = 1$,

$$d\sin\theta = 3\lambda/2$$

Now,

$$\alpha = \frac{\pi}{\cancel{\lambda}} * \frac{3\cancel{\lambda}}{2} = \frac{3\pi}{2}$$

Again, from eq^n (iii),

$$I_\theta = \left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2 I_m$$

$$\Rightarrow \boxed{I_\theta = \frac{I_m}{22}}$$

ⓑ For 2nd order secondary maxima, $n=2$

$$d\sin\theta = 5\lambda/2$$

then,

$$\alpha = \frac{\pi}{\lambda} * \frac{5\lambda}{2} = \frac{5\pi}{2}$$

Hence,

$$I_\theta = \left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 I_m$$

$$\boxed{I_\theta = \frac{I_m}{61}}$$

Case - III, For secondary minima,

$$d \sin\theta = n\lambda$$

Then, $\alpha = \frac{\pi}{\lambda} * n\lambda = n\pi$

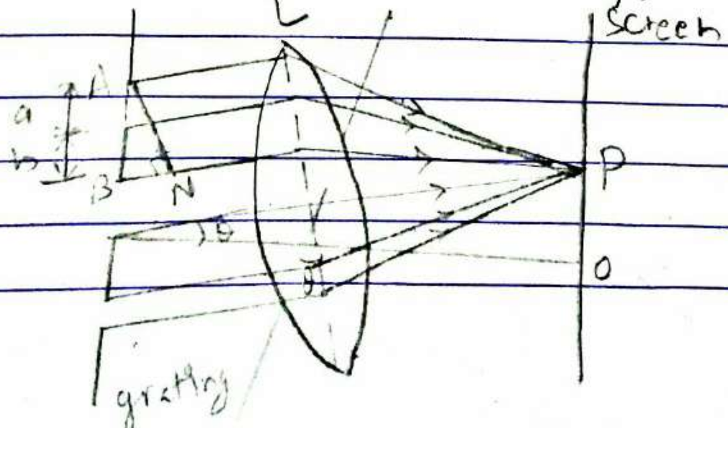
Also,

$$I_\theta = \left(\frac{\sin n\pi}{n\pi}\right)^2 I_m$$

$\boxed{I_\theta = 0}$ For every value of n i.e. dark fringe.

(4) Diffraction Grating:

↳ The arrangement of combination of large number of parallel slits having same width and separated one another by same distance is called diffraction grating.



Consider a parallel beam of light of wavelength 'λ' is incident on a plane diffraction grating having 'N' number of lines per unit length. 'a' be the width of the slit and 'b' be the distance betn each slit. $(a+b)$ is known as grating element. The undiffracted beam of light are converged at point 'O' on the screen which is known as central bright fringe. When we rotate the lens anticlockwise by an angle of 'θ' then the diffracted beams with angle of diffraction 'θ' are converged at 'P' on the screen.

To find the path difference betn the ray of light from point 'A' and 'B'. Let us draw a perpendicular 'AN' on light 'BP'.

Now,

The path difference is,

$$\text{path difference} = BN = (a+b)\sin\theta$$

For maximum Intensity,

$$\text{path difference} = n\lambda \quad \text{where } n = 0, 1, 2, 3, 4, 5, \ldots$$

Here,

$$(a+b)\sin\theta = n\lambda$$

or $$\boxed{(a+b)\sin\theta_n = n\lambda}$$

Again,

$$\frac{1}{N}\sin\theta_n = n\lambda$$

or $$\boxed{\sin\theta_n = nN\lambda}$$

X-ray diffraction :-

The condition of diffraction of a wave is, the wavelength of the wave must be comparable of width of the slit. Since, the wavelength of x-ray is 1000 times smaller than the wavelength of light, the ordinary grating cannot diffract the x-ray. We have suggested that the crystal can be used for the diffraction of x-ray. In the crystal atoms are regularly arranged in planes called lattice plane. The distance betn two adjacent atoms act as a grating element.

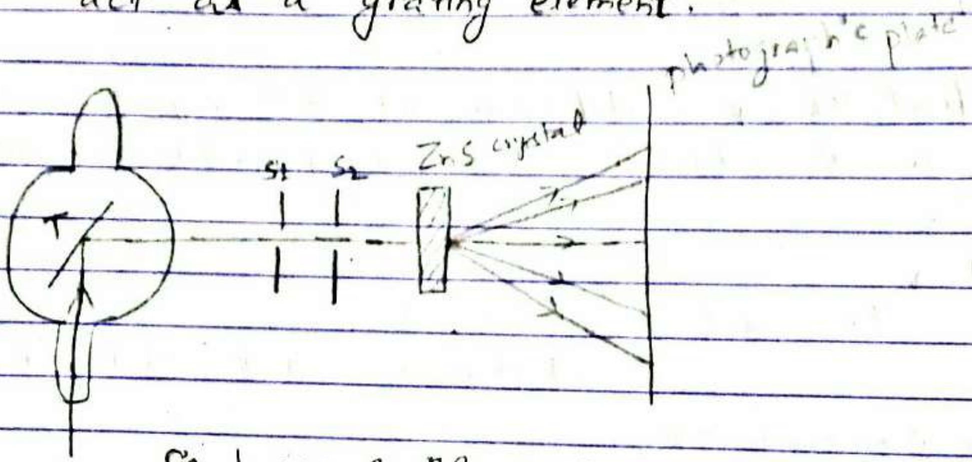


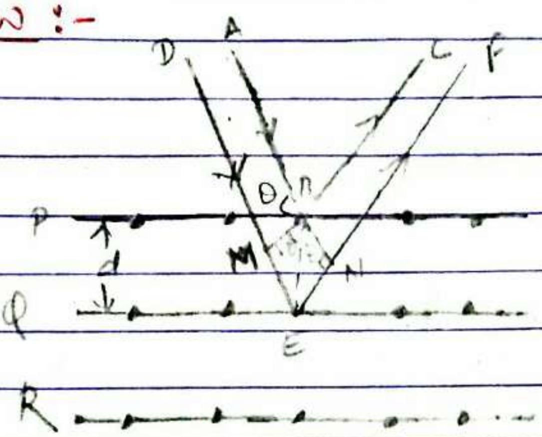
fig Laue Experiment for x-ray diffraction through crystal

fig (b). Laue pattern

Laue experimentally showed that the x-ray can be diffracted through crystal. The experimental arrangements is shown in above fig (a). The x-ray from x-ray tube is made fine by two slits 's1' and 's2' after then allowed to fall on 'ZnS' crystal. After the expose of several hours and under the suitable arrangement many faint but regulerly arranged pattern are seen on the photographic plate, which are known Laue pattern as in fig (b)

This is due to the diffraction of x-ray through crystal.

(4) Bragg's law :-



⇒ Consider 'P', 'Q' and 'R' are lattice planes of a crystal which are separated by a distance 'd' from each other. A parallel beam of X-ray of wavelength 'λ' is incident on the glancing angle 'θ'. The x-ray 'AB' is incident on plane 'P' and reflected along 'BC' also the x-ray 'DE' is incident on plane 'Q' and reflected along 'EF'. To find the path difference betn two reflected beams 'BC' and 'EF'. Let us draw perpendiculars 'BM' on 'DE' and 'BN' on 'EF'.

The path difference between 'BC' and 'EF' is,

$$\text{path difference} = ME + EN$$

From △BME,

$$\sin\theta = \frac{ME}{BE} \Rightarrow ME = d\sin\theta$$

and,

From △BNE,

$$\sin\theta = \frac{EN}{BE} \Rightarrow EN = d\sin\theta$$

Now, the path difference is,

$$\text{path difference} = d\sin\theta + d\sin\theta = 2d\sin\theta$$

for max^m intensity,

path difference $= n\lambda$, $n = 0, 1, 2, 3, \ldots$

Hence, $2d\sin\theta = n\lambda$

or $\boxed{2d\sin\theta_n = n\lambda}$

→ This is called Bragg's law or Bragg's eq^n.

Numericals:

① How many orders will be visible if the wavelength of incident radian is 5000 Å and the number of lines on the grating is 1025 in 1 cm.

⇒ sol^n

$n_{max} = ?$

wavelength $(\lambda) = 5000\,Å = 5 \times 10^{-7}\,m$

$N = 1025$ per cm $= 102500$ per m

∵ we know that,

$\sin\theta = Nn\lambda$

for n_{max}, $\sin\theta = 1$

or, $N n_{max} \lambda = 1$

or $n_{max} = \dfrac{1}{N\lambda}$

or $n_{max} = \dfrac{1}{102500 \times 5 \times 10^{-7}}$

∴ $n_{max} = 19.51 = \underline{\underline{19}}$ Ans

(8) A plane diffraction grating have 6000 lines/cm is used to obtain a spectrum of light from sodium lamp in 2nd order. Calculate the ~~angular~~ angular separation betn two sodium lights whose wavelength are 5890 Å & 5896 Å.

⇒ Soln Given,

diffraction grating $(N) = 6000$ line/cm
$= 600000$ line/m

1st Wavelength $(\lambda_1) = 5890\ A^0$
2nd wavelength $(\lambda_2) = 5896\ A^0$

Find, Angular separation $(\Delta\theta) = ?$

Now, We know that,

$$\sin\theta = N \cdot n\lambda \quad \text{--- (i)}$$

∴ For 1st wavelength, $n = 2$ (2nd order)

$$\sin\theta = 600000 \times 5890 \times 10^{-10} \times 2$$

or $\sin\theta = 0.7068$

$$\therefore \theta = \sin^{-1}(0.7068)$$
$$= 44.98^\circ.$$

And, For 2nd wavelength, $(n=2)$,

$$\sin\theta' = Nn\lambda_2$$

or, $\sin\theta' = 600000 \times 2 \times 5896 \times 10^{-10}$

or, $\theta' = \sin^{-1}(0.7075)$

$$= 45.03^0$$

$$\therefore \Delta\theta = \theta' - \theta = 45.03 - 44.98 = 0.05^0 \text{ Ans.}$$

Q③ If X-ray of wavelength 0.5 Å are diffracted at an angle of 5° in first order. What is the spacing betⁿ the adjacent planes of crystal? At what angle the 2nd order maxm occurs?

⇒ Solⁿ Given

Wavelength (λ) = 0.5 Å

$= 0.5 \times 10^{-10}$ m

angle of diffraction (θ) = 5°

1st order diff. (n) = 1

Hints.

$2d\sin\theta = n\lambda$

1st order, $\theta = ?$ $n = 1$

2nd, $\theta = ?$ for $n = 2$

⇒ ∵ We know that,

$$2d\sin\theta = n\lambda$$

$$\Rightarrow d = \frac{\lambda}{2\sin\theta} \quad [\because n=1]$$

$$= \frac{0.5 \times 10^{-10}}{2 \times \sin 5^\circ} = 2.87 \times 10^{-10}\ m$$

For 2nd order Maximum, (n = 2),

$$\because 2d\sin\theta = n\lambda$$

$$\text{or, } \sin\theta = \frac{\cancel{2} \times 0.5 \times \cancel{10^{-10}}}{\cancel{2} \times 2.87 \times \cancel{10^{-10}}}$$

$$\text{or } \theta = \sin^{-1}\left(\frac{0.5}{2.87}\right)$$

$$\boxed{\therefore \theta = 10.03^\circ}$$

∴ The 2nd order maxima occurs at an angle of 10.03° Ans.

Q④ Light of wavelength 600nm is incident normally on a slit of width 0.1 mm, what is the intensity at $\theta = 0.2^\circ$?

⇒ Given,

$\lambda = 600nm = 6 \times 10^{-7}$ m

$d = 0.1mm = 0.1 \times 10^{-3}$ m

$\theta = 0.2^\circ$

Now,

$$\alpha = \frac{\pi}{\lambda} d\sin\theta$$

$$= \frac{0.1\times10^{-3}\times \sin 0.2^\circ}{6\times10^{-7}}\times\pi$$

$$= 0.58\,\pi$$

Again,

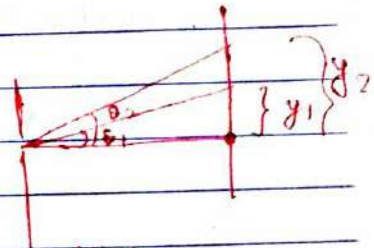
$$I_\theta = \left(\frac{\sin\alpha}{\alpha}\right)^2 I_m$$

$$= \left(\frac{\sin 0.58\pi}{0.58\pi}\right)^2 I_m$$

$$= (0.53)^2 I_m$$

$$\boxed{\therefore I_\theta = 0.28\, I_m}$$

Q. A slit 1mm wide is illuminated by light of wavelength 589nm. We see a diffraction pattern on a screen 3m away. What is the difference betn first two minima on the same same side of central maxima?



⇒ We know that,

For secondary minima,

$$d\sin\theta_n = n\lambda$$

For 1st two minima,

$n=1$	$n=2$
$d\sin\theta_1 = \lambda$	$d\sin\theta_2 = 2\lambda$
or $\sin\theta_1 = \lambda/d$	or $\sin\theta_2 = \frac{2\lambda}{d}$
For small angle,	For small angle,
$\theta_1 \approx \lambda/d$	$\theta_2 \approx \frac{2\lambda}{d}$

Hints

$d\sin\theta_1 = \lambda$

$\approx \theta_1 = \lambda/d$

$d\sin\theta_2 = 2\lambda$

$\approx \theta_2 = 2\lambda/d$

From fig

$\theta_1 = \frac{y_1}{D}$

$\theta_2 = \frac{y_2}{D}$

Again from fig:-

$$\theta_1 = \frac{y_1}{D} \quad \& \quad \theta_2 = \frac{y_2}{D}$$

Now,

$$\frac{y_1}{D} = \frac{\lambda}{d} \quad \& \quad \frac{y_2}{D} = \frac{2\lambda}{d}$$

or, $y_1 = \frac{\lambda D}{d}$ or, $y_2 = \frac{2\lambda D}{d}$

Also, $y_2 - y_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$

$$= \frac{\lambda D}{d}$$

$$= \frac{589 \times 10^{-9} \times 3}{1 \times 10^{-3}}$$

$$= 1.767 \times 10^{-3}\ m$$

Q. The distance betn 1st and 5th minima of a single slit diffraction pattern is 0.35 mm with the screen 40 cm away from the slit, when the light of wave length 550 nm is used. Find the slit width and the angle of the 1st order diffraction minima? (0.01°)

⇒ Soln, Given

difference betn 5th & 1st minima $(y_5 - y_1) = 0.35\ mm$
$= 0.35 \times 10^{-3}\ m$

Wavelength $(\lambda) = 550\ nm = 550 \times 10^{-9}\ m$

distance betn slit and screen $(D) = 40\ cm = \frac{40}{100}\ m$
$= 0.4\ m$

Now, we know that, For minima,

$$y_5 = \frac{5\lambda D}{d} \quad \& \quad y_1 = \frac{\lambda D}{d}$$

$$\therefore y_5 - y_1 \Rightarrow \frac{5\lambda D}{d} - \frac{\lambda D}{d} = \frac{4\lambda D}{d}$$

or, $0.35 \times 10^{-3} = \frac{4\lambda D}{d}$

$$\Rightarrow d = \frac{4 \times 550 \times 10^{-9} \times 0.4}{0.35 \times 10^{-3}} = 2.51 \times 10^{-3}\ m$$

Now,

For 1st order minima, $(n=1)$

$d\sin\theta = n\lambda$

or $d\sin\theta = 1\cdot\lambda$

or $\sin\theta = \frac{\lambda}{d}$

or $\sin\theta = \frac{550\times10^{-9}}{2.51\times10^{-3}}$

or $\theta = \sin^{-1}(2.19\times10^{-4})$

$\boxed{\therefore \theta = 0.012^{\circ}}$ Ans.

Q.7 Fraunhoffer's diffraction pattern is obtained with a slit of width 0.28 mm and the light source of wave length 6000Å. Determine the angle at which the 1st dark band to next bright band are formed?

⇒ Given,

width of slit $(d) = 0.28\,mm = 0.28\times10^{-3}\,m$

wavelength of light $(\lambda) = 6000Å = 6000\times10^{-10}\,m$

For 1st dark band $(n=1)$,

$d\sin\theta_n = n\lambda$

$\Rightarrow \theta_1 = \sin^{-1}\left(\frac{1\cdot\lambda}{d}\right)$

$= \sin^{-1}\left(\frac{6000\times10^{-10}}{0.28\times10^{-3}}\right)$

$= \sin^{-1}(2.143\times10^{-3})$

$\therefore \theta_1 = 0.122^{\circ}$

For next bright band $(n=1)$,

$d\sin\theta_n = (2n+1)\lambda/2$

$\Rightarrow (\theta_1') = \sin^{-1}\left[(2\times1+1)\frac{\lambda}{2d}\right]$

$\Rightarrow (\theta_1)' = \sin^{-1}\left(\frac{3\lambda}{2d}\right)$

$\Rightarrow (\theta_1)'$

$\Rightarrow (\theta_1)' = \sin^{-1}\left(\frac{3\lambda}{2d}\right)$

$= \sin^{-1}\left(\frac{3 \times 6000 \times 10^{-10}}{2 \times 0.28 \times 10^{-3}}\right)$

$= \sin^{-1}(3.214 \times 10^{-3})$

$= 0.184^\circ$

$\therefore$ Angle betn 1st dark band to next bright band is,

$\Rightarrow (\theta_1)' - (\theta_1) = 0.184^\circ - 0.122^\circ = 0.062^\circ$

$\boxed{\text{i.e. } \Delta\theta = 0.062^\circ}$

Q. Fraunhoffer's diffraction pattern is observed using light of wavelength 550 nm and a single slit. The bright band next to the 1st dark band is formed at an angle of 15 min. Calculate the width of the slit.

$\Rightarrow$ Soln.

Here, Given, Wavelength of light $(\lambda) = 550\,nm = 550 \times 10^{-9}\,m$

$\Delta\theta = \theta_1' - \theta_1 = 15\,min. = \frac{15}{60} \times \frac{\pi}{180} = 4.365 \times 10^{-3}$ rad.

Now, For 1st dark to next bright band, [bright $\Rightarrow (2n+1)\lambda/2$ dark $n\lambda$; $d\sin\theta$]

$\frac{3\lambda}{2d} - \frac{\lambda}{d} \Rightarrow \frac{3\lambda - 2\lambda}{2d}$

$\Rightarrow \frac{\lambda}{2d} = 4.365 \times 10^{-3}$

$\Rightarrow d = \frac{\lambda}{2 \times 4.365 \times 10^{-3}} = \frac{550 \times 10^{-9}}{2 \times 4.365 \times 10^{-3}}$

$\boxed{d = 6.30 \times 10^{-5}\,m}$

Q. The wavelength of visible spectrum are approximately 400 nm (violet) and 700 nm (red). Find the angular width of 1st order visible spectrum produced by a plane grating with 600 slits per mm

When white light fall normally on the grating.

=) Given,

wavelength of voilet (λ_v) = 400 nm
$= 400 \times 10^{-9}$ m

wavelength of red (λ_r) = 700 nm
$= 700 \times 10^{-9}$ m

diffraction grating (N) = 600 /mm
$= 600 \times 10^{3}$ /m

To find angular width of 1st order visible, $(n = 1)$

$$\boxed{d \sin\theta_{1r} = n\lambda_r \Rightarrow \text{or, } \theta_{1r} = \frac{1 \cdot \lambda_r}{d}}$$

$\sin\theta_r = N \cdot n \cdot \lambda_r$
$= 600 \times 10^3 \times 1 \times 700 \times 10^{-9}$
$\sin\theta_r = 0.42$
$\Rightarrow \theta_r = \sin^{-1}(0.42) = 24.83°$.

And, For voilet,
$\sin\theta_v' = N \cdot n \cdot \lambda_v$
or, $\sin\theta_v = 600 \times 10^3 \times 1 \times 400 \times 10^{-9}$
or $\sin\theta_v = 0.24$
$\therefore \theta_v = \sin^{-1}(0.24) = 13.886°$

$\therefore$ Angular width for 1st order visible spectrum $= 24.830 - 13.886$
$= \underline{10.944°}$

(10) The path difference betn the intensity at central maxima and the point on the screen at a point on the screen is $\left(\frac{1}{8}\right)^{th}$ of the wavelength. Find the ratio of intensity of this point to that at the centre of the central maxima.

=) Given, path diff. $= \frac{1}{8} \times \lambda$

Here,

$$\alpha = \frac{\pi}{\lambda} \times \text{path diff.}$$

$$= \frac{\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{8} = \frac{\pi}{8}$$

Now,

$$\frac{I_\theta}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin \pi/8}{\pi/8}\right)^2$$

$$= \left(\frac{\sin \frac{180}{8}}{\frac{3.1428}{8}}\right)^2$$

$$= 0.949$$

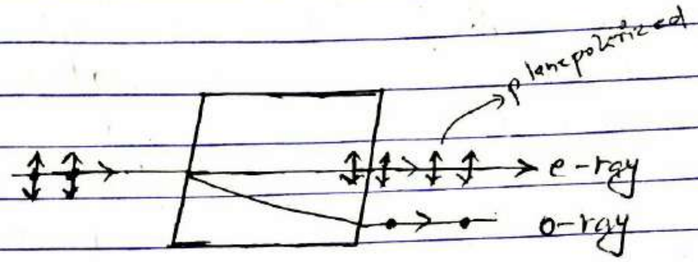
$\therefore$ $\boxed{\frac{I_\theta}{I_m} = 0.949}$

Polarization :-

↳ The phenomenon of restriction of vibration of a light wave to vibrate in a particular direction is called polarization.

Double refraction:-

⇒ When a ray of light is incident on a some crystals (like quartz, calcite) we get two refracted beams of light of equal intensity this phenomenon is known Double refraction. One of the refracted beam obey the law of refraction called ordinary ray (o-ray) which is partially polarized. And another ray doesn't obey the laws of refraction called extra-ordinary ray (e-ray) which is completely plane polarized.



(II) Nichol prism:-

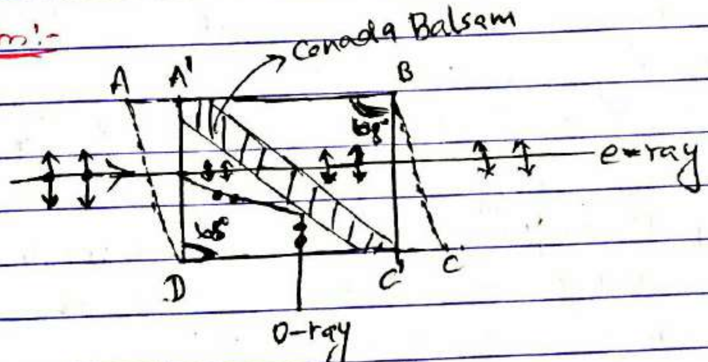


Fig: Nichol prism

The Nichol prism is very important device to produce and analyze the plane polorized light. When the Nichol prism is used to produce the plane polarized light, it is called polarizer and when it is used to analyze the plane polarized light, it is called analyzer.

The Nichol prism is a double refracting calcite crystal having length 3 times than its breadth. The face AD and BC of the crystal are ~~cutted~~ cut so that its angle reduced to 68°. The resulting crystal is again cut along the axis A'C' and then these two parts are connected together by canada balsam, whose refractive index is lies between e-ray and o-ray for the calcite.

When a ordinary ray of light is incident on the prism it refracted into e-ray and o-ray the adjustment is so that the o-ray total internally reflected which is absorbed blacken surface of the prism (lower surface). And we get only plane polarized e-ray output from Nichol prism.

(4) Half wave plate :-

↳ It is a double refracting crystal having optic axis parallel to its refracting face and thickness is so adjusted that it introduces half of wavelength of path difference betn e-ray and o-ray. If t be the thickness of the half wave plate. μ_o and μ_e are refractive indices of plate for o-ray and e-ray. Then the path difference between e-ray and o-ray is,

$$\text{Path diff} = \mu_e t - \mu_o t$$
$$= t(\mu_e - \mu_o)$$

Also, we know that,

for half wave plate,

$$\text{Path difference} = \lambda/2$$

Now,

$$t(\mu_e - \mu_o) = \lambda/2$$

$$\Rightarrow t = \frac{\lambda}{2(\mu_e - \mu_o)} \quad \text{for positive crystal}$$

and

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} \quad \text{for negative crystal.}$$

(#) Quater wave plate :-

↳ It is a double refracting crystal having optic axis parallel to its refracting face and thickness is so adjusted that it introduce quarter of wavelength of path difference betn o-ray and e-ray. If 't' be the thickness of the quarter wave plate. μ_o and μ_e are refractive indices of plate for o-ray and e-ray. Then, the path difference betn e-ray and o-ray is,

$$\text{path diff} = \mu_e t - \mu_o t$$
$$= t(\mu_e - \mu_o)$$

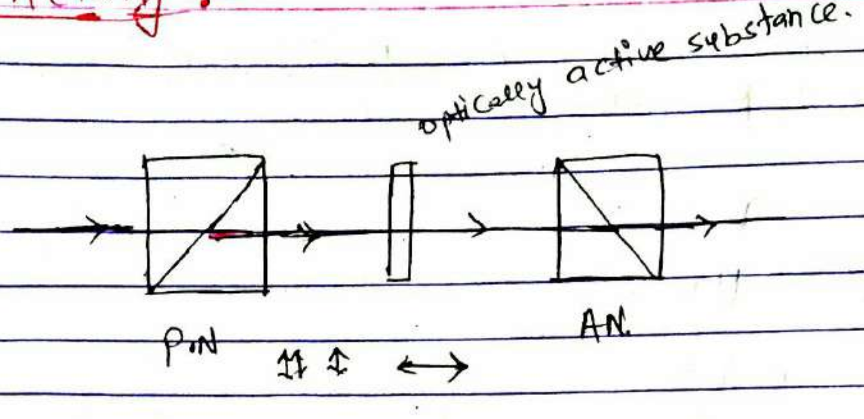
Also, we know that,
For quarter wave plate,

$$\text{path difference} = \lambda/4$$

Now,
$$t(\mu_e - \mu_o) = \lambda/4$$

$$\Rightarrow t = \frac{\lambda}{4(\mu_e - \mu_o)} \quad \text{for}$$

(#) Optical Activity :-



⇒ When two Nichol prism PN and AN are placed such that their optic axis are perpendiculer to each other, then we do not obtaine output from AN but some substances are placed between PN and AN, we get output from AN. This is because the substance rotates the plane of vibration of light. This substances are known as optically active substance and the phenomenon is known as Optical activity.

(x) Specific Rotation :-

↳ Some optically active substance can rotate the plane of vibration of light, the angle of rotation depends on wave-length, temperature, length of the substance and concentration of the substance.

The specific rotation for a given wavelength of light and at a given temperature is defined as the angle of rotation produced by the solution of length 1 decimetre (10cm) and concentration 1 gm/cc.

i.e.

$$\boxed{[S]_{\lambda}^{\theta} = \frac{10\,\theta}{l \cdot C}}$$

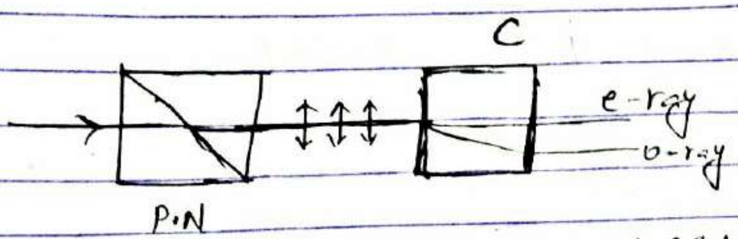
Where,

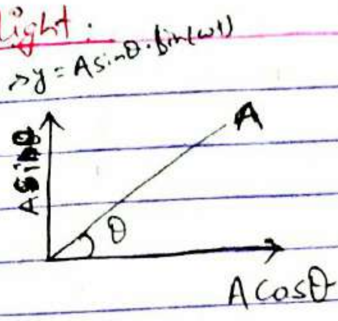
$\theta \rightarrow$ Angle of rotation

$l \rightarrow$ length in cm

$C \rightarrow$ Concentration in gm/cc.

④ Linearly, circularly and elliptically polarized light:





$\Rightarrow$ Consider an unpolarized beam of light is incident on the Nichol prism then we get plane polarized light which is again allowed to incident on a double refracting crystal whose optic axis is parallel to refracting faces. $\because y = a\sin(\omega t + \phi)$

The crystal splits the plane polarized light into e-ray and o-ray. They move in same direction but with different velocities. So, they have some phase difference let it be 'ϕ'.

Let 'A' be the amplitude of the incident light(radiation) on the crystal and 'θ' be the angle made by incident light to the optic axis of crystal.

From above fig:-

For o-ray,

$$x = A\cos\theta \sin(\omega t + \phi)$$

and,

$$y = A\sin\theta \cdot \sin\omega t$$

Let,

$A\cos\theta = a$ and $A\sin\theta = b$; we get,

$$x = a(\sin\omega t \cos\phi + \cos\omega t \cdot \sin\phi)$$

and, $y = b\sin\omega t$

$\Rightarrow \quad y/b = \sin\omega t.$

Now

$$\frac{x}{a} = \sin\omega t \cdot \cos\phi + \cos\omega t \cdot \sin\phi$$

or $$\frac{x}{a} = \frac{y}{b} \cdot \cos\phi + \left(\sqrt{1 - \frac{y^2}{b^2}}\right) \sin\phi$$

or $$\left(\frac{x}{a} - \frac{y}{b}\cos\phi\right) = \left(\sqrt{1 - \frac{y^2}{b^2}}\right) \sin\phi$$

squaring both sides ; we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\phi - \frac{2xy\cos\phi}{ab} = \left(1 - \frac{y^2}{b^2}\right)\sin^2\phi$$

or, $$\frac{x^2}{a^2} + \frac{y^2}{b^2}\left(\cos^2\phi + \sin^2\phi\right) - \frac{2xy}{ab}\cos\phi = \sin^2\phi$$

or, $$\boxed{\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\phi + \frac{y^2}{b^2} = \sin^2\phi}$$

This is the general eqn of ellipse.

[phase diff = 0 means $(\phi) = 0$] with same phase

Case I, If $\phi = 0$ and $a \neq b$

$$\frac{x^2}{a^2} - \frac{2xy}{ab} + \frac{y^2}{b^2} = 0$$

[$\because \sin 0 = 0$, $\cos 0 = 1$]

or, $$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$\Rightarrow$ $\boxed{x = \frac{a}{b}y}$, This is the eqn of st. line, so the light is linearly polarized if phase difference betn e-ray and o-ray is zero.

Case - II, If $\phi = \pi/2$ and $a \neq b$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

[$\because \sin 90 = 1$
$\cos 90^\circ = 0$]

This is the eqn of ellipse, so, the light is elliptically polarized if phase difference betn o-ray and e-ray is $(\pi/2)$. and 'a' doesn't equal to 'b'.

Case - III, If $\phi = \pi/2$ and $a = b$

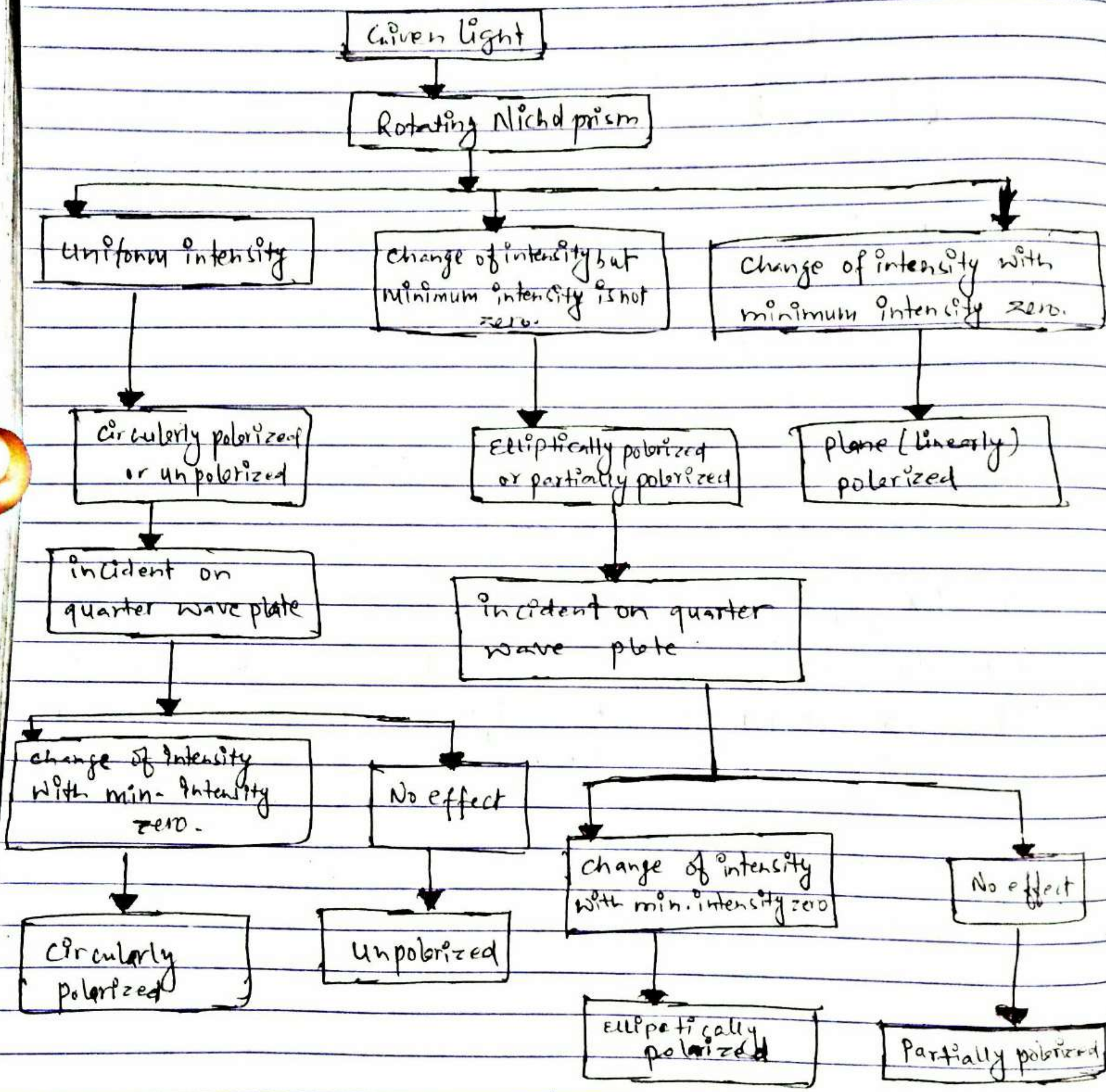
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow \boxed{x^2 + y^2 = a^2}$$

This is the eqn of circle, so, the light is circularly polarized if phase difference betn o-ray and e-ray is $(\pi/2)$ and 'a' is equal to 'b'.

⊛ Determination of linearly, circularly and elliptically polarized light:



Geometrical optics

(1) Combination of lenses :-

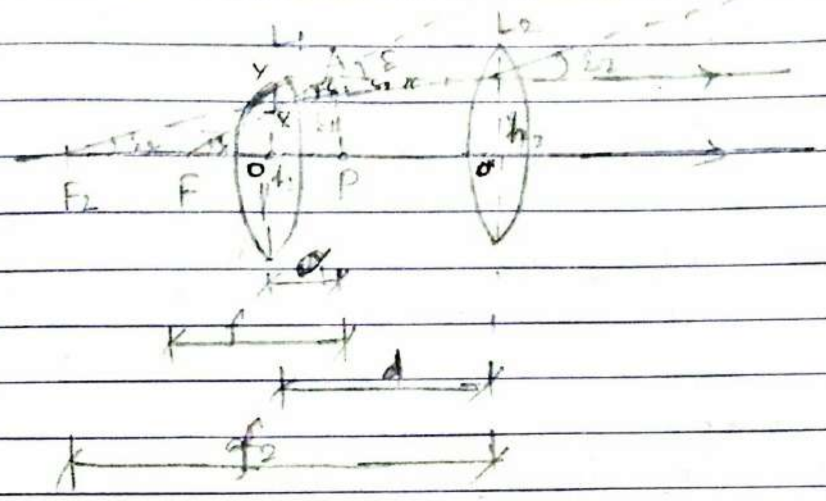
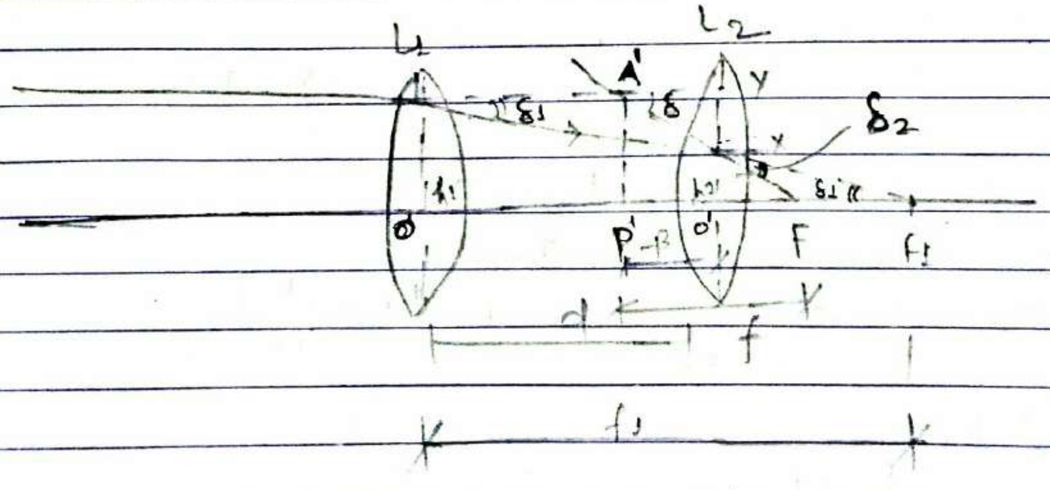


fig (a)



⇒ Consider two convex lenses L_1 and L_2 of focal length f_1 and f_2 are placed coaxially at a distance of 'd'. Let 'δ_1', 'δ_2' and 'δ' are angle of deviation produced by lens L_1, L_2 and resultant of L_1 & L_2 respectively.

The refracting surface AP and A'P' of focal length 'f' represents the resultant of combination of "L_1" ~~and~~ and 'L_2'.

Now from fig (a),

$$\delta = \delta_1 + \delta_2$$

For small angles, δ_1, δ_2 and δ,

$$\tan\delta = \tan\delta_1 + \tan\delta_2$$

or, $$\frac{h_2}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \quad \text{--- (i)}$$

~~Again from ΔXYB~~, Again,

from fig (a),

$$h_1 = h_2 - XY$$

From ΔXYB,

$$\delta_2 = \frac{XY}{d} \qquad [\because BY = d]$$

or, $$XY = d\delta_2 = \frac{d h_2}{f_2}$$

or, $$h_1 = h_2 - \frac{d h_2}{f_2} = h_2\left(1 - \frac{d}{f_2}\right)$$

Then, eqn (i), becomes

$$\frac{h_2}{f} = \frac{h_2(1 - d/f_2)}{f_1} + \frac{h_2}{f_2}$$

or, $$\frac{1}{f} = \frac{1}{f_1}\left(1 - \frac{d}{f_2}\right) + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{f_1} - \frac{d}{f_1 \cdot f_2} + \frac{1}{f_2}$$

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}}$$

This ~~eqn~~ is the expression of equivalent focal length of combination of two lenses.

If L_1 and L_2 are in contact, $d=0$.

$$\Rightarrow \boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}} \quad //$$

To determine the position of principle points 'p' and p'

From fig (a), ΔFOX and ΔFPA are similar,

$$\frac{h_1}{h_2} = \frac{FO}{f}$$

$$\text{or, } \frac{h_1}{h_2} = \frac{f-\alpha}{f}$$

We know that

$$h_1 = h_2\left(1 - \frac{d}{f_2}\right)$$

$$\Rightarrow \frac{h_2 - \frac{d h_2}{f_2}}{h_2} = \frac{f-\alpha}{f}$$

$$\Rightarrow \frac{h_2 f_2 - d h_2}{f_2 h_2} = \frac{f-\alpha}{f}$$

$$\Rightarrow 1 - \frac{d}{f_2} = 1 - \frac{\alpha}{f} \qquad \Rightarrow \boxed{\alpha = \frac{df}{f_2}}$$

From fig(b),

In $\Delta FO'X$ and $\Delta FP'A'$,

$$\frac{h_2}{h_1} = \frac{FO'}{f}$$

or, $$\frac{h_2}{h_1} = \frac{f-(-\beta)}{f}$$

or, $$\frac{h_2}{h_1} = \frac{f+\beta}{f}$$

Again, $h_2 = h_1\left[1 - d/f_1\right]$ [similar from fig(b)]

$$\Rightarrow \frac{h_1 - \frac{h_1 d}{f_1}}{h_1} = \frac{f+\beta}{f}$$

or, $$\frac{h_1 f_1 - d h_1}{f_1 h_1} = \frac{f+\beta}{f}$$

or, $$\cancel{1} - \frac{d \cancel{h_1}}{f_1 \cdot \cancel{h_1}} = \cancel{1} + \frac{\beta}{f}$$

or, $$\frac{d f}{f_1} = -\beta$$

$$\therefore \boxed{\beta = -\frac{df}{f_1}}$$

$$\therefore \boxed{\alpha = \frac{fd}{f_2} \quad \text{and} \quad \beta = \frac{-fd}{f_1}}$$

⊗ Cardinal points :-

⇒ In case of thin lens, we neglect the thickness of the lens to calculate the various formulas but in case of thick lens or combination of lenses, we can not neglect the thickness of lens or distance between two lenses. In fact, there are six major points

(i) Two principal points (ii) Two focal points

(iii) Two Nodal points,

These six points are known as cardinal points. Planes ~~passing~~ passes through this points and perpendicular to principal axis are known as cardinal planes. These cardinal points and cardinal planes are intrisic properties of lenses.

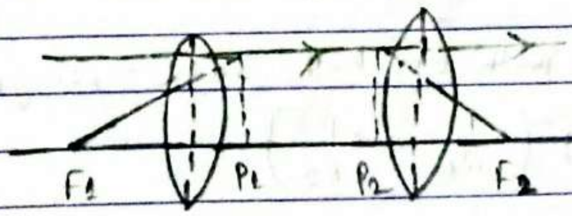


fig:-(a)

In above figure P_1 and P_2 are two principal points and and F_1 and F_2 are two focal points.

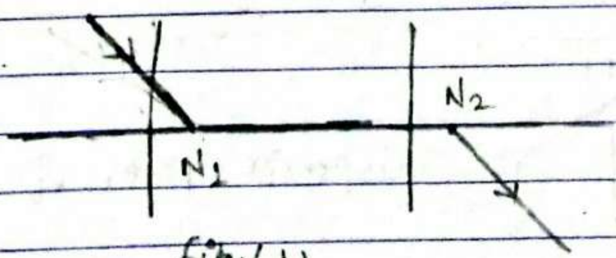


fig:-(b)

In this fig N_1 and N_2 are two Nodal points.

Chromatic Aberration :-

↳ The inability of a lens to make single image of an object is called chromatic aberration.

Achromatism :-

↳ The removal of chromatic aberration by suitable arrangement of combination of lenses is called Achromatism.

We know that from lens makers formula;

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \text{ --- (i)}$$

Where,

$f \rightarrow$ focal length of lenses.

$\mu \rightarrow$ Refractive index of lenses

R_1 and $R_2 \rightarrow$ Radii of curvature.

differentiating eqn (i) ~~with~~ on both sides; we get,

$$\frac{-df}{f^2} = (d\mu - 0)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

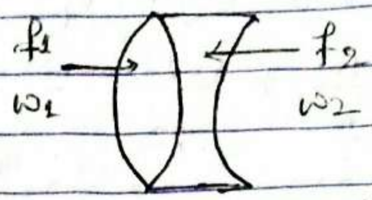
or $$\frac{-df}{f^2} = d\mu * \frac{1}{f(\mu - 1)} \quad \text{[from eqn(i)]}$$

or $$\frac{-df}{f} = \frac{d\mu}{(\mu - 1)}$$

$$\Rightarrow \boxed{\omega = \frac{-df}{f}}$$ is dispersive power of lens.

(i) When two lenses are in contact:

Consider two lenses of focal lengths f_1 and f_2. W_1 and W_2 are dispersive power of material of lenses respectively. are placed in contact with each other.



The equivalent focal length of combination of lenses is,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

differentiating both sides; we get

$$\frac{-df}{f^2} = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

or $$-\frac{df}{f^2} = \left(\frac{-df_1}{f_1}\right)\cdot\frac{1}{f_1} + \left(\frac{df_2}{f_2}\right)\times\frac{1}{f_2}$$

or $$\frac{-df}{f^2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$$

<u>for Achromatism,</u>

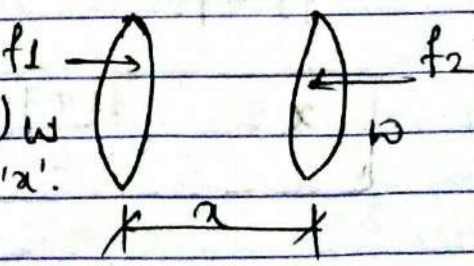
$$df = 0$$

$$\Rightarrow \boxed{\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0}$$

↳ This is the condition of Achromatism when two lenses are in contact

(ii) <u>When two lenses are separated by some distance :-</u>

⇒ Consider two convex lenses of focal lengths f_1 and f_2 made from same material (i.e. $\omega_1 = \omega_2 = \omega$) are separated by some distance 'x'.



The equivalent focal length of combination is,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 \cdot f_2}$$

differentiating both sides we get,

$$-\frac{df}{f^2} = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2} - x\left(\frac{-df_1}{f_1^2 \cdot f_2} - \frac{df_2}{f_1 \cdot f_2^2}\right)$$

or, $$\frac{-df}{f^2} = \left(\frac{-df_1}{f_1}\right) \times \frac{1}{f_1} + \left(\frac{-df_2}{f_2}\right)\frac{1}{f_2} - x\left[\left(\frac{-df_1}{f_1}\right)\cdot\frac{1}{f_1 \cdot f_2} + \left(\frac{-df_2}{f_2}\right)\frac{1}{f_1 \cdot f_2}\right]$$

or $$\frac{-df}{f^2} = \frac{\omega}{f_1} + \frac{\omega}{f_2} - x\left[\frac{\omega}{f_1 \cdot f_2} + \frac{\omega}{f_1 \cdot f_2}\right]$$

or $$\frac{-df}{f^2} = \frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{2x\omega}{f_1 \cdot f_2}$$

For Achromatism, $df = 0$

or, $$\frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{2x\omega}{f_1 \cdot f_2} = 0$$

or $$\cancel{\frac{\omega f_2 + \omega f_1 - 2x\omega}{f_1 \cdot f_2}} \Rightarrow \frac{\cancel{\omega}(f_1+f_2)}{\cancel{f_1 \cdot f_2}} = \frac{2x\cancel{\omega}}{\cancel{f_1 \cdot f_2}}$$

or $$f_1 + f_2 = 2x$$

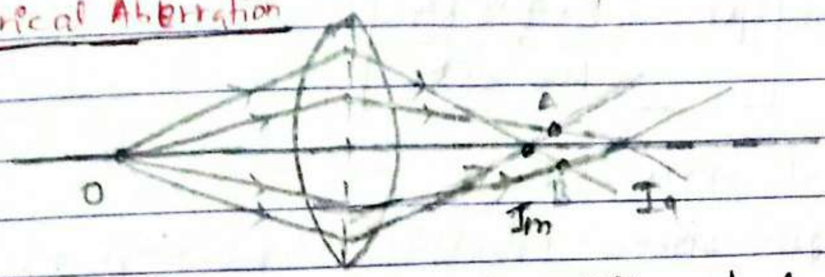
or, $$\boxed{x = \frac{f_1+f_2}{2}}$$, This is the condition of Achromatism, when two lenses are separated at a distance of 'x'.

(a) Mono chromatic Aberration :-

⇒ Aberration produced by the lens when the monochromatic light is used is called monochromatic aberration.

Types of Mono chromatic Aberration :-

(i) Spherical Aberration



Spherical Aberration is inability of lens to focus the marginal and axial rays coming from the point source placed on the principal axis at a point.

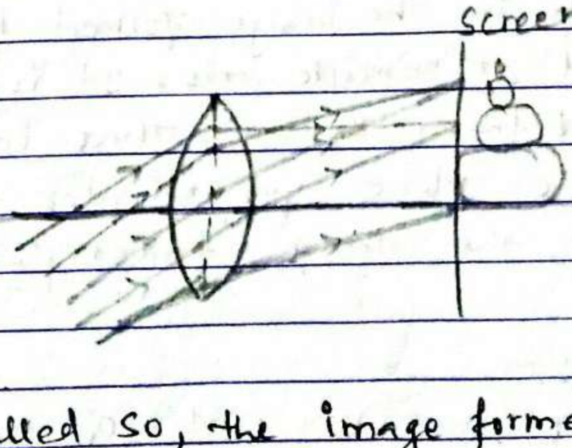
When a screen is placed perpendicular to AB, the circular image is formed

Reduction/ Remedies

→ (i) By using stops or aperture

→ (ii) By using the combination of lenses which satisfy the condition. $\boxed{d = f_1 - f_2}$.

(ii) Coma :-



⇒ Aberration produced by the lens when a point object is not situated on the principal axis is called Coma. It is called so, the image formed is comet shape.

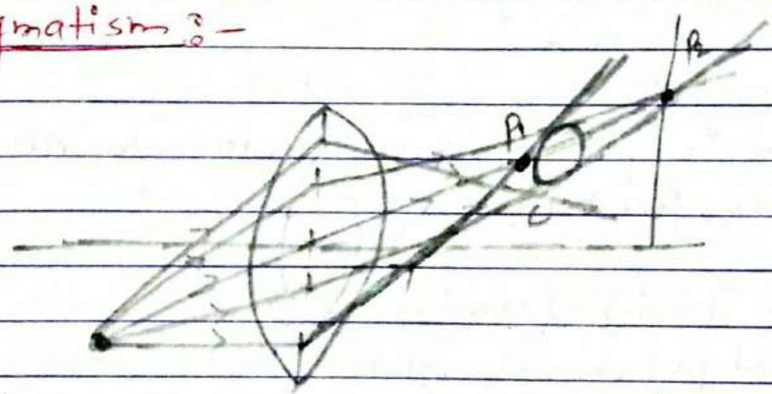
Remedies :-

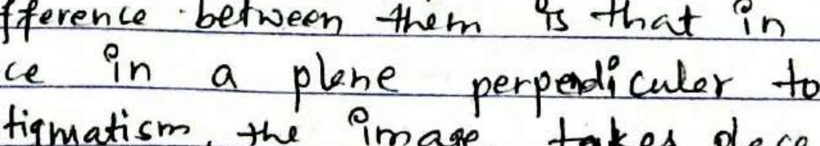
(i) By choosing proper radii of curvature of lens surface.
(ii) By using stops at proper position.
(iii) Using the lens con satisfy the Abb's sine law.

i.e. $$\boxed{M = \frac{\mu_1 \sin\theta_1}{\mu_2 \sin\theta_2}}$$

μ_1 = object space R.I.
μ_2 = Image space R.I.
θ_1 = Incident beam angle with principle axis
θ_2 = Refracted beam angle with principle axis.

(iii) Astigmatism :-



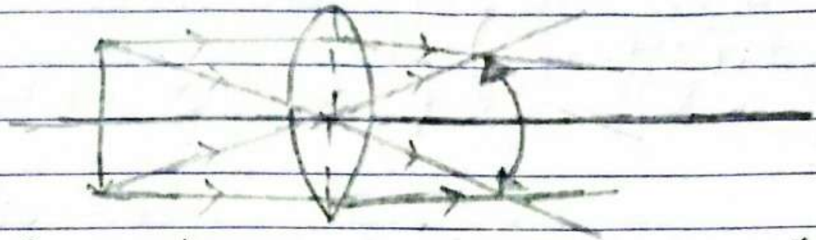


The aberration in the image formed by the lens of an point object not situated at principle axis. It is similor to coma, the difference between them is that in coma the image takes place in a plane perpendiculer to principal axis and in Astigmatism, the image takes place along the principal axis.

Remedies :-

(1) By using a convex and a concave lens of suitable focal lengths and ~~by suitable~~ separated by a distance.

(iv) Curvature of field

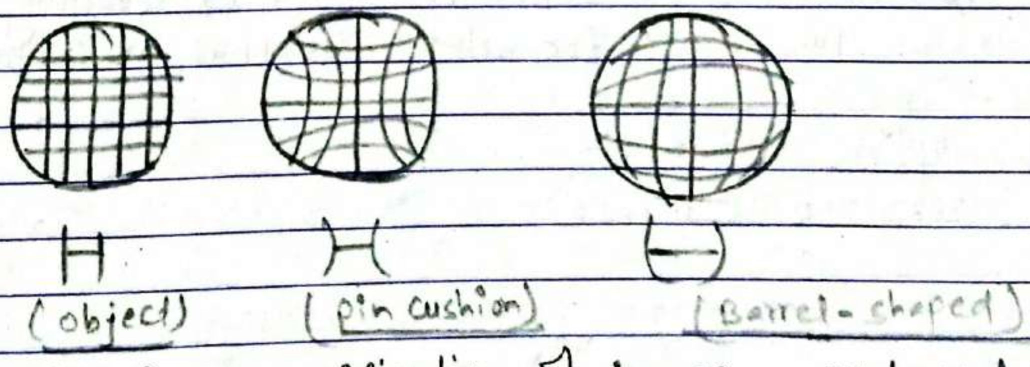


When the image formed by a single lens is curved, then this type of aberration is called curvature of field. The central portion of the image near the axis is in focus and the outer region of image is away from ~~region~~ axis are blurred.

(i) By using the combination of concave and con~~vex~~ lens which satisfy the petzwal's condition

i.e. $\boxed{\frac{f_1}{f_2} = \frac{-\mu_2}{\mu_1}}$

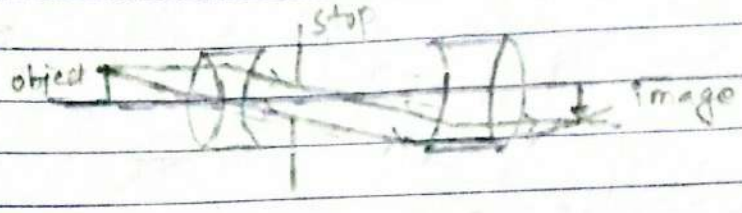
(v) Distorsion



The variation in magnification of image produced by a lens for different axial distances results an aberration called distorsion. They are two types of distorsion:→ (i) pin-cushion and (ii) Barrel-shaped distorsion

Remedies :-

(i) By using ~~stop~~ stop between two symmetrical lenses.



(11) Optical Fiber :-

↳ An optical fiber is a transparent conduit as thin as human hair, made of glass or class plastic, designed to guide light waves along its length. It works on the principle of total internal reflection.

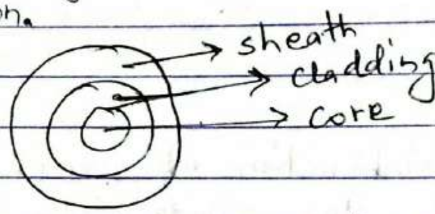


Fig: cross-sectional view of optical fiber

The optical fiber has three layers. The inner most layer in core which is light guiding region. It is surrounded by co-axial middle region known as cladding. The outer region is sheath. The refractive index of cladding is always less than the core for total internal reflection.

(x) Types of Optical fiber :-

(a) Step index optical fibers :-

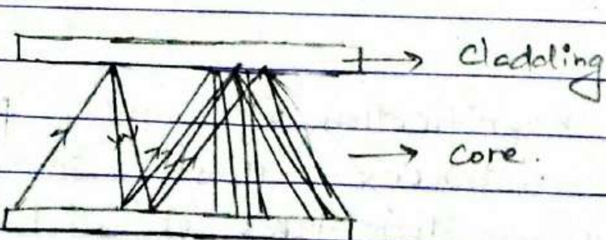


Fig: Step index optical fiber

⇒ The optical fiber in which the refractive index of core and cladding is always constant is called step index optical fiber. It has high transmission loss.

(b) Graded index optical fiber :-

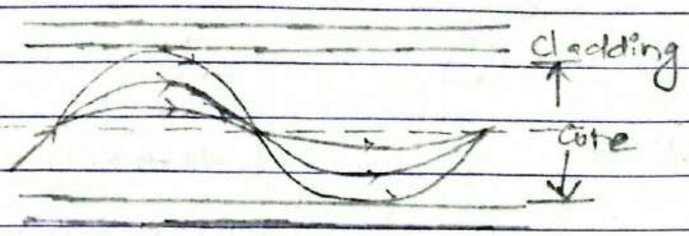


Fig: Graded index optical fiber

⇒ The optical fiber in which the refractive index of core decreases parabollically from centre to the cladding to a constant value is called graded index optical fiber. It has low transmission loss.

(#) Self focusing of optical fiber :-

⇒ The data received at the other end of a fiber can have error due to spreading of light signals as no light is perfectly monochromatic. The waves reached the other end at different time. This defect can be found in the step index fiber. In ~~graded~~ graded index fiber, this defect is minimized i.e. all the wavelengths are meet at a point on ~~the~~ the axial line of core. This is known as self focusing of optical fiber.

(#) Laser :-

⇒ Laser is a device which produces highly intense, monochromatic and coherent beam of light. The laser stands for light amplification by stimulated emission of radiations.

(*) Useful Term in Lasers:-

(a) Induced absorption:-

An atom in lower level absorbs a photon of frequency 'hν' and moves to an upper level.

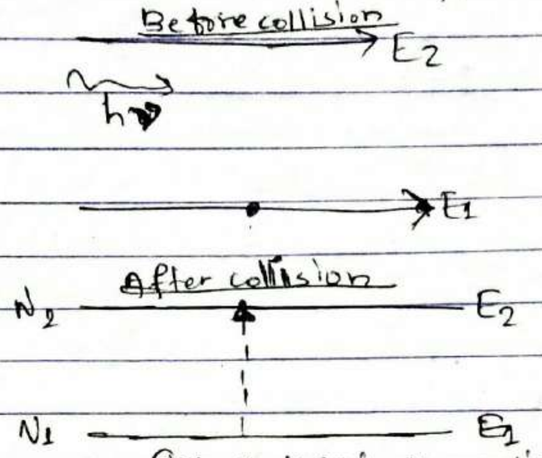


Fig: Induced Absorption

$$\left.\frac{dN_2}{dt}\right|_{In} = B_{12}\,\rho_{21} N_1$$

$N_1 \Rightarrow$ No. of atoms with electrons in the state N_1

$B_{12} \Rightarrow$ Einstein coefficient for Induced absorption.

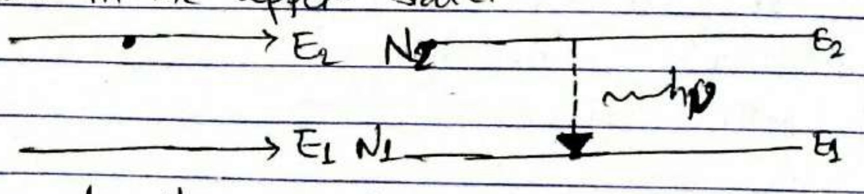
$\rho_{21} \rightarrow$ density of Electromagnetic radiation

Induced absorption transition rate is proportional to the no. of atoms with electrons in the lower state and to the density of the radiant energy incident on these atoms.

(b) Spontaneous Emission:-

An atom in upper level can decay spontaneously to the lower level and emit a photon of frequency 'hν' if the transition between E_2 and E_1 is radiative. The photon has random phase and direction.

⇒ Spontaneous transition rate is proportional to the no. of atoms with electrons in the upper state.



$$\left.\frac{dN_1}{dt}\right|_{sp} = A_{21} N_2$$

Where, N_2 = no. of atoms with electrons in the state 2.
A_{21} = Einstein coefficient for spontaneous emission

(c) Stimulated or Induced emission:-

The atoms of special elements can stay comparatively in higher energy state for a longer time and such higher energy state is called meta-stable state. The repeated interaction of photons with the excited atoms emits the highly intense, monochromatic, coherent and unidirectional beam of light which is called laser light. This process is called ~~spontaneous~~ stimulated emission of radiation.

→ Transition rate for stimulated emission is proportion to N_2 and to the density of radiation incident on the atoms with energy equal to the energy different betn two states. i.e. $\left.\frac{dN_1}{dt}\right|_{st.} = B_{21} \rho_{21} N_2$

Where, B_{21} → Einstein coefficient. , ρ_{21} → density of incident radiation

N_2 → no. of atoms in the state $n=2$

(iv) Population Inversion :-
The establishment of situation in which the no. of atoms in the higher energy level is greater than that of the lower energy level is called population inversion. In this system, atoms is in temperature equilibrium and there is more atoms in low states than in higher states.

(v) Optical pumping :- Optical pumping is a process in which light is used to raise (or pump) electrons from a lower energy level in an atom or molecule to higher one by supplying photons carrying the energy ($h\nu' = E_3 - E_1$) which is termed as optical pumping.

He-Ne Laser :-

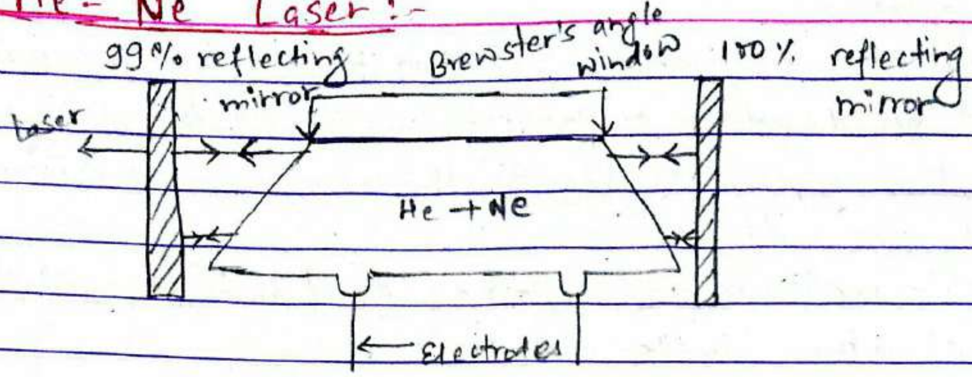


fig (a) He-Ne Laser

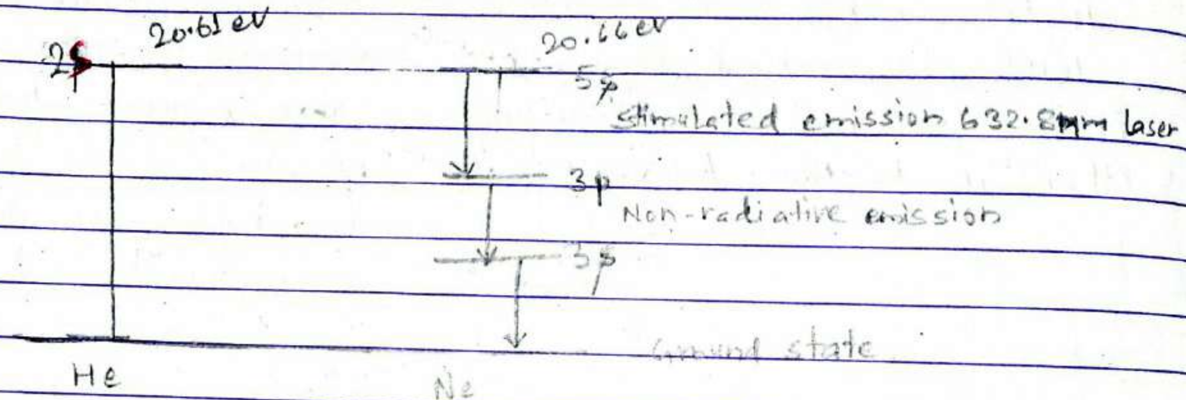


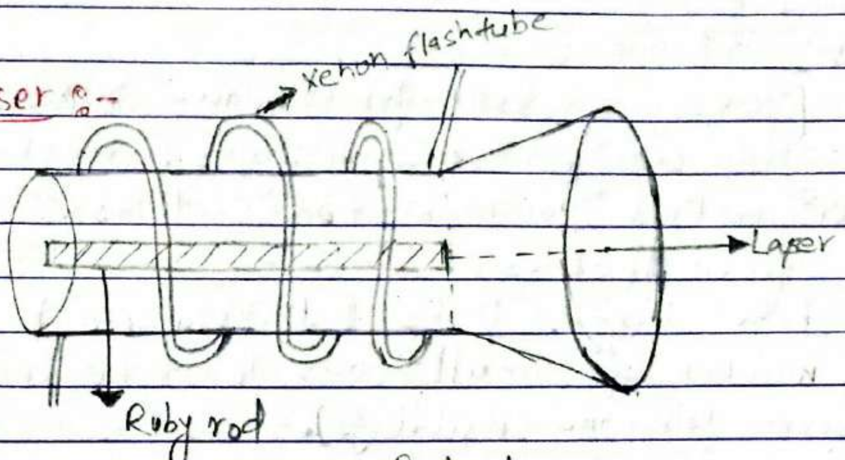
fig (b) Energy level diagram of He-Ne laser

⇒ The simple construction of He-Ne laser is shown in above fig (a). It consists of a discharge tube of length about 5 cm and diameter about 5mm. The mixture of He & Ne about 5:1 and at a pressure of 1 torr is kept inside the tube two ~~edge~~ ends of the tubes are cut at the Brewster's angle (45°). There are two reflecting mirrors at the ends of the tube, one of them is 100% reflecting and another is 99% reflecting. The gas inside the tube is ionized by passing the current from electrodes.

When the current is passed through the electrodes. The He atom is ionized and goes to the excited state (2's state) of energy 20.61 eV. The energy of 5's state of Neon is 20.66 eV

When the excited He atom collides with Ne atom, the energy transfer to Ne and it excited to 5's state. The excited Ne atom goes to 3p state ~~which~~ with stimulated emission by producing the laser of wavelength 632.8 nm. The photons which are parallel to the length of the tube are reflected by two reflecting mirrors and rapidly build to the intense beam & escape out from 99% reflecting mirror in the form of highly intense beam of laser.

(H) Ruby laser :-



Fig(a) Ruby Laser

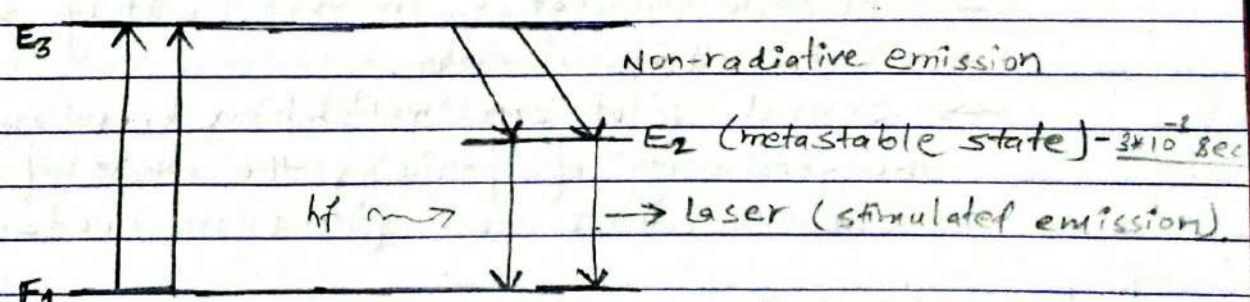


fig.(b) Energy level diagram of chromium

→ The simple construction of ruby laser is shown in above fig (a). It consists of a ruby rod (Mixture of Al_2O_3 and 0.05% of Cr_2O_3), which is placed inside a glass tube. The glass tube is wounded by Xenon flash tube.

When the light is produced from xenon flash tube. The photon is absorbed by the ruby rod and the chromium atom is excited to energy state (E_3). Then, the excited chromium atom goes to E_2 state with non-radiative emission, which is metastable state. Now, the chromium atom at E_2 goes to ground state E_1 with stimulated emission and the laser is produced which is escape out from the glass tube.

Applications of optical fiber

⇒ Optical fibers are used for various purposes such as:

→ The light pipes as diagnascopes.

→ Fiber optical communication systems.

→ As fiber cables.

→ Telephone trunk links (links capable of carrying a large number of simultaneous telephone conversations between telephone buildings).

→ Used in undersea links,

→ Missile guidance in the military sphere.

→ Video transmission.

→ Several pilot experiments have been run to examine the feasibility of providing the whole of a community's communications or information needs.

Applications of laser :-

⇒ Laser can be used in :-

→ In drilling holes in hard metal and diamond because of its intense energy.

→ In surgery for the treatment purpose because it produces the localized heat.

→ In radios and television because, being coherent, it is modulated to send hundred of messages simultaneously.

→ For automatic control of rockets and satellites.

→ In war for detecting and destroying aeroplanes, missiles and tanks.

→ In scientific research.

→ In fiber optics and in holography ~~to~~ produce three diamensional optical images.

Numericals :-

① Given the dispersive power of crown and flint glass edges 0.02 and 0.04 respectively. Find the focal lengths of two components of an achromatic doublet of focal length 20cm.

⇒ Soln. Given,

dispersive power of 1st $(\omega_1) = 0.02$

" " of 2nd $(\omega_2) = 0.04$

focal length of 1st $(f_1) = ?$

focal length of 2nd $(f_2) = ?$

Equivalent wavelength of doublet $(f) = 20cm$

We know that,

for achromatism,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\text{or, } \frac{\cancel{0.02}}{f_1} = -\frac{\cancel{0.04}\,2}{f_2}$$

$$\therefore \boxed{f_2 = -2 \cdot f_1} \quad \text{--- ①}$$

Now,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\text{or } f = \frac{f_1 \cdot f_2}{f_1 + f_2} \quad \Rightarrow \quad 20 = \frac{f_1 \cdot (-2f_1)}{f_1 - 2f_1}$$

or $20 = \frac{-2f_1 \cdot f_1}{-f_1}$

or, $f_1 = 20/2 = 10\,cm$

$\therefore$ $\boxed{f_1 = 10\,cm}$

And from eqn (i),

$f_2 = -2f_1$

or $f_2 = -2 \times 10 = -20\,cm$

$\therefore$ $\boxed{f_2 = -20\,cm}$

Q.② Two thin convex lenses having focal lengths 10 cm and 4 cm are co-axially separated by distance of 5 cm, find the equivalent focal lengths of combination. Also, determine the positions of principal points.

⇒ Soln: Here given,

Focal lengths of 1st lens (f_1) = 10 cm

focal length of 2nd lens (f_2) = 4 cm

separation distance (d) = 5 cm

Equivalent focal length (f) = ?

positions of principal points,

α = ? β = ?

$\because$ We know that,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}$$

$$= \frac{1}{10} + \frac{1}{4} - \frac{\cancel{5}}{\cancel{10}_2 \times 4}$$

$$= \frac{4 + 10 - 5}{40}$$

$$= \frac{9}{40}$$

or $f = 1/\frac{9}{40} =$

$\therefore f = \frac{40}{9}\,cm$

Now,

$\alpha = \frac{df}{f_2}$ & $\beta = \frac{-df}{f_1}$

or $\alpha = \frac{5 \times \frac{40}{9}}{4}$

or $\beta = \frac{-5 \times \frac{40}{9}}{10}$

or $\alpha = \frac{\cancel{200}^{50}}{9} \times \frac{1}{\cancel{4}}$

or $\beta = -\frac{20\cancel{0}}{9} \times \frac{1}{\cancel{10}}$

$\boxed{\therefore \alpha = \frac{50}{9} \text{ cm}}$

$\boxed{\therefore \beta = -\frac{20}{9} \text{ cm}}$

③ Two thin lenses of focal lengths 8 cm each are separated by 4 cm determine the equivalent ~~length~~ focal length of the combination and illustrate the principal points in fig and also find the distance between two principal points.

⇒ Soln: Here Given,

focal length of 1st lens (f_1) = 8 cm

focal length of 2nd lens (f_2) = 8 cm

separation distance (d) = 4 cm

Equivalent focal length (f) = ?

~~positi~~ principal points,

$\alpha = ?$ $\beta = ?$

distance betn principal points = ?

Now, we know that,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{8} + \frac{1}{8} - \frac{\cancel{4}}{8 \times \cancel{8}2}$$

$$= \frac{2+2-1}{16} = 3/16$$

$\therefore f = 1/3/16 = \frac{16}{3}$ cm

And, $\alpha = \frac{df}{f_2} = \frac{4 \times 16/3}{8} = \frac{\cancel{64}^{8}}{3} \times \frac{1}{\cancel{8}} = 8/3$ cm

$$\beta = -\frac{df}{f_1} = -\frac{4 \times 16/3}{8} = -\frac{64}{3} \times \frac{1}{8}$$

$$\boxed{\beta = -8/3 \text{ cm}}$$

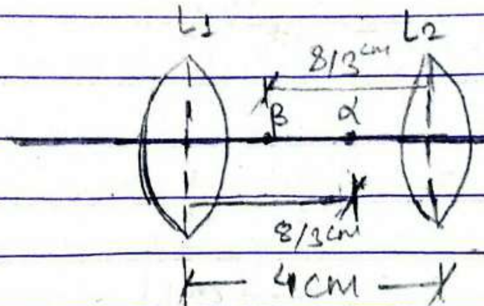


Fig: illustration of principal points

Now,

Here distance betn 'α' and 'β' is given by;

$$= \alpha + \beta - d$$

$$= \frac{8}{3} + 8/3 - 4$$

$$= 16/3 - 4$$

$$= \frac{16-12}{3}$$

$$= 4/3 \text{ cm Ans.}$$

∴ The distance betn principal points 'α' and 'β' is $4/3$ cm.

(4) Two thin convex lenses having focal lengths 3 cm and 4 cm are co-axially separated by distance of 2 cm. Find the equivalent focal length. Also determine the position of principal points. Find the position and nature of image, if an object is placed 4 cm infront of the first lens.

⇒ Soln, Given,

Focal length of 1st convex lens $(f_1) = 3$ cm

focal length of 2nd convex lens $(f_2) = 4$ cm

separation distance of lenses $(d) = 2$ cm

Equivalent focal length $(f) = ?$

positions of principal points ($\alpha = ?$ and $\beta = ?$)

And, Nature of image at 4 cm infront of 1st lens.

→ We know that,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}$$

$$= \frac{1}{3} + \frac{1}{4} - \frac{2}{3 \times 4 \, 2}$$

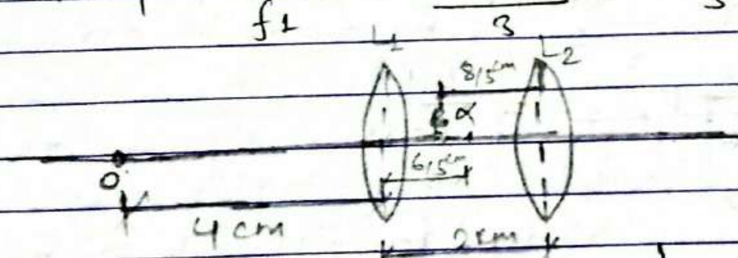
$$= \frac{4+3-2}{12}$$

$$= 5/12 \text{ cm}$$

$\therefore f = 12/5$ cm

And, $\alpha = \frac{df}{f_2} = \frac{2 \times 12/5}{4} = \frac{24^{6}}{5} \times 1/4 = 6/5$ cm

and, $\beta = -\frac{df}{f_1} = \frac{-2 \times 12/5}{3} = -\frac{24^{8}}{5} \times 1/3 = -8/5$ cm



Object distance $(u) = -(4 + 6/5)$ cm $= -\frac{26}{5}$ cm

Image distance $(v) = ?$

∵ We know that,

$$1/f = 1/u + 1/v$$

or, $1/f = 1/{-26/5} + 1/v$

or, $\frac{5}{12} = -\frac{5}{26} + 1/v$

or, $\frac{1}{v} = \frac{65+30}{156}$

or, $v = \frac{156}{95}$

$\boxed{\therefore v = 1.642 \text{ cm}}$

∵ $v < u$,

i.e. Image distance < Object distance.

∴ diminished.

Nature of image :-

→ Real, inverted and diminished.

→

5. Two thin lenses of 30 cm and 20 cm are used to reduced the spherical abberation calculate the position of principal points.

⇒ Soln, Here Given,

focal length of 1st lens $(f_1) = 30cm$

focal length of 2nd lens $(f_2) = 20$ cm.

position of principal points $(\alpha = ?\ \&\ \beta = ?)$

∵ We know that,

separation $(d) = |f_1 - f_2| = 30-20 = 10cm$

Now,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{30} + \frac{1}{20} - \frac{10}{\cancel{20}_2 \times 30}$$

$$= \frac{2+3-1}{60} = \frac{\cancel{4}^1}{\cancel{60}_{15}}$$

$$\therefore \boxed{f = 1/\tfrac{1}{15} = 15cm}$$

Now, principal points:-

$$\alpha = \frac{df}{f_2} = \frac{10 \times 15}{\cancel{20}_2} = 7.5cm$$

$$\& \ \beta = -\frac{df}{f_1} = \frac{-10 \times 15}{\cancel{30}_2} = -5\ cm$$

Ans.

6. Two thin lenses of focal lengths f_1 and f_2 separated by a distance 'd' have an equivalent focal length of 50cm. The combination satisfy the conditions for no chromatic abberation and spherical abberation, find the values of f_1, f_2 and d.

⇒ Soln, Here, Given, Equivalent focal length of combined lens $(f) = 50cm$

focal lengths are f_1 and f_2 separated by distance 'd'.

1st For, no chromatic abberation $(d) = \frac{f_1+f_2}{2}$

2nd For, no spherical abberation $(d) = f_1 - f_2$

Now,

$$\frac{f_1+f_2}{2} = f_1 - f_2$$

or, $f_1 + f_2 = 2f_1 - 2f_2$

or, $f_2 + 2f_2 = 2f_1 - f_1$

or, $3f_2 = f_1$ —— (i)

Again we know that;

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}$$

or $f = \frac{f_1 \cdot f_2}{f_1 + f_2 - d}$

or $f = \frac{f_1 \cdot f_2}{\cancel{f_1} + f_2 - \cancel{f_1} + f_2}$ $[\because d = f_1 - f_2]$

or, $f = \frac{f_1 \cdot \cancel{f_2}}{2\cancel{f_2}}$

or, $f_1 = 2 \times 50$

$\therefore f_1 = 100\ cm$

Now, $3f_2 = f_1$

$\Rightarrow f_2 = \frac{100}{3}\ cm$

And, $d = f_1 - f_2 = 100 - \frac{100}{3} = \frac{300-100}{3} = 200/3\ cm$

$\therefore f_1 = 100\ cm$, $f_2 = \frac{100}{3}\ cm$ & $d = \frac{200}{3}\ cm$ Ans.

Midterm (Before this chapter)

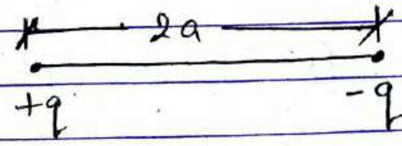
Electrostatics

Electric dipole :-

Combination of two equal and opposite charges separated by some small distance is called electric dipole.

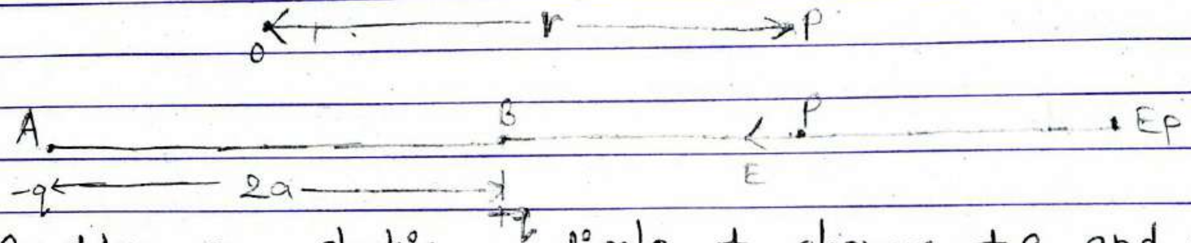
Dipole Moment $(p) = q * 2a$

$p = 2qa$



Electric field due to dipole

A) At a point on axial line :-



Consider an electric dipole of charges +q and -q separated by a distance '2a'. Take a point 'p' on the axial line at distance 'r' from centre of dipole at which the electric field is to be determined.

The electric field at 'p' due to charge +q is,

$$E_+ = \frac{q}{4\pi\varepsilon_0 (r-a)^2} \text{ along BP.}$$

and due to charge -q is

$$E_- = \frac{q}{4\pi\varepsilon_0 (r+a)^2} \text{ along PA.}$$

Now, The resultant electric field at 'p' is,

$$E = E_+ - E_-$$

$$\text{or, } E = \frac{q}{4\pi\varepsilon_0 (r-a)^2} - \frac{q}{4\pi\varepsilon_0 (r+a)^2}$$

$$\text{or, } E = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2 (r+a)^2}\right]$$

ed

$$\text{or, } E = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cancel{r^2}+2ar+\cancel{a^2}-\cancel{r^2}+2ar-\cancel{a^2}}{(r^2-a^2)^2}\right)$$

$$\text{or, } E = \frac{q}{4\pi\varepsilon_0} * \frac{4ar}{r^4\left(1-\frac{a^2}{r^2}\right)^2}$$

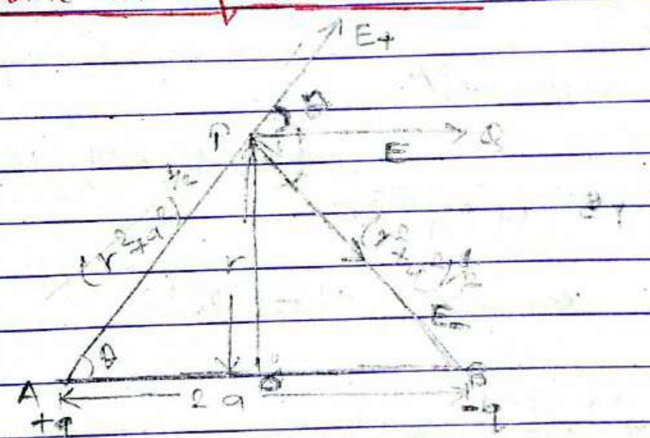
since, $a << r$, so $\frac{a^2}{r^2} << 1$, the term a^2/r^2 can be neglected.

$$\text{or, } E = \frac{q}{4\pi\varepsilon_0} * \frac{4a}{r^3}.$$

$$= \frac{2 \cdot 2qa}{4\pi\varepsilon_0 r^3}$$

$$\boxed{E = \frac{2P}{4\pi\varepsilon_0 r^3}}$$ where, $p = 2qa$ is dipole moment.

(b) At a point in equatorial line:-



→ Consider an electric dipole of charges +q and −q separated by some distance '2a'. Take a point 'P' on the equatorial line at distance of 'r' from the centre of dipole at which the electric field is to be determined.

The electric field at 'P' due to +q is

$$E_+ = \frac{q}{4\pi\varepsilon_0(r^2+a^2)} \text{ along } AP$$

and due to $-q$ is,

$$E_- = \frac{q}{4\pi\varepsilon_0 (r^2+a^2)} \text{ along 'PB'.}$$

'E_+' and 'E_-' can be resolve into two components. The vertical components are equal and opposite so they cancel to each other and the resultant electric field is the sum of horizontal components.

i.e. $E = 2E_+ \cos\theta$ along PQ.

or, $$E = \frac{2 \times q}{4\pi\varepsilon_0 (r^2+a^2)} \cos\theta$$

or $$E = \frac{2q}{4\pi\varepsilon_0 (r^2+a^2)} \cdot \frac{a}{(r^2+a^2)^{1/2}}$$

or, $$E = \frac{2qa}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}$$

or $$E = \frac{p}{4\pi\varepsilon_0 r^3 (1+a^2/r^2)^{3/2}}$$ [$\because$ $p = 2aq$ is dipole moment.]

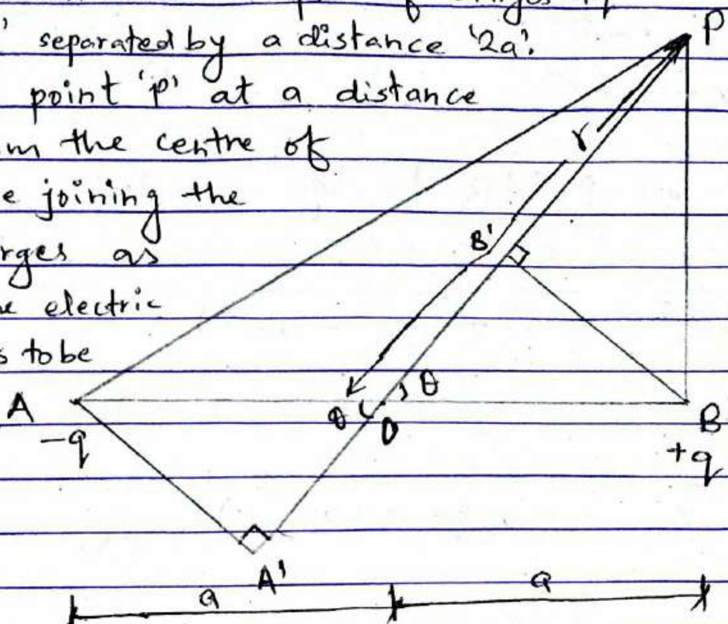
Since, $\frac{a^2}{r^2} \ll 1$, ~~[illegible]~~

so, a^2/r^2 can be neglected.

$$\therefore \boxed{E = \frac{p}{4\pi\varepsilon_0 r^3}}$$

(4) Electric field due to dipole:-

→ Consider an electric dipole of charges '+q' and '-q' separated by a distance '2a'. Take a point 'p' at a distance 'r' from the centre of the line joining the two charges as where the electric potential is to be determined.



Since electric potential obeys superposition principle so potential due to electric dipole as a whole would be sum of potential due to both the charges +q and -q.

The line AA' perpendicular to OP is drawn and BB' perpendicular to OP as shown in above fig.

From ~~figure~~ in $\Delta AA'O$

$$\cos\theta = \frac{A'O}{a}$$

or, $A'O = a\cos\theta$

and in $\Delta BB'O$,

$$\cos\theta = \frac{B'O}{a}$$

or $B'O = a\cos\theta$

Here,

$$PA' = r + a\cos\theta$$

$$PB' = r - a\cos\theta$$

Now,

$PA \approx PA' = r + a\cos\theta$

$PB \approx PB' = r - a\cos\theta$

The electric potential at P due to +q is,

$$V_+ = \frac{q}{4\pi\varepsilon_0 PB} = \frac{q}{4\pi\varepsilon_0 (r - a\cos\theta)}$$

& due to −q is,

$$V_- = \frac{q}{4\pi\varepsilon_0 PA} = \frac{q}{4\pi\varepsilon_0 (r + a\cos\theta)}$$

Again,

Total electric potential at 'P' due to dipole is,

$$V = V_+ + V_-$$

$$\text{or, } V = \frac{q}{4\pi\varepsilon_0 (r - a\cos\theta)} - \frac{q}{4\pi\varepsilon_0 (r + a\cos\theta)}$$

$$\text{or, } V = \frac{q}{4\pi\varepsilon_0}\left[\frac{r + a\cos\theta - r + a\cos\theta}{(r - a\cos\theta)(r + a\cos\theta)}\right]$$

$$\text{or, } V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{(r^2 - a^2\cos^2\theta)}$$

$$\text{or, } V = \frac{2aq\cos\theta}{4\pi\varepsilon_0 r^2\left(1 - \frac{a^2\cos^2\theta}{r^2}\right)}$$

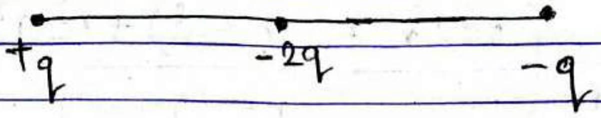
since, $\frac{a^2\cos^2\theta}{r^2} << 1$, so it can be neglected.

$$\therefore \boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$$ where, $p = 2aq$ is a dipole moment

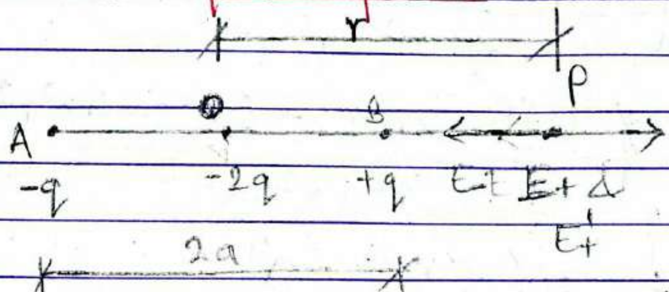
⊛ Quadrapole:-

→ The combination of two electric dipole is called quadrapole

$$\boxed{\text{Quadrapole moment } (Q) = 2qa^2}$$



⊕ Electric field due to quadrapole:-



→ Consider a quadrapole of charges '+q' and '-q' separated by a distance '2a'. Take a point 'p' at a distance 'r' from the center of the quadrapole.

The electric field at 'p' due to '+q' at 'B' is,

$$E_+ = \frac{q}{4\pi\epsilon_0 (r-a)^2} \text{ along BP.}$$

due to '+q' at 'A' is,

$$E_+' = \frac{q}{4\pi\epsilon_0 (r+a)^2} \text{ along AP.}$$

due to '-2q' at 'O' is,

$$E_- = \frac{2q}{4\pi\epsilon_0 r^2} \text{ along PO.}$$

Now the resultant electric field at 'p' due to quadrapole is

$$E = E_+ + E_+' - E_-$$

$(r+a)(r+a)\ (r-a)(r-a)$
$(r^2-a^2)\cdot(r^2-a^2)$

$\dfrac{r^4-2a^2r^2+a^4}{r^2-2ar-a^2}$

$$E = \frac{q}{4\pi\varepsilon_0(r-a)^2} + \frac{q}{4\pi\varepsilon_0(r+a)^2} - \frac{2q}{4\pi\varepsilon_0 r^2}$$

or $$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2(r+a)^2 + r^2(r-a)^2 - 2(r-a)^2(r+a)^2}{r^2(r^2-a^2)^2}\right]$$

or $$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2(r^2+2ar+a^2) + r^2(r^2-2ar+a^2) - 2(r^2-a^2)^2}{r^2(r^2-a^2)^2}\right]$$

or $$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2\{2r^2+2a^2\} - 2r^4 + 4a^2r^2 - 2a^4}{r^2(r^2-a^2)^2}\right]$$

or $$E = \frac{q}{4\pi\varepsilon_0}\left\{\frac{\cancel{2r^4} + 2a^2r^2 - \cancel{2r^4} + 4a^2r^2 - 2a^4}{r^2(r^2-a^2)^2}\right\}$$

or $$E = \frac{q}{4\pi\varepsilon_0}\left\{\frac{6a^2r^2 - 2a^4}{r^2(r^2-a^2)^2}\right\}$$

Since, $a << r$, so, a^4 and $\frac{a^2}{r^2}$ can be neglected.

$$E = \frac{6qa^2r^2}{4\pi\varepsilon_0 r^6}$$

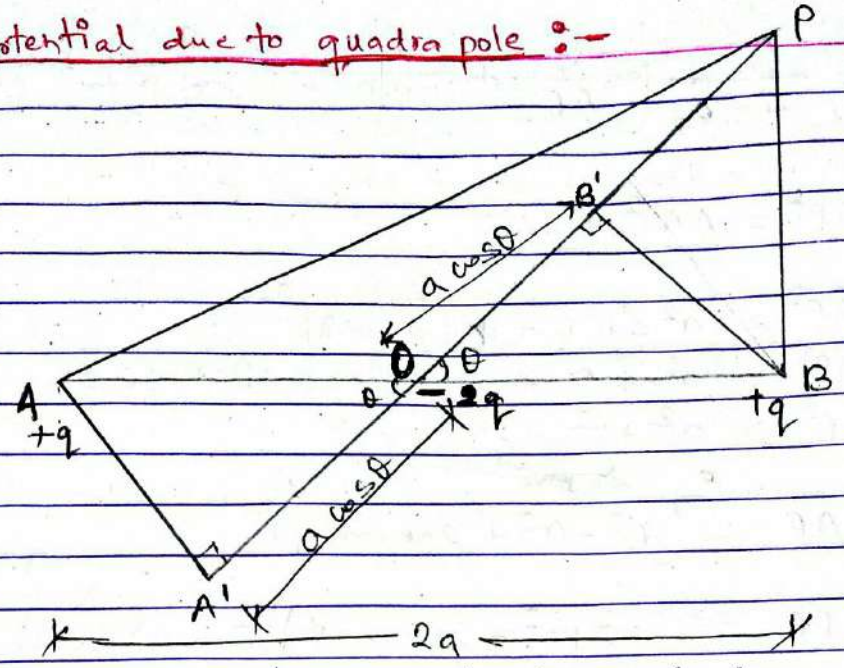
Rough,
$r^2\{(r^2)^2 - 2r^2a^2 + \cancel{(a^2)^2}\}$
$r^2 \cdot r^4 -$

or, $$E = \frac{3 \cdot 2qa^2}{4\pi\varepsilon_0 r^4}$$

$$\therefore \boxed{E = \frac{3Q}{4\pi\varepsilon_0 r^4}}$$, where, $Q = 2qa^2$ is a quadrapole moment

$\frac{A'a}{AO}$
a

Electric potential due to quadrapole :-



→ consider a quadrapole of charges '+q' and '-q' separate by a distance '2a'. Take any point 'p' at a distance of 'r' from the centre of quadrapole at which the electric potential is to be determined.

The potential at 'p' due to '+q' charge at 'A' is,

$$V_+ = \frac{q}{4\pi\varepsilon_0 Ap}$$

Due to '+q' charge at 'B' is,

$$V_+' = \frac{q}{4\pi\varepsilon_0 BP}$$

Due to '-2q' charge at 'O' is

$$V_- = \frac{2q}{4\pi\varepsilon_0 OP}$$

$AB = 2a$
$OP = r$
$OA' = a\cos\theta$
$OB' = a\cos\theta$

Now, the total potential at 'p' due to whole quadrapole is,

$$V = V_+ + V_+' + V_-$$

$$\text{or, } V = \frac{q}{4\pi\varepsilon_0 Ap} + \frac{q}{4\pi\varepsilon_0 BP} - \frac{2q}{4\pi\varepsilon_0 OP}$$

or $V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{AP} + \frac{1}{BP} - \frac{2}{OP}\right)$ ---- (i)

Here,

$$AP^2 = AA'^2 + PA'^2$$

or, $AP^2 = a^2\sin^2\theta + (r + a\cos\theta)^2$

or $AP^2 = a^2\sin^2\theta + r^2 + 2ar\cos\theta + a^2\cos^2\theta$

or $AP^2 = a^2 + r^2 + 2ar\cos\theta$

or $AP = (r^2 + a^2 + 2ar\cos\theta)^{1/2}$

Similarly, $BP = (r^2 + a^2 - 2ar\cos\theta)^{1/2}$

and $OP = r$

putting the values of AP, BP and OP in eq^n (i), we get,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r^2 + a^2 + 2ar\cos\theta)^{1/2}} + \frac{1}{(r^2 + a^2 - 2ar\cos\theta)^{1/2}} - \frac{2}{r}\right]$$

or, $V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)^{1/2}} + \frac{1}{r\left(1 + a^2/r^2 - \frac{2a}{r}\cos\theta\right)^{1/2}} - \frac{2}{r}\right]$

or $V = \frac{q}{4\pi\varepsilon_0 r}\left[\left(1 + a^2/r^2 + \frac{2a}{r}\cos\theta\right)^{-1/2} + \left(1 + a^2/r^2 - \frac{2a}{r}\cos\theta\right)^{-1/2} - 2\right]$ --- (ii)

Using binomial expansion,

$$\left\{1 + \left(\frac{a^2}{r^2} + \frac{2a}{r}\cos\right)\right\}^{-1/2} = 1 - \frac{1}{2}\left(a^2/r^2 + \frac{2a}{r}\cos\theta\right) + \frac{-\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!}\left(a^2/r^2 + \frac{2a}{r}\cos\theta\right)^2 + \cdots$$

$$= 1 - \frac{a^2}{2r^2} - \frac{a}{r}\cos\theta + \frac{3}{8}\left(\frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)^2$$

$$= 1 - \frac{a^2}{2r^2} - \frac{a}{r}\cos\theta + \frac{3}{8}\left\{\frac{a^4}{r^4} + \frac{2a^2}{r^2}\cdot\frac{2a}{r}\cos\theta + \frac{4a^2\cos^2\theta}{r^2}\right\}$$

$$= 1 - \frac{a^2}{2r^2} - \frac{a}{r}\cos\theta + \frac{3}{8}\frac{a^4}{r^4} + \frac{3a^3\cos\theta}{2r^3} + \frac{3}{2}\frac{a^2}{r^2}\cos^2\theta$$

$$= 1 - \frac{a^2}{2r^2} - \frac{a}{r}\cos\theta + \frac{3a^2\cos^2\theta}{2r^2}$$ [$\frac{a^4}{r^4}$ & $\frac{a^3}{r^3}$ are neglected as it is very small]

Again,

$$\left\{1 + \left(\frac{a^2}{r^2} - \frac{2a\cos\theta}{r}\right)\right\}^{-1/2}$$

$$= 1 - \frac{a^2}{2r^2} + \frac{a\cos\theta}{r} + \frac{3a^2}{2r^2}\cos^2\theta$$

Now eqn (ii) becomes;

$$V = \frac{q}{4\pi\varepsilon_0 r}\left(1 - \frac{a^2}{2r^2} - \frac{a\cos\theta}{r} + \frac{3a^2\cos^2\theta}{2r^2} + 1 - \frac{a^2}{2r^2} + \frac{a}{r}\cos\theta + \frac{3a^2}{2r^2}\cos^2\theta - 2\right)$$

$$\text{or, } V = \frac{q}{4\pi\varepsilon_0 r}\left(\frac{-2a^2}{2r^2} + \frac{3\times 2a^2\cos^2\theta}{2r^2}\right)$$

$$\text{or } V = \frac{q}{4\pi\varepsilon_0 r}\left(-\frac{a^2}{r^2} + \frac{3a^2\cos^2\theta}{r^2}\right)$$

$$\text{or } V = \frac{a^2 q}{4\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right)$$

(ii)

$$\text{or } V = \frac{2qa^2}{8\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right)$$

$$\therefore \boxed{V = \frac{Q}{8\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right)}$$ Where, $Q = 2a^2q$ is quadrapole moment.

Case-1, At axial line $\theta = 0°$,

$$\boxed{V = \frac{2Q}{8\pi\varepsilon_0 r^3}}$$

Case-2 At equatorial line $\theta = 90°$

$$V = \frac{-Q}{8\pi\varepsilon_0 r^3}$$

$$V = \frac{Q}{8\pi\varepsilon_0 r^3}.$$

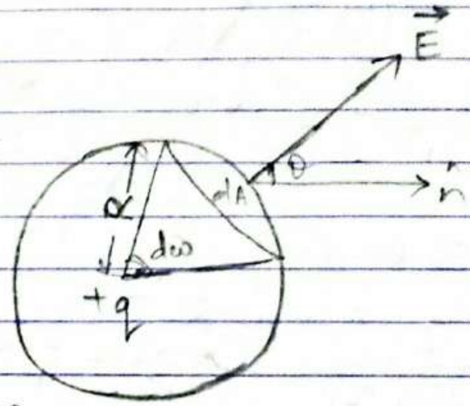
(4) Gauss Theorem:

→ It states that," The electric flux passing through a surface is equal to the $\frac{1}{\varepsilon_0}$ times the charge enclosed by the surface."

i.e electric flux (ϕ) = $\frac{q}{\varepsilon_0}$

$$\boxed{\int_s \vec{E} \cdot dA = \frac{q}{\varepsilon_0}}$$

proof:-



Consider a charge '+q' is enclosed by a closed surface 'S'. Take a small area element '$\vec{dA}$' on the surface of the 'S'. The normal '$\hat{n}$' of surface makes an angle 'θ' with electric field $\vec{E}$. 'R' be the radius of spherical surface.

Now,

$$\int_s \vec{E} \cdot \vec{dA} = \int_s E\,dA\cos\theta$$

$$= \int_s \frac{q}{4\pi\varepsilon_0 R^2}\,dA\cos\theta \qquad \left[\because E = \frac{q}{4\pi\varepsilon_0 R^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\int_s \frac{dA\cos\theta}{R^2}$$

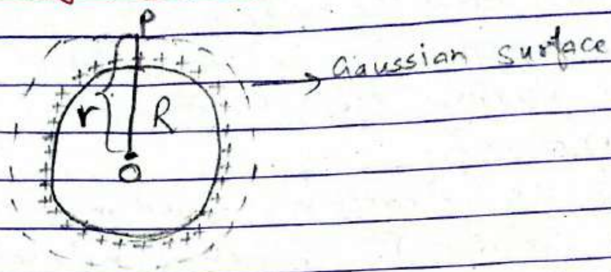
$$= \frac{q}{4\pi\varepsilon_0}\int_s d\omega$$

$$= \frac{q}{4\pi\varepsilon_0} \times 4\pi$$

$$\boxed{\therefore \int_s \vec{E} \cdot \vec{dA} = \frac{q}{\varepsilon_0}}$$ which is gauss theorem.

Application of Gauss theorem :-

(1) Electric field due to charged sphere :-



⇒ consider a charged sphere of radius 'r' which consists the positive charge '+q'. Take a point 'P' at a distance of 'r' from the centre of the sphere at which an electric field is to be determined. For this, let us draw gaussian ~~surface~~ spherical surface of radius 'r' and the centre 'O' which enclose the charged sphere.

The area of gaussian surface is,

$$A = 4\pi r^2$$

Electric flux is given by,

$$\phi = E * A$$

$$\phi = E * 4\pi r^2 \quad --- (i)$$

Now, from gauss theorem,

$$\phi = \frac{q}{\varepsilon_0} \quad --- (ii)$$

equating eqn (i) & (ii); we got,

$$E * 4\pi r^2 = \frac{q}{\varepsilon_0}$$

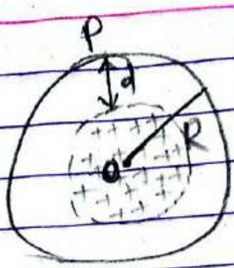
or, $$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

at the surface of the sphere, r = R.

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}}$$

(ii) Inside the sphere:

→ Take a point 'p' at a distance of 'd' from the surface of the sphere inside at which the electric field is to be determined.



Let us draw a gaussian surface of radius 'R−d' from the point 'p' which enclosed the charge 'q''.
If the charge is uniformly distributed then the volume charge density is constant.

i.e. $$\frac{q'}{\frac{4}{3}\pi(R-d)^3} = \frac{q}{\frac{4}{3}\pi R^3}$$

or, $$q' = \frac{q(R-d)^3}{R^3}$$

The area of gaussian surface, $4\pi(R-d)^2$.

The electric flux is given by,

$$\phi = E * A$$
$$= E * 4\pi(R-d)^2$$

From gauss theorem,

$$\phi = \frac{q'}{\varepsilon_0}$$

or, $$E * 4\pi(R-d)^2 = \frac{q'}{\varepsilon_0}$$

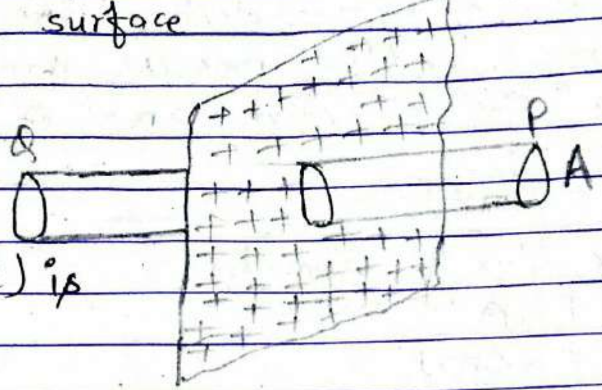
or, $$E * 4\pi(R-d)^2 = \frac{1}{\varepsilon_0} * \frac{q(R-d)^3}{R^3}$$

or, $$E = \frac{q(R-d)}{4\pi\varepsilon_0 R^3}$$

or, $$\boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}\left(1 - \frac{d}{R}\right)}$$

(2) Electric field due to infinite sheet of charge

→ Consider an infinite plane sheet of charge '+q' having surface charge density (σ).



Take a point 'p' near the sheet at which the electric field (E) is to be determined.

Let us draw cylinderical gaussian surface of cross-sectional area 'A' on the both side of the sheet.

The charge enclosed by the gaussian surface is,

$$q = \sigma \times A$$

Now,

From gauss theorem, the electric flux is given by,

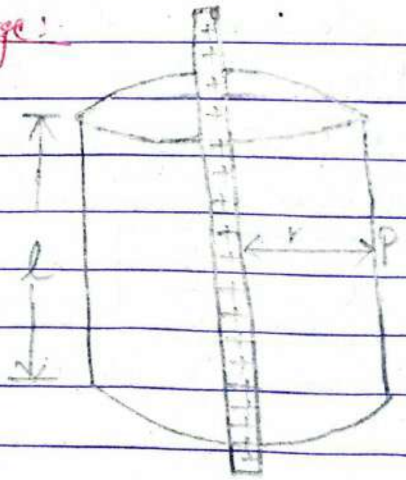
$$\phi = q/\varepsilon_0$$

or, $$E \times 2A = \frac{\sigma \times A}{\varepsilon_0}$$

or, $$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

(iii) Electric field due to linear charge:

→ Let us consider a long straight conductor having linear charge density ($\lambda = q/l$).



Take a point 'P' at a distance 'r' from the conductor at which electric field is to be determined

Let us consider a gaussian surface (cylinderical) of radius 'r' and length 'l', since, the electric line of forces are perpendicu to the charge, no flux passess through the construction o the cylinder. The effective area of gaussian surface is,

$$A = 2\pi r l$$

The electric flux is given by,

$$\phi = E * A$$
$$= E * 2\pi r l$$

The charged enclosed by gaussian surface is

$$q = \lambda * l$$

From gauss theorem,

$$\phi = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \quad \text{--- (ii)}$$

From eqn (i) & (ii), we get,

$$E * 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

or $\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 r}}$

#) Capacitor :-

➡ The electronic device used to store the charge is called capacitor.

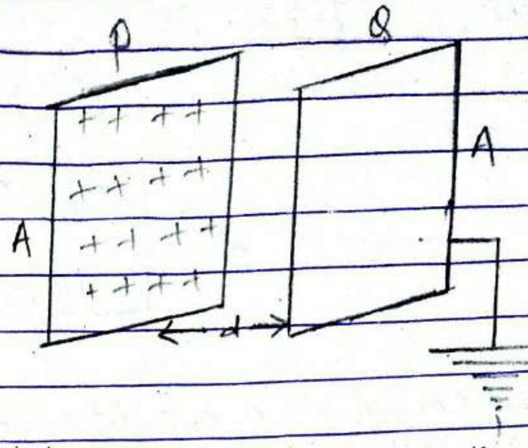
→ The ability of a capacitor to store the charge is called capacitance (C).

$$\boxed{C = \frac{q}{V}}$$

→ Its SI unit is Farad (F)

(x) Parallel plate capacitor:-

→ Two parallel metal plate having same cross-sectional and separated by a small distance one of them contain positive charge and another is grounded. The combination is called parallel plate capacitor.



Consider two metallic plate 'P' and 'Q' having same crossectional area 'A' and separated by a small distance 'd'. The plate 'P' is positively charged and 'Q' is grounded.

The electric field between plate is given by,

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{q}{A\varepsilon_0} \quad \left[\because \sigma = \frac{q}{A}\right]$$

Again,

$$E = \frac{V}{d}$$; where, 'V' is p.d. between plates.

or, $$\frac{V}{d} = \frac{q}{A\varepsilon_0}$$

or, $$\frac{A\varepsilon_0}{d} = \frac{q}{V}$$

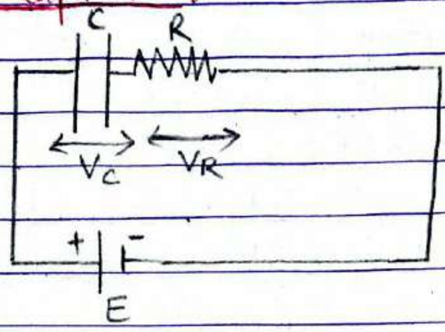
or, $$\boxed{C = \frac{A\varepsilon_0}{d}} \quad \left[\because C = q/V\right]$$

If there is dielectric medium between plates, then,

$$\boxed{C = \frac{A\varepsilon}{d}}$$ where, $\varepsilon \rightarrow$ permittivity of the medium.

(4) Charging and discharging of capacitor:-

(i) Charging of capacitor:-



→ Consider a capacitor of capacitance 'C' and resistor of resistance 'R' are connected in series with a battery of emf 'E'.

Here,

$$E = V_C + V_R$$

or, $$E = \frac{q}{C} + IR$$

or, $$C \cdot E = q + R \cdot C \cdot I$$

or, $$q_0 = q + R \cdot C \cdot \frac{dq}{dt} \quad [q_0 \rightarrow \text{max}^m \text{ charge on capacitor}]$$

or, $$q_0 - q = R \cdot C \cdot \frac{dq}{dt}$$

or, $$\frac{dt}{RC} = \frac{dq}{q_0 - q} \quad \text{------ (i)}$$

At $t = 0$, $q = 0$

At $t = t$, $q = q$

Integrating eqn (i); we get,

$$\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{q_0 - q}$$

or, $$\frac{t}{RC} = \left[-\log(q_0 - q)\right]_0^q$$

or, $$-\frac{t}{RC} = \log(q_0 - q) - \log q_0$$

or, $$-t/_{RC} = \log\left(\frac{q_0 - q}{q_0}\right)$$

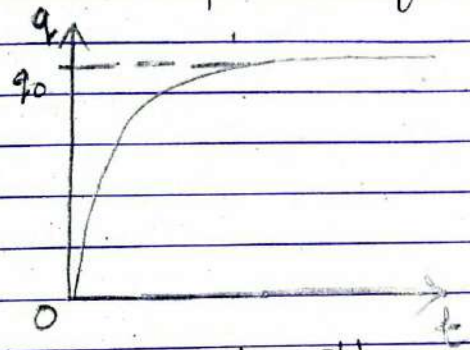
or $$e^{-t/RC} = \frac{q_0 - q}{q_0}$$

or $$q_0 e^{-t/RC} = q_0 - q$$

or $$q = q_0 - q_0 e^{-t/RC}$$

$$\therefore \boxed{q = q_0 \left(1 - e^{-t/RC}\right)}$$

This is the expression of charging of capacitor.



If $t = RC$, $q = q_0(1 - e^{-1})$

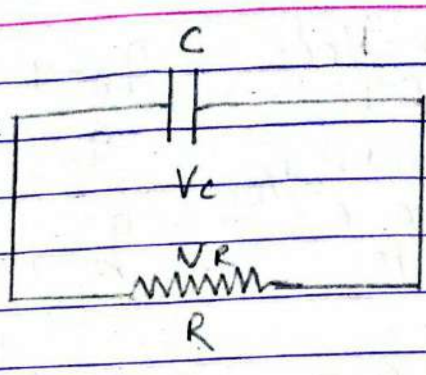
$$q = q_0\left(1 - \frac{1}{e}\right)$$

$$q = 63\% \text{ of } q_0$$

~~for charging of capacitor~~ Hence, the RC time constant for charging ~~of~~ capacitor is defined as the time taken by the capacitor to charge 63% of its maximum capacity.

(ii) Discharging of capacitor:-

→ Consider a capacitor of capacitance 'C' is initially full charged (q_0) which is discharge through a resistor of resistance 'R'.



Here, $V_C = V_R$

or, $\frac{q}{C} = IR$

or, $\frac{q}{C} = \frac{-dq}{dt} R$ (-ve sign indicates that charge decreases on increase in time).

or, $\frac{-dt}{RC} = \frac{dq}{q}$ ---- (1)

At $t=0$, $q = q_0$

At $t=t$, $q=q$

Integrating eqn (1), we get,

$$\int_0^t \frac{-dt}{RC} = \int_{q_0}^{q} \frac{dq}{q}$$

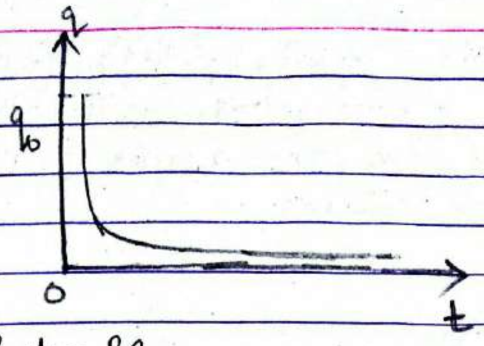
or, $\frac{-t}{RC} = [\log q]_{q_0}^{q}$

or, $\frac{-t}{RC} = \log q - \log q_0$

or, $-t/RC = \log\left(\frac{q}{q_0}\right)$

or, $\frac{q}{q_0} = e^{-t/RC}$

$$\therefore \boxed{q = q_0 e^{-t/RC}}$$



If $t = RC$

$q = 37\%$ of q_0

Hence, the RC time constant for discharging of capacitor is defined as time taken by the capacitor to discharge 37% of its maximum capacity.

Numerical :-

① What is the electric potential at the surface of a gold nucleus? The radius is 6.6×10^{-15} m and $Z = 79$.

⇒ Sol^n

Given,

atomic (z) = 79

radius of nucleus (r) = 6.6×10^{-15} m

∴ Electric potential (V) = ?

∵ We know that,

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

$$= \frac{Ze}{4\pi\varepsilon_0 r} \quad [\because q = Ze]$$

$$= \frac{79 \times 1.6 \times 10^{-19} \times 9 \times 10^{9}}{6.6 \times 10^{-15}} \quad \left[\because e = 1.6 \times 10^{-19} \text{ (charge of proton)}, \ \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\right]$$

$$= 17236363.64$$

$$\boxed{V = 1.72 \times 10^{7} \text{ V.}}$$

(2) Each of two small spheres is charged positively by the combined charge 5×10^{-5} C. If each sphere is repelled from other by a force 1 N when the spheres are 2m apart. How is the total charge distributed between spheres.

⇒ Soln. Given,

force (F) = 1 N

distance (d) = 2m

Combined charge $(q_1 + q_2) = 5 \times 10^{-5}$ C

Now, $F = \frac{q_1 . q_2}{4\pi \varepsilon_0 d^2}$

or, $1 = \frac{9 \times 10^9 . q_1 (5 \times 10^{-5} - q_1)}{2^2}$

or, $\frac{4}{9 \times 10^9} = q_1 \times 5 \times 10^{-5} - q_1^2$

or $q_1^2 - 5 \times 10^{-5} q_1 + 4.44 \times 10^{-10} = 0$

on solving,

Either, $q_1 = 3.84 \times 10^{-5}$ or 1.15×10^{-5}

then,

$q_2 = 1.16 \times 10^{-5}$

or $q_2 = 3.85 \times 10^{-5}$ } Ans.

(3) Two conducting spheres of radii R_1 and R_2 are at same potential. Compare their charges and surface charge densities.

→ Soln:

∵ Given as both spheres having same potential.
i.e. $V_1 = V_2$

or, $$\frac{q_1}{4\pi\varepsilon_0 R_1} = \frac{q_2}{4\pi\varepsilon_0 R_2}$$

or, $$\frac{q_1}{R_1} = \frac{q_2}{R_2}$$

or, $$\boxed{\frac{q_1}{q_2} = \frac{R_1}{R_2}} \quad \text{------} \ldots (i)$$

Now,
for surface charge density, we have

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/A_1}{q_2/A_2}$$

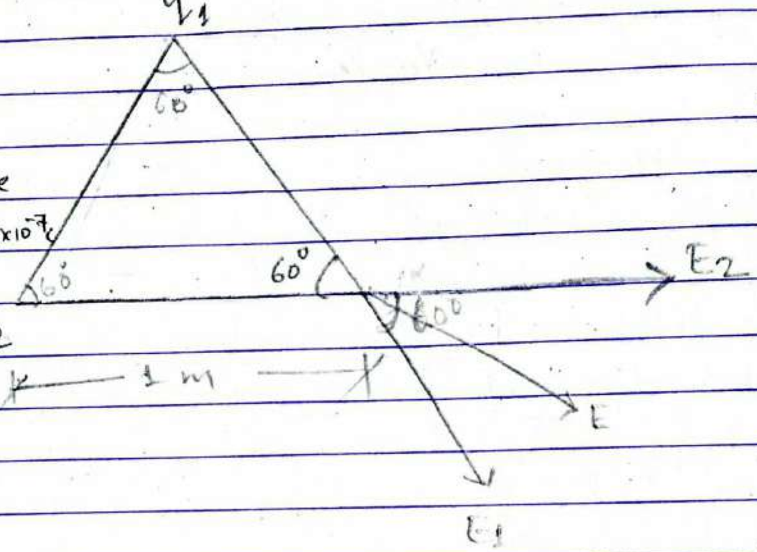
or, $$\frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2} \times \frac{A_2}{A_1}$$

Using the value from eqn (i);

or, $$\frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2} \times \frac{4\pi R_2^2}{4\pi R_1^2}$$

$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}}$$

(4) Three point charges each of 3×10^{-7} c are placed at a corner of an equilateral triangle whose side is 1m. What is the electric field at one of the vertex of the triangle due to these charges.



⇒ Sol^n:

given,

three point charges are equal and their value = 3×10^{-7} c

i.e. $q_1 = q_2 = q_3 = 3\times10^{-7}$ c

~~radius~~ $r = 1$ m

$$\therefore E_1 = E_2 = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{3\times10^{-7}\times9\times10^{9}}{1^2} \quad \left[\because \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\right]$$

$$= 2700 \text{ N/C} \quad [\text{V/m}]$$

Now,

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1 \cdot E_2 \cos\theta}$$

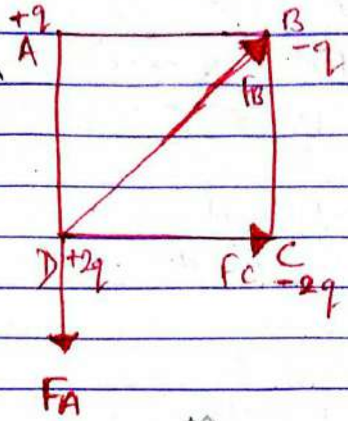
$$= \sqrt{(2700)^2 + (2700)^2 + 2\times2700\times2700\times\cos 60^\circ}$$

$$= \sqrt{(2700)^2(1+1+2\times\tfrac{1}{2})}$$

$$= 2700\sqrt{3} \text{ V/m}$$

$$\boxed{\therefore E = 4.676\times10^{3} \text{ V/m or N/C}}$$

(3) In fig what is the resultant force on the charge at D of the square ABCD. Assume, $q = 10^{-7}$ C and $a = 5$ cm.



⇒ Sol^n Given sides(a) = 5 cm = 5×10^{-2} m
charge(q) = 10^{-7} C

$$F_x = (F_A)_x + (F_B)_x + (F_C)_x$$

$$F_y = (F_A)_y + (F_B)_y + (F_C)_y$$

$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Here,

$$F_A = \frac{q \times 2q}{4\pi\varepsilon_0 \times AD^2}$$

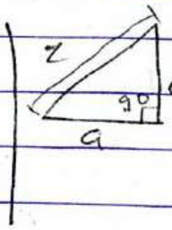
$$= \frac{2\times(10^{-7})^2 \times 9\times10^9}{(5\times10^{-2})^2}$$

$$= 0.072\ \text{N}$$

Now, $(F_A)_x = 0$

and $(F_A)_y = -$ ~~FA~~ $= -0.072$ N

And, $$F_B = \frac{q \times 2q}{4\pi\varepsilon_0\, BD^2}$$



$\Rightarrow Z = \sqrt{a^2+a^2}$
$= \sqrt{2}\cdot a$

$$= \frac{2\times(10^{-7})^2 \times 9\times10^9}{(\sqrt{2}\,a)^2}$$

$$= \frac{2\times(10^{-7})^2 \times 9\times10^9}{2\times(5\times10^{-2})^2}$$

$$= 0.036\ \text{N}$$

∴ $(F_B)_x = F_B \cos45^\circ = 0.025$ N
$(F_B)_y = F_B \sin45^\circ = 0.025$ N

Again,

$$F_c = \frac{2q * 2q}{4\pi \varepsilon_0 (D)^2}$$

$$= \frac{4 * (10^{-7})^2 * 9*10^9}{(5*10^{-2})^2}$$

Now $= 0.144 \text{ N}$

$\therefore (F_c)_x = F_c = 0.144 \text{ N}$

$(F_c)_y = 0$

$\therefore \quad F_x = (F_A)_x + (F_B)_x + (F_c)_x$

$= 0 + 0.025 + 0.144$

$= 0.169 \text{ N}$

& $F_y = (F_A)_y + (F_B)_y + (F_c)_y$

$= -0.072 + 0.025 + 0$

$= -0.047 \text{ N}$

Also, resultant force,

$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

$$= \sqrt{(0.169)^2 + (-0.047)^2}$$

$= 0.175 \text{ N}$

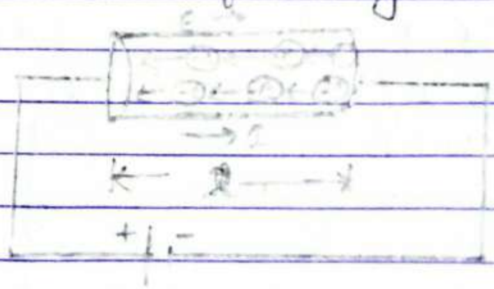
& $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.047}{0.169}\right)$

$= -15.54^\circ$ //

Electricity and Magnetism

Drift velocity :-

→ When an external electric field is applied to a conductor, the free electrons of the conductor gains constant velocity opposite to the applied field. This constant velocity of free electrons is called drift velocity.



→ Consider a conductor of cross-sectional area 'A' and length 'l'. When electric field is not applied, electrons are moved in random direction. So, there is no net current on the conductor. When the electric field is applied by connecting the battery to the conductor, the free electrons gain drift velocity 'V_d' to the opposite direction to the applied field.

Now, the current flows through the conductor is,

$$I = \frac{q}{t}$$

or, $$I = \frac{Ne}{t}$$

or, $$I = \frac{nAle}{t} \qquad \left[\because n = \frac{N}{V} = \frac{N}{A \times l}\right]$$

or, $$I = nAe\left(\frac{l}{t}\right)$$

$\therefore$ $\boxed{I = neAV_d}$ where, $V_d = l/t$ is drift velocity

n = no. of free electron per unit volume.

or, $$\frac{I}{A} = neV_d$$

or, $\boxed{J = neV_d}$ where, $\vec{J} = I/A$ is current density.

Resistance :-

→ The property of a conductor which oppose the flow the flow of current through it. It was found that the resistance of a conductor.

(i) Directly proportional to the length.
i.e. $R \propto l$ ---(i)

(ii) Inversely proportional to the cross-sectional area.
i.e. $R \propto 1/A$ ---(ii)

Combining eq^n (i) and (ii); we get

$$R \propto l/A$$

or, $$\boxed{R = \frac{\rho l}{A}} \quad \text{------(iii)}$$

$$\Rightarrow \rho = \frac{RA}{l}$$

Where, 'ρ' is a proportionality constant called resistivity. It is defined as the resistance of a conductor having unit cross-sectional area and unit length. Its SI unit is 'Ωm'. Its value depends on nature of material and temperature.

We know that, From ohm's law,

$$V = IR$$

or, $E \times l = I \times \frac{\rho l}{A}$ $\quad \left[\because V = \overset{\text{electric field}}{E} \times \overset{\text{distance}}{l} \text{ and } R = \frac{\rho l}{A}\right]$

or, $E = \frac{I}{A} \times \rho$

or, $E = J \times \frac{1}{\sigma}$ $\quad \left[\because \sigma = 1/\rho \text{ is conductivity}\right]$

or, $\boxed{J = \sigma . E}$

This is the relation between current density and electric field.

⊛ Ohm's law :-

It states that, "The current passing through a conductor is directly proportional to the potential difference betn two ends of the conduct

If physical parameter are constant (i.e. Temperature, length, diameter and nature of material).

i.e. $I \propto V$

or, $V \propto I$

$\Rightarrow \boxed{V = IR}$ where, 'R' is resistance of conductor.

⊛ Joule's law of Heating :-

⇒ When a current passes through a conductor, heat is developed on the conductor. This effect ~~is~~ of current is called Heating effect of current. The amount of heat produce/ ~~heat~~ developed on a current carrying conductor is,

(i) Directly proportional to the square of current.

i.e. $H \propto I^2$ ---- (i)

(ii) Directly proportional to the resistance.

i.e. $H \propto R$ ---- (ii)

(iii) Directly proportional to the time for which the current passes.

i.e. $H \propto t$ ---- (iii)

combining eq^n (i), (ii) and (iii); we get,

$$H \propto I^2 R t$$

$$\therefore \boxed{H = \frac{I^2 R t}{J}}$$

Where, 'J' is joule's mechanical equivalent of heat.

$$\boxed{J = 4.2\ J/cal}$$

(11) Verification of joule's Law of Heating :-

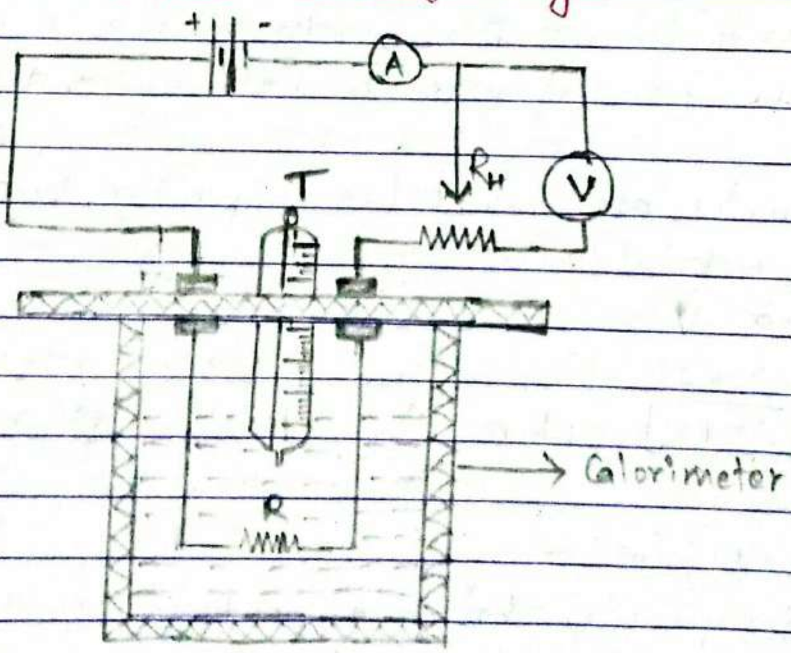
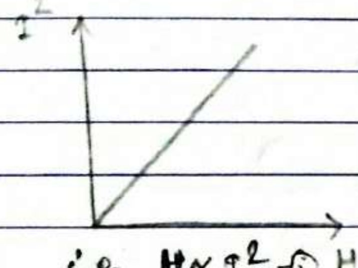


fig: Calorimeter

→ Consider a colorimeter filled with water half of its volume. A resistance wire is immersed in the water which is connected to the external battery through a Rheostat.

The experiment is done in three steps. At first, the current through the resistor is varying with the help of Rheostat and the corresponding heat developed is measured by the help of temperature difference measured by the thermometer. When we plot the graph between heat developed and square of current, we get a straight line passes through the origin which is shown in fig. below.

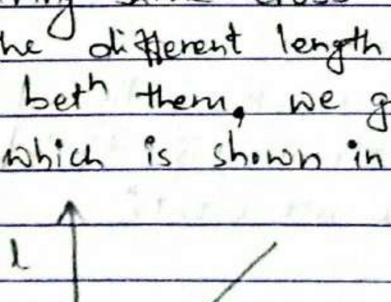


i.e. $H \propto I^2$ —(i)

At constant 'R' and 't'.

<u>In 2nd step,</u>

The experiment is repeated for same amount of current and same interval of time with the resistance wire of different length but having same cross-section. The corresponding heat developed for the different length of wire are noted and the graph is plotted betn them, we get a straight line passes through the origin which is shown in fig. below.

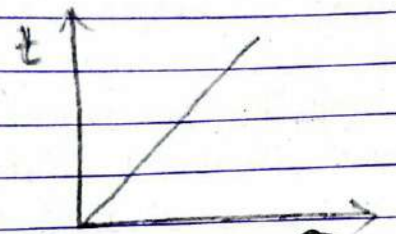


i.e. $H \propto l$

Also, we know that, $R \propto l$,

$\Rightarrow H \propto R$ --- (ii) at constant 'I' and 't'.

At last, the experiment is again repeated for the same amount of current and same wire but for different interval of time and the corresponding heat developed is noted. The graph between heat developed and time interval is plotted which is the straight line passes through the origin is shown in fig. below:-



i.e. $H \propto t$ -- (iii) at constant 'I' and 'R'.

combining eqn (i), (ii) & (iii); we get,

$$H \propto I^2Rt$$

or $$\boxed{H = \frac{I^2Rt}{J}}$$

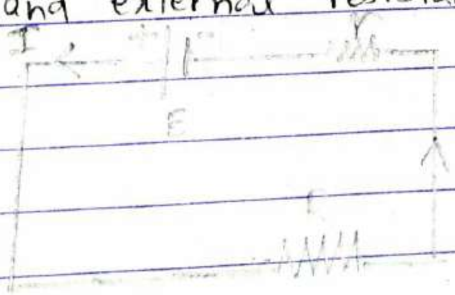
Where,
'J' is joule's mechanical equivalent of heat.

$\boxed{J = 4.2 \ J/cal}$

Which verify the joule's low of heating.

Maximum power transfer Theorem:-

It states that, " The power delivered by a source to the external resistance is maximum, if the internal resistance of source and external resistance are equal."



Consider a battery of emf of 'E' and internal resistance 'r' is connected to a external resistance 'R'.

The current flowing through the circuit is,

$$I = \frac{E}{R+r}$$

Now, the power is,

$$P = I^2 R$$

or, $$P = \left(\frac{E}{R+r}\right)^2 \times R$$

$$\frac{R}{(R+r)^2}$$

$$\frac{(R+r)^2 \cdot 1 - R \cdot 2(R+r) \cdot R}{(R+r)^4}$$

For power to be maximum,

$$\frac{dP}{dR} = 0$$

or, $$E^2 \cdot \frac{(R+r)^2 \cdot 1 - R \cdot 2(R+r) \cdot 1}{(R+r)^4} = 0$$

or, $$(R+r)^{\cancel{2}} = 2R\,\cancel{(R+r)}$$

or, $$R+r = 2R$$

or, $$\boxed{R = r}$$

Again, $$\frac{d^2P}{dR^2} = -ve$$

So, the power is maximum,

When $\boxed{R=r}$

This is maximum power transfer theorem.

Date.................................
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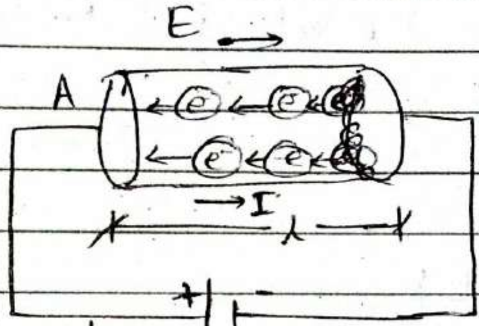
Electricity & Magnetism :-

Drift velocity :-

When an external electric field is applied to a conductor, the free electrons of the conductor gains constant velocity opposite to the applied field. This constant velocity of free electrons is called drift velocity.



consider a conductor of cross-sectional area 'A' and length 'l'. When electric field is not applied, electrons are ~~made~~ moved in random direction. So there is no net current on the conductor. When the electric field is applied by ~~conducting~~ connecting & the battery to the conductor, the free electrons gain drift velocity 'V_d' to the opposite direction to the applied field.

Now the current flows through the conductor is,

$$I = q/t$$

$$\text{or } I = \frac{Ne}{t}$$

$$\text{or } I = \frac{nAle}{t} \quad \left[\because n = \frac{N}{V} = \frac{N}{Al} \right)$$

$$\text{or } I = nAe\left(l/t\right)$$

or $\boxed{I = neAV_d}$ where, $V_d = l/t$ is drift velocity.

n = no. of free electrons per unit volume.

or, $\frac{I}{A} = neV_d$

or, $\boxed{J = neV_d}$ where, $\vec{J} = I/A$ is current density

(*) ~~Resistance~~ :

Resistances:-

↳ The property of a conductor which oppose the flow of current through it. It was found that the resistance of a conductor is,

(i) Directly proportional to the length

i.e. $R \propto l$ --- (i)

(ii) Inversely proportional to cross-sectional area,

i.e. $R \propto \frac{1}{A}$ --- (ii)

combining eqn (i) & (ii) we get

$R \propto \frac{l}{A}$

or $R = \frac{\rho l}{A}$ --- (iii)

$\Rightarrow \rho = \frac{RA}{l}$

Where, 'ρ' is a proportionality constant called resistivity. It is defined as the resistance of a conductor having unit cross-sectional area and unit length. Its SI unit is Ωm. Its value depends on nature of material and temperature.

We know that, from ohm's law,

$$V = IR$$

$$\text{or, } E \times l = I \times \frac{\rho l}{A} \qquad \left[\because V = E \times l \text{ and, } R = \frac{\rho l}{A} \right]$$

$E = V/d(\text{or } l)$ electric field

$$\text{or, } E = \frac{I}{A} \times \rho$$

$$\text{or } E = J \times \frac{1}{\sigma} \qquad \left[\sigma = 1/\rho \text{ is conductivity} \right]$$

$$\text{or } \boxed{J = \sigma E}$$

This is the relation between current density & electric field.

Joule's law of Heating:

The heat developed on a current carrying conductor is,

(i) Directly proportional to the square of current,

i.e. $H \propto I^2$ --- (i)

(ii) Directly proportional to resistance,

i.e. $H \propto R$ --- (ii)

(iii) Directly proportional to the time for which the current passes.

i.e. $H \propto t$ ... (iii)

Combining eqn (i) (ii) & (iii), we get,

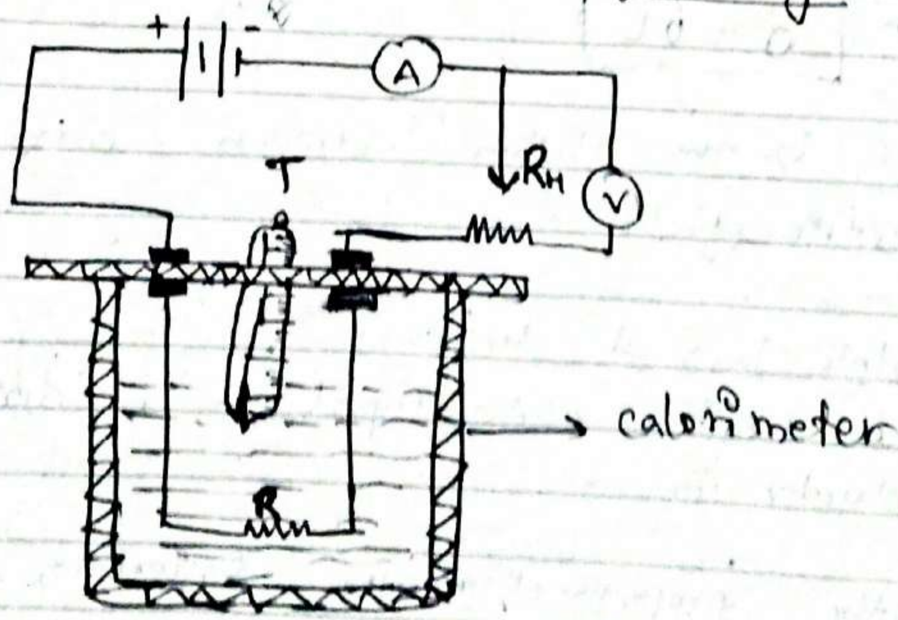
$H \propto I^2 Rt$

or, $\boxed{H = \frac{I^2 Rt}{J}}$

Where, 'J' is joule's mechanical equivalent of heat.

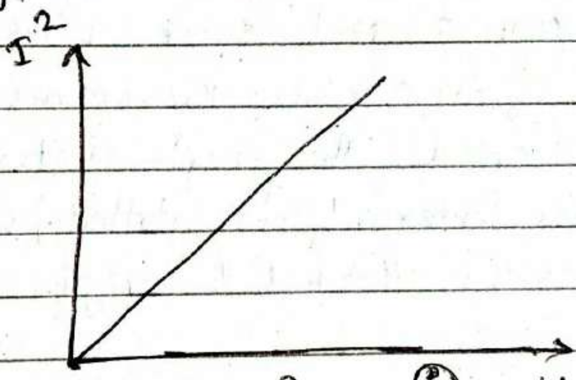
$\boxed{J = 4.2 \text{ J/cal}}$

(A) Verification of joule's law of Heating:



Consider a calorimeter filled with water half of its volume. A resistance wire is ~~emerged~~ immersed in the water which is connected to the external batt through a Rheostat.

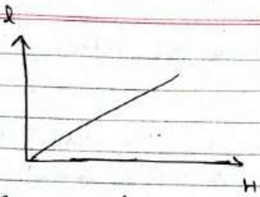
The experiment is done in three steps. At first the current through the resistor is ~~varied~~ varying with the help of Rheostat and the corresponding heat developed is measured by the help of temperature difference measured by the thermometer. When we plot the graph between heat developed and square of current, we get a straight line passes through the origin which is shown in fig. below:-



i.e. $H \propto I^2$ --(i) H

at constant 'R' and 't'.

In second step, the experiment is repeated for same amount of current and same interval of time with the resistance wire of different length but having same cross-section. The corresponding heat developed for the different length of wire are noted and the graph is plotted betn them, we get a straight line passes through the origin which is shown in fig. below.

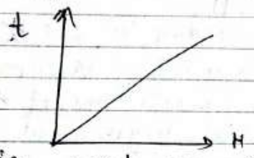


i.e. $H \propto l$

Also, we know that, $R \propto l$,

$\Rightarrow$ $H \propto R$ ---- (ii) at constant 'I' and 't'.

At last, the experiment is again repeated for the same amount of current and same wire but for different interval of time and the corresponding heat developed is noted. The graph between heat developed and time interval is plotted, which is the straight line passes through the origin is shown in fig. below,



i.e. $H \propto t$ ---- (iii) at constant 'I' & 'R'.

combining eqn (i), (ii) and (iii); we get,

$H \propto I^2Rt$

or, $\boxed{H = \frac{I^2Rt}{J}}$

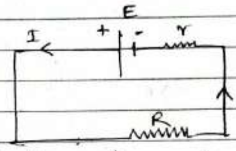
Where, 'J' is joule's mechanical equivalent of heat

$\boxed{J = 4.2 J/cal}$

Which verify the joule's law of heating.//

(*) Maximum power transfer Theorem:-

It states that, " The power delivered by a source to the external resistance is maximum, if the internal resistance of source and external resistance are equal."



Consider a battery of e.m.f of 'E' and internal resistance 'r' is connected to a external resistance 'R'. The current flowing through the circuit is,

$$I = \frac{E}{R+r}$$

Now, the power is,

$$P = I^2R$$

or, $$P = \left(\frac{E}{R+r}\right)^2 R$$

For power to be maximum;

$$\frac{dP}{dR} = 0$$

or, $E^2 \frac{(R+r)^2 - R\cdot 2(R+r)}{(R+r)^4} = 0$

or, $(R+r)^2 = 2R\cancel{(R+r)}$

or, $R+r = 2R$

or, $\boxed{R = r}$

Again,

$$\frac{d^2P}{dR^2} = -ve$$

So, the power is maximum, when $R = r$

This is maximum power transfer theorem.

Atomic view of resistivity.

⇒ let 'm' and 'e' are mass and charge of an electron respectively. If 'E' be the applied electric field on the conductor, the force on each electron is given by

$$F = eE$$

If 'a' be the acceleration produced on the conductor then from Newton's 2nd law of motion,

$$F = ma$$

or, $ma = eE$

or, $\boxed{a = \frac{eE}{m}}$ ---(i)

'Vd' be the drift velocity of electrons and 'τ' be average time between collision.

$$V_d = a\tau$$

or, $\boxed{V_d = \frac{eE\tau}{m}}$

The electron mobility is defined as the drift velocity per unit applied electric field.

i.e. electron mobility $(\mu) = \frac{V_d}{E}$

or, $\boxed{\mu = \frac{e\tau}{m}}$

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Also the current density is given by,

$J = neV_d$

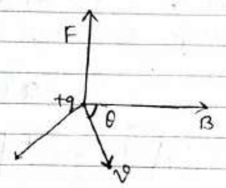
or, $\sigma E = \frac{ne \cdot eE\tau}{m}$ $[\because J = \sigma E]$

or, $\sigma = \frac{ne^2\tau}{m}$

or, $\rho = \frac{1}{\sigma}$

or, $\boxed{\rho = \frac{m}{ne^2\tau}}$ //.

Force on a charge moving in Magnetic field:
(Lorentz force)



(i) $F \propto q$
(ii) $F \propto B$
(iii) $F \propto v$
(iv) $F \propto \sin\theta$

combining them,

$F \propto Bqv\sin\theta$

$F = Bqv\sin\theta$

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In vector form,

$\vec{F} = q(\vec{v} \times \vec{B})$

(*) Force experienced by current carrying conductor in magnetic field:-

⇒ Consider a conductor of length 'l' and cross-sectional area 'A' carrying a current 'I' is placed in uniform magnetic field 'B' at an angle of 'θ' as shown in fig above.

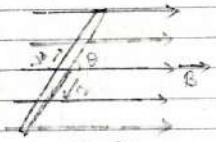


fig. (1)

The Lorentz force experienced by each electrons of the conductor is given by,

$F_e = BeV_d\sin\theta$

Where, V_d → drift velocity of electron

If 'n' be the no. of free electrons per unit volume of the conductor then the total no. of free electrons is,

$N = nAl$

Now, the force experienced by the conductor is,

$F = F_e \times N$

or, $F = BeV_d\sin\theta \times N$

or, $F = BeV_d\sin\theta \times nAl$

or, $F = B(neAV_d) \cdot l\sin\theta$

or, $\boxed{F = B \cdot I \cdot l\sin\theta}$ $[\because I = neV_dA]$

In vector form,

$\vec{F} = I(\vec{l} \times \vec{B})$, the direction of 'F' is determine by using Fleming's left hand rule.

(#) Torque experienced by current carrying ~~conductor~~ rectangular coil placed in magnetic field:-

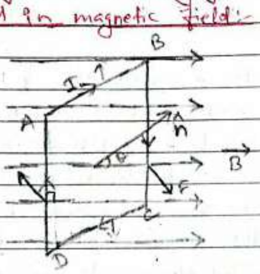


fig (a)

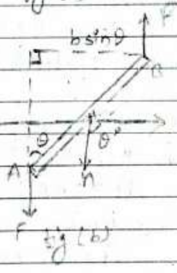


fig (b)

⇒ Consider a rectangular coil of length 'l' and breadth 'b' carrying a current 'I' is placed in uniform magnetic field 'B'. 'θ' be the angle made by normal of the coil to the magnetic field. Sides 'AB' and 'CD' experienced equal and opposite forces, they cancel to each other.

The force experienced by side AD is,

$F_1 = BIl \sin 90^\circ$

$\boxed{F_1 = BIl}$,

The direction of this force is determine using Flemings left hand rule which is perpendicular to plane of paper and directed inward.

The force experienced by the side BC is,

$F_2 = BIl \sin 90^\circ$

$\boxed{F_2 = BIl}$

The direction of this force is determine using Fleming's left hand rule which is perpendicular to plane of paper and directed outward.

These forces 'F_1' and 'F_2' are equal and opposite but their point of action is not same so they constitute a couple. The torque due to couple is given by,

$\tau = F_1 \times b \sin\theta$

or, $\tau = BIl \cdot b \sin\theta$

or $\tau = B \cdot IA \sin\theta \quad [\because A = l \times b]$

If the coil contain 'N' no. of turns, then the torque is given by,

$\boxed{\tau = BINA \sin\theta}$

(#)
Normal → $\sin\theta$
coil angle → $\cos\theta$

Hall Effect:

⇒ When a current carrying specimen is placed perpendicularly to uniform magnetic field B, an electric field or voltage is induced on the specimen perpendicular to both current and magnetic field. The voltage is called Hall-voltage and the phenomenon is called Hall-Effect.

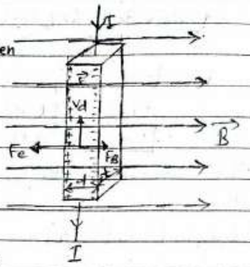


Fig (i)

Consider a current carrying conductor of cross-sectional area 'A', width 'd' and thickness 't' carrying a current 'I' placed in a uniform magnetic field 'B' perpendicularly. The free electrons of the conductor experienced a Lorentz force and collected to the right edge of the conductor leaving positive charge to the left edge which create an electric field 'E' due to which electron experienced a force 'Fe' which is opposite to the force due to magnetic field.

At equilibrium,

$$F_e = F_B$$

$$\text{or, } \cancel{e}E = B\cancel{e}V_d$$

$$\text{or, } E = B.V_d \quad \text{--- (i)}$$

At this condition, the electric field is called hall field and the voltage is called hall voltage.

We know that, the current density is given by,

$$J = -neV_d$$

$$V_d = \frac{-J}{ne}$$

Also, the hall voltage is,

$$V_H = E*d$$

$$\text{or, } \boxed{E = \frac{V_H}{d}}$$

putting the value of V_d in eqn (i)

$$E = B*\left(\frac{-J}{ne}\right)$$

$$\text{or, } \frac{E}{JB} = \left(-\frac{1}{ne}\right) \quad \text{--- (ii)}$$

The term $\frac{E}{JB}$ is called hall coefficient (R_H)

$$\text{i.e. } \boxed{R_H = \frac{E}{JB} = \frac{-1}{ne}}$$

Again,

$$I = neAV_d$$

$$\text{or, } V_d = \frac{I}{neA}$$

putting the value of 'E' and 'V_d' in eqn (i), we get,

$$\text{or, } \frac{V_H}{d} = B.\frac{I}{neA}$$

$$\text{or, } \frac{V_H}{\cancel{d}} = \frac{B*I}{ne\cancel{d}t}$$

$\Rightarrow \boxed{V_H = \frac{BI}{net}}$, This is Hall voltage.

(4) Biot - Savart law :

$\Rightarrow$ Consider a conductor carrying current 'I'. The magnetic field is induced around it. The magnetic field at a point 'P' at a distance 'r' from current carrying length element AB of length 'dl' is,

(i) Directly proportional to the length of length element.

i.e. $dB \propto dl$ --- (i)

(ii) Directly proportional to current,

i.e. $dB \propto I$ --- (ii)

(iii) Directly proportional to $\sin\theta$

i.e. $dB \propto \sin\theta$.

(iv) Inversely proportional to square of the distance,

i.e. $dB \propto \frac{1}{r^2}$ --- (iv)

Combining eqn (i), (ii), (iii) & (iv) ; we get,

$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

$$\boxed{dB = \frac{\mu_0}{4\pi} \times \frac{I\,dl\,\sin\theta}{r^2}}$$

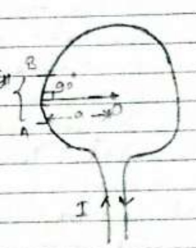
In vector form,

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \times \frac{I\,(\vec{r} \times d\vec{l})}{r^3}}$$

Applications :-

(1) Magnetic field at the centre of current carrying circular coil :

Consider a circular coil of radius 'a' carrying a current 'I' which produces the magnetic field around it. To find the magnetic field at the centre of coil. Take a length element 'AB' of length 'dl' of the coil.



According to Biot-Savart law, the magnetic field at 'O' due to the current on the length element is

$$dB = \frac{\mu_0}{4\pi} \times \frac{I\,dl\,\sin 90^\circ}{a^2}$$

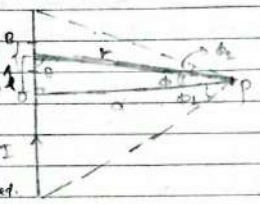
or, $$dB = \frac{\mu_0}{4\pi} \times \frac{I\,dl}{a^2}$$

The total magnetic field at 'O' due to current on whole coil is, $B = \int_0^{2\pi a} dB$

If the coil contains 'N' no. of turns,

$$\boxed{B = \frac{\mu_0 N I a^2}{2(r^2+a^2)^{3/2}}}$$

(iii) Magnetic field due to long straight conductor:



⇒ Consider a long straight conductor carrying a current 'I'. Take a point 'P' at a distance 'a' from the conductor at which the magnetic field is to be determined.
For this, Take a length element 'AB' of length 'dl' at a distance 'l' from 'O'.
According to Biot-Savart law. Magnetic field at 'P' due to current on 'AB' is,

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2} \quad \text{--- (1)}$$

From fig;

$$\sin\theta = \cos\phi = a/r$$

$$\text{or, } r = \frac{a}{\cos\phi} = a\sec\phi$$

Also,

$$\tan\phi = l/a$$

$$\text{or, } l = a\tan\phi$$

$$\text{or, } dl = a\sec^2\phi \cdot d\phi$$

Now, eqn (1) ; becomes,

$$dB = \frac{\mu_0 I}{4\pi} \times \frac{a\,\cancel{\sec^2}\phi\, d\phi \cdot \cos\phi}{a^2 \cancel{\sec^2}\phi}$$

$$= \frac{\mu_0 I}{4\pi a} \cos\phi\, d\phi$$

The total magnetic field due to the current in conductor is,

$$B = \int_{-\phi_1}^{\phi_2} dB$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \int_{-\phi_1}^{\phi_2} \cos\phi\, d\phi$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \left[\sin\phi\right]_{-\phi_1}^{\phi_2}$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \left[\sin\phi_2 - \sin(-\phi_1)\right]$$

$$\therefore \boxed{B = \frac{\mu_0 I}{4\pi a} \left[\sin\phi_2 + \sin\phi_1\right]}$$

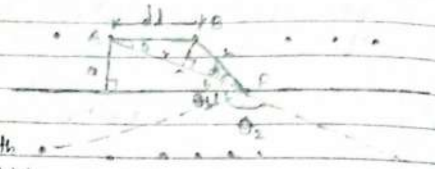
For infinitely long st. conductor,

$$\phi_1 = \phi_2 = 90^\circ$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a}[1+1]$$

$$\text{or, } \boxed{B = \frac{\mu_0 I}{2\pi a}}$$

Magnetic field due to solenoid :-



⇒ Consider a long solenoid having 'n' number of turns per unit length carrying a current 'I' and radius 'a' at a point 'P' on the axis of solenoid at which the magnetic field is to be determined. Let a length element 'AB' of length 'dl' at a distance of 'r' from 'P'

The magnetic field at 'P' due to a circular coil on length element 'AB' is,

$$\frac{\mu_0 I \, \sin\theta}{2 r^2}$$

The element AB contain 'ndl' no. of turns.

The magnetic field at 'P' due to AB is,

$$dB = \frac{\mu_0 I a}{2r^2} * \sin\theta * ndl \quad \text{--- (1)}$$

$$\left\{ \begin{array}{l} \underline{\text{cowm,}} \\ B = \dfrac{\mu_0 I a^2}{2(r^2+x^2)^{3/2}} \\ = \dfrac{\mu_0 I a}{2(r^2+x^2)} \cdot \dfrac{a}{(a^2+r^2)^{1/2}} \\ = \dfrac{\mu_0 I a}{2(r^2+x^2)} \cdot \sin\theta \end{array} \right.$$

In $\triangle ABC$, $\sin\theta = \frac{BC}{AB}$

or, $\sin\theta = \frac{r d\theta}{dl}$ $\left[\because d\theta = \frac{BC}{BP}\right]$ $\left[\Rightarrow BC = BP \cdot d\theta\right]$

or, $dl = \frac{r d\theta}{\sin\theta}$

putting the value of dl in eqn (1),

$$dB = \frac{\mu_0 I a}{2r^2} * \sin\theta * n * \frac{r d\theta}{\sin\theta}$$

$$= \frac{n\mu_0 I}{2}\left(\frac{a}{r}\right) d\theta$$

$$= \frac{n\mu_0 I}{2} \sin\theta \, d\theta$$

Now,

total magnetic field at 'P' due to current in whole solenoid

$$B = \int_{\theta_1}^{\theta_2} dB$$

or, $B = \frac{n\mu_0 I}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$

or $B = \frac{n\mu_0 I}{2} \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$

or, $B = \frac{n\mu_0 I}{2} \left[\cos\theta_1 - \cos\theta_2\right]$

For infinitely long st solenoid,

$\theta_1 = 0$, and $\theta_2 = 180°$.

or, $B = \frac{n\mu_0 I}{2}\left[1 - (-1)\right]$

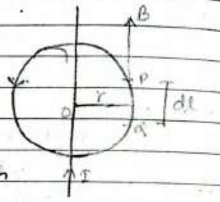
$$\boxed{B = n\mu_0 I}$$

Ampere's circuital law:

⇒ It states that, "The line integral of magnetic field around a closed loop is equal to the 'μ_0' times current enclosed by it."

i.e. $$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

proof:



Consider a straight conductor carrying a current 'I'. The magnetic field is produced around it. Draw a circular loop of radius 'r' which enclose the conductor. Take a length element 'PQ' of length 'dl' which is parallel to the magnetic field. (i.e. $\theta=0$).

Now,

$$\oint \vec{B}\cdot d\vec{l} = \int_0^{2\pi r} B\,dl\cos 0^\circ$$

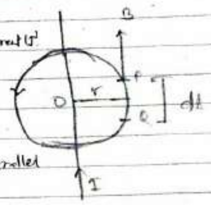
or $$\oint \vec{B}\cdot d\vec{l} = \int_0^{2\pi r} \frac{\mu_0 I}{2\pi r}\cdot dl \quad \left[\because B = \frac{\mu_0 I}{2\pi r} \text{ for st. conductor}\right]$$

or $$\oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi r} \times 2\pi r$$

$\therefore$ $$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$$ proved.

Applications:

Magnetic field due to straight conductor:-



⇒ Consider a straight conductor carrying current 'I'. The magnetic field is produced around it. Draw a circular loop of radius 'r' which enclose the conductor. Take a length element 'PQ' of length 'dl' which is parallel to the magnetic field. (i.e. $\theta=0°$).

$\therefore$ We know that, from Ampere's circuital law;

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

or $$\int_0^{2\pi r} B\,dl\cos 0^\circ = \mu_0 I$$

or $$B \times 2\pi r = \mu_0 I$$

or $$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

(4) Ampere's law in differential form:-

We know that, the ampere's law is,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$J = I/A$

Also,

Current in terms of current density,

$$I = \iint_S \vec{J}\cdot d\vec{s}$$

or, $$\oint \vec{B}\cdot d\vec{l} = \mu_0 \iint_S \vec{J}\cdot d\vec{s} \quad \text{---- (i)}$$

Using stoke's theorem,

$$\oint \vec{B}\cdot d\vec{l} = \iint_S (\nabla\times\vec{B})\cdot d\vec{s}$$

$\iint_S \vec{A}\cdot d\vec{s} = \iiint_V (\nabla\cdot\vec{A})$

Now, eq^n (i) becomes,

$$\iint_S (\nabla\times\vec{B})\cdot d\vec{s} = \mu_0 \iint_S \vec{J}\cdot d\vec{s}$$

or, $$\iint_S (\nabla\times\vec{B} - \mu_0\vec{J})\cdot d\vec{s} = 0$$

since, $d\vec{s} \neq 0$, so,

$$\nabla\times\vec{B} - \mu_0\vec{J} = 0$$

$$\Rightarrow \boxed{\nabla\times\vec{B} = \mu_0\vec{J}}$$

This is Ampere's law in differential form.

(4) Faraday's law of electromagnetic induction:-

(1) Whenever the magnetic flux linked with a closed circuit changes an emf is induced in the circuit.

(2) The induced emf lasts as long as the change in magnetic flux takes place.

(3) The magnitude of induced emf is directly proportional to the rate of change of magnetic flux.

ie $$\varepsilon \propto \frac{d\phi}{dt}$$

or, $$\varepsilon = -\frac{d\phi}{dt}$$

$\because \phi = B\cdot A$

$\Rightarrow \iint_S B\, d\vec{s} = \phi$

Also, $$\phi = \iint_S \vec{B}\cdot d\vec{s}$$

$$\Rightarrow \boxed{\varepsilon = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{s}}$$

This is faraday's law in integral form.

We know that, the emf is given by,

$$\varepsilon = \oint \vec{E}\cdot d\vec{l}$$

$$\left\{ \begin{array}{l} V = E \times d \\ d = \oint d\vec{l} \end{array} \right.$$

or, $$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\iint_s \vec{B}\cdot d\vec{s}$$

or, $$\oint \vec{E}\cdot d\vec{l} = -\iint_s \frac{d\vec{B}}{dt}\cdot d\vec{s} \quad \text{---- (i)}$$

Using stokes theorem;

$$\oint \vec{E}\cdot d\vec{l} = \iint_s (\nabla\times\vec{E})\cdot d\vec{s}$$

Now, eqn (i); becomes,

$$\iint_s (\nabla\times\vec{E})\cdot d\vec{s} + \iint_s \frac{d\vec{B}}{dt}\cdot d\vec{s} = 0$$

or, $$\iint_s \left(\nabla\times\vec{E} + \frac{d\vec{B}}{dt}\right)\cdot d\vec{s} = 0$$

Since, $d\vec{s} \neq 0$, so,

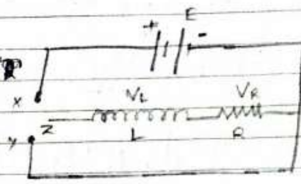
$$\nabla\times\vec{E} + \frac{d\vec{B}}{dt} = 0$$

or, $$\boxed{\nabla\times\vec{E} = -\frac{d\vec{B}}{dt}}$$

This is faraday's low in differential form.

(4) LR Circuit :-

⇒ Consider an inductor of inductance 'L' and resistor of resistance 'R' are connected in series through a battery of emf 'E'. When point 'x' is connected to 'z', the current through the circuit increases due to the self inductance of conductor, the flow of current opposes.



$$E = V_L + V_R$$

or, $$E = L\cdot\frac{dI}{dt} + IR$$

or, $$\frac{E}{R} = \frac{L}{R}\cdot\frac{dI}{dt} + I$$

or, $$I_0 = \frac{L}{R}\frac{dI}{dt} + I$$

or, $$I_0 - I = \frac{L}{R}\cdot\frac{dI}{dt}$$ [where $I_0 = E/R$ is maximum current in circuit]

Let, $$L/R = \tau$$

or, $$I_0 - I = \tau\cdot\frac{dI}{dt}$$

or, $$\frac{dI}{I_0 - I} = \frac{dt}{\tau} \quad \text{---- (i)}$$

For, $t = 0$, $I = 0$ and $t = t$, $I = I$.

Integrating eqⁿ ①, we get,

$$\int_0^I \frac{dI}{I_0 - I} = \int_0^t \frac{dt}{\tau}$$

or $\left[-\log(I_0 - I)\right]_0^I = \frac{t}{\tau}$

or $-\log(I_0 - I) + \log I_0 = \frac{t}{\tau}$

or $\log\left(\frac{I_0}{I_0 - I}\right) = \frac{t}{\tau}$

or $\frac{I_0}{I_0 - I} = e^{t/\tau}$

or $\frac{I_0 - I}{I_0} = e^{-t/\tau}$

or $1 - \frac{I}{I_0} = e^{-t/\tau}$

or $1 - e^{-t/\tau} = \frac{I}{I_0}$

or $\boxed{I = I_0 (1 - e^{-t/\tau})}$

This is the increase in current.

Fig:

The connection between 'x' and 'z' is removed and the point 'y' is connected to 'z', then the current start to decrease which is also opposed by self inductance of inductor.

Here,

$V_L = V_R$

or $-L \frac{dI}{dt} = IR$

or $\frac{dI}{I} = \frac{-dt}{L/R}$

or $\frac{dI}{I} = \frac{-dt}{\tau}$ $\left[\because \tau = L/R\right]$

for $t = 0$, $I = I_0$ and $t = t$, $I = I$

Integrating both sides we get,

$$\int_{I_0}^{I} \frac{dI}{I} = -\int_0^t \frac{dt}{\tau}$$

or $\left[\log I\right]_{I_0}^{I} = \frac{-t}{\tau}$

or $\log I - \log I_0 = \frac{-t}{\tau}$

or $\log\left(\frac{I}{I_0}\right) = -t/\tau$

or $\frac{I}{I_0} = e^{-t/\tau}$

or $\boxed{I = I_0 e^{-t/\tau}}$; This is decrease in current

Next sunday
20761

(1) Assignment Questions:

(1) State and explain Kirchhoff's law. Derive the balanced condition of wheatstone bridge.
(2) What is semiconductor. Discuss its types in detail.
(3) What is superconductor. Write its application.
(4) What is self inductance. Derive the expression of self inductance of solenoid.

(2) Numerical:-

(1) A conductor of uniform radius 1.2 cm carries a current of 3A due to potential gradient of 120 V/m. What the specific resistance of the material.?

(2) A wire resistance of 6 Ω is drawn out so that its new length is 3 times its original length. Find the resistance of the longer wire.

(3) Two conductors are made of same material and have the same length. Consider A is a solid wire of diameter 1mm. Conductor B is a hollow tube of outside and inside diameter 2mm and 1mm. Find the ratio of resistance of two wires R_A/R_B

(4) What percentage change in resistance, if its length increased by 1%.?
(5) A copper strip 0 cm wide and 1mm thick is placed in a magnetic field 1.5T. If a current 200 A is set up in the strip. Given hall voltage & hall mobility. If $n = 8.4\times10^{28} m^{-3}$ & $\rho = 1.72\times10^{-8} \Omega m$

Abhishek

Q(1) Soln: Given
radius $(r) = 1.2$ cm $= 1.2\times10^{-2}$ m

current $(I) = 3$ A
Potential gradient $(E) = 120$ V/m $= (V/l)$
specific resistance $(\rho) = ?$

We know that
$$\rho = \frac{RA}{l}$$
$$= \frac{\frac{V}{I}\cdot\pi r^2}{l} \qquad [\because R = V/I]$$
$$= \left(\frac{V}{l}\right)\cdot\frac{\pi r^2}{I} \qquad [\because A = \pi r^2]$$
$$= 120 \times \frac{\pi\times(1.2\times10^{-2})^2}{3}$$
$$= 0.0181$$

Q(2) Soln: Given
$R_1 = 6\Omega$
$l_1 = l$ (say)
$l_2 = 3l$
$R_2 = ?$

∴ We know that,
$$R \propto \frac{\rho l}{A}\times\frac{l}{l} = \frac{\rho l^2}{V}$$
i.e. $\boxed{R \propto l^2}$

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Hence,

$$\Rightarrow \frac{R_1}{R_2} = \frac{l_1^2}{l_2^2}$$

$$\Rightarrow \frac{6}{R_2} = \left(\frac{l}{3l}\right)^2$$

$$\Rightarrow \frac{6}{R_2} = \frac{1}{9}$$

$$\Rightarrow R_2 = 54\ \Omega$$

Q.(3) Sol^n

Given,

$$\rho_A = \rho_B, \quad d_o = 2\text{ mm}$$

$$l_A = l_B, \quad d_i = 1\text{ mm}$$

$$A_A = \frac{\pi d^2}{4} = \frac{\pi}{4}$$

$$A_B = \frac{\pi d_o^2}{4} - \frac{\pi d_i^2}{4}$$

$$= \frac{\pi}{4}(2^2 - 1^2)$$

$$= \frac{3}{4}\pi \text{ mm}^2$$

Now,

$$\frac{R_A}{R_B} = \frac{\rho_A \frac{l_A}{A_A}}{\rho_B \frac{l_B}{A_B}} = \frac{A_B}{A_A} = \frac{\frac{3}{4}\pi}{\pi/4} = 3:1$$

$$\therefore \boxed{R_A : R_B = 3:1}$$

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Q.(4) Sol^n, Given,

$$l_1 = l \quad (\text{let})$$

$$l_2 = l + 2\% \text{ of } l$$

$$= l + \frac{2}{100} \times l$$

$$= 1.02\, l$$

$$\%\text{ change in Resistance} = \left(\frac{R_2 - R_1}{R_1}\right) \times 100\%$$

$$= \left(\frac{l_2^2 - l_1^2}{l_1^2}\right) \times 100\%$$

$$= \frac{(1.02)^2 - (1)^2}{l^2} \times 100\%$$

$$= 0.0404$$

$$= 4.04\%$$

Q.(5) Sol^n,

$$v = \frac{BI}{net}$$

$$\mu = \frac{\sigma}{ne} = \frac{1}{\rho ne}$$

$$\because \mu = \frac{\frac{J}{ne}}{E}$$

$$= \frac{\sigma E}{neE}$$

$$= \frac{\sigma}{ne}$$

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Q.(6) A long circuler coil consisting 100 turns with diameter 1.2 m carries a current of 5A. (a) find the magnetic field at a point along the axis 80 cm from the centre. (b) At what distance from the centre along the axis is the field magnitude 1/8 as greater as at its centre.

Q.(7) If a wire of length 88 cm bends to form a square which carries a current of 50 A. Calculate the magnetic field at the centre of sphere.

Q.(8) A solenoid 1.3 long 2.6 cm in diameter carries a current of 18A. The magnetic field inside the solenoid is 23 mT. Find the length of the wire forming the solenoid.

Q.(9) A horizontal rod with mass 2kg and length 3m is aligned in a north-south direction. The rod carries current 2A in the direction towards the north. When the magnetic field is applied to the entire length, The force on the rod keeps it at rest. Find the magnitude of magnetic field.

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Q.(6) ⇒ Solⁿ: Given

$N = 100$

$d = 1.2\ m$

$a = 0.6\ m$

(a) $B = ?$ at $x = 80\ cm = 0.8\ m$

∵ We know that $B = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$

=

Given:-

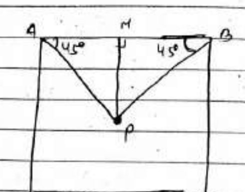
(b) $B_c = \frac{1}{8} B$, $x = ?$

$\Rightarrow \frac{\mu_0 N I}{2a} = \frac{1}{8} \times \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$

Theory

Q. (7) Sol^n

(Imp.)



Magnetic field at point 'P' due to AB is,

$= \frac{\mu_0 I}{4\pi (MP)} [\sin\phi_1 + \sin\phi_2]$

$= \frac{\mu_0 I}{4\pi l/2} (\sin 45° + \sin 45°)$

$= \frac{\mu_0 I}{2\pi l} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right)$

$= \frac{\mu_0 I}{\sqrt{2}\pi l}$

Now

the magnetic field at 'P' due to whole conductor,

$B = \frac{4 \times \mu_0 I}{\sqrt{2}\pi l}$

$\because 4l = 88$ cm

$\Rightarrow l = 22$ cm $\Rightarrow 0.22$ M

Q. (8) Sol^n:

R.

$d = 2.6$ cm

$a = 1.3$ cm $= 1.3 \times 10^{-2}$ m

$I = 18$ A

$B = 23$ mT $= 23 \times 10^{-3}$ T

Now, $B = \mu_0 n I$

or, $n = \frac{B}{\mu_0 I} = \frac{23 \times 10^{-3}}{4\pi \times 10^{-7} \times 18}$

$\therefore n =$

Again,

$N = n \times l =$

Also, length of wire $(L) = 2\pi a \times N =$

Here,

Q ⑨ Sol^n:- mass(m) = 2 kg

length(l) = 3m

current (I) = .2A

$\because$ For rod is at rest,

$mg = F_m$

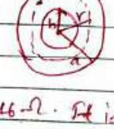
or, $mg = BIl$

or, $B = \dfrac{mg}{Il}$

or $B = \dfrac{2 \times 9.81}{2 \times 3}$

or $\boxed{B = 3.27 \text{ Tesla}}$

⑩ The figure shows a cross-section of hollow cylinder of radii 'a' and 'b' carrying current 'I'. Using the circular amperial loop verify $B = \dfrac{\mu_0 I (r^2 - b^2)}{2\pi r(a^2 - b^2)}$ for $b < r < a$



⑪ A wire has a resistance of 16 Ω. It is melted and drawn into a wire of half of its original length. Calculate the resistance of new wire. What is the %age change in resistance.

⑫ A p.d. of 1V applied to 100 mm long copper wire of diameter of 0.04 cm. Calculate current, current density, electric field strength and rate of joule's heating. ($\rho = 1.72 \times 10^{-8}$ Ωm

Q.⑩

Sol^n: $\Rightarrow$ Given, condition

$J = J'$

or, $\dfrac{I}{A} = \dfrac{I'}{A'}$ $\Rightarrow$ $I' \rightarrow$ current enclosed by circular amperial loop.

or, $\dfrac{I}{\pi(a^2 - b^2)} = \dfrac{I'}{\pi(r^2 - b^2)}$

or, $I' = \dfrac{I(r^2 - b^2)}{(a^2 - b^2)}$

Applying ampere's law,

$\oint \vec{B} \cdot \vec{dl} = \mu_0 I'$

or $\displaystyle\int_0^{2\pi r} B\,dl = \dfrac{\mu_0 I (r^2 - b^2)}{(a^2 - b^2)}$

or, $B \cdot 2\pi r = \dfrac{\mu_0 I (r^2 - b^2)}{(a^2 - b^2)}$

or, $\boxed{B = \dfrac{\mu_0 I (r^2 - b^2)}{2\pi r (a^2 - b^2)}}$ proved.

(11) Sol^n Here given

resistance of wire $(R) = 16\,\Omega$

$\because$ resistivity $(\rho) = \frac{RA}{l}$

$\Rightarrow R = \frac{\rho l}{A}$

$= \frac{\rho l^2}{Al} = \frac{\rho l^2}{V}$

(11) Electromagnetic wave :-

(1) Equation of continuity :-

We know that, the current is given by,

$$I = -\frac{dq}{dt}$$, '-ve' sign indicates that the current losses.

Also, the charge density is given by,

$$\rho = \frac{q}{V}$$

$$\Rightarrow q = \rho . V$$

$$\Rightarrow q = \iiint_V \rho . d\vec{v}$$

$$\text{or, } I = -\iiint_V \frac{d\rho}{dt} . d\vec{v}$$

$$\text{or, } \iint_S \vec{J} . d\vec{s} + \iiint_V \frac{d\rho}{dt} . d\vec{v} = 0$$

Using divergence theorem,

$$\iiint_V \nabla . \vec{J} \, d\vec{v} + \iiint_V \frac{d\rho}{dt} . d\vec{v} = 0$$

$$\text{or, } \iiint_V \left(\nabla . \vec{J} + \frac{d\rho}{dt} \right) . d\vec{v} = 0$$

Since, $d\vec{v} \neq 0$

$$\Rightarrow \boxed{\nabla . \vec{J} + \frac{d\rho}{dt} = 0}$$

This is eqn of continuity.

(4) Maxwell's Equation :-

(i) $\nabla . \vec{D} = \rho$; Gauss's law (where, $\vec{D} = \epsilon_0 \vec{E}$)

(ii) $\nabla . \vec{B} = 0$; Gauss's law of magnetism

(iii) $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$; Faraday's Law

(iv) $\nabla \times \vec{H} = \vec{J}$; Ampere's law (where, $H = \frac{B}{\mu_0}$)

These four set of equations are known as Maxwell's equations.

Here, first three equations are correct but fourth equation is only valid for steady state condition. This eqn is modified by Maxwell.

To modify the ampere's law maxwell introduce the concept of displacement current.

The ampere's law in ~~given by~~ differential form is,

$$\nabla \times \vec{H} = \vec{J}$$

taking divergence on both sides,

$$\nabla . (\nabla \times \vec{H}) = \nabla . J$$

The curl of a divergence of a vector qty. is zero.

i.e. $\nabla . (\nabla \times \vec{H}) = 0$

$$\Rightarrow \nabla . \vec{J} = 0 \quad --- (i)$$

But, from continuity eq^n;

$$\nabla \cdot \vec{J} + \frac{d\rho}{dt} = 0 \quad \text{---} \; (ii)$$

* eq^n (ii) only valid for steady state.

Again, we know that,

$$\nabla \cdot \vec{D} = \rho$$

putting the value of 'ρ' in eq^n (ii)

$$\nabla \cdot J + \frac{d(\nabla \cdot \vec{D})}{dt} = 0$$

$$\text{or, } \nabla \cdot \left(\vec{J} + \frac{d\vec{D}}{dt}\right) = 0 \quad \text{---} \; (iii)$$

Maxwell replaced $\vec{J}$ in ampere's by $\left(J + \frac{d\vec{D}}{dt}\right)$

Then, the modified ampere's law by maxwell's is,

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}}$$

The added $\frac{d\vec{D}}{dt}$ is known as displacement current.

Now, the modified maxwell's eq^n are:

(i) $\nabla \cdot \vec{D} = \rho$

(ii) $\nabla \cdot \vec{B} = 0$

(iii) $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$

(iv) $\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$

Maxwell's eq^n in integral form:

(i) $\nabla \cdot \vec{D} = \rho$

$\Rightarrow$ Taking volume integral in both sides;

$$\iiint_V \nabla \cdot \vec{D} \cdot d\vec{V} = \iiint_V \rho \cdot d\vec{V}$$

$$\Rightarrow \iiint_V \nabla \cdot \vec{D} \cdot d\vec{V} = q$$

Using divergence theorem,

$$\iint_S \vec{D} \cdot d\vec{S} = q$$

$$\text{or, } \iint_S \varepsilon_0 \vec{E} \cdot d\vec{S} = q$$

$$\text{or, } \boxed{\iint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}}$$

(ii) $\nabla \cdot \vec{B} = 0$,

$\Rightarrow$ ~~Using~~ taking volume integral on both sides;

$$\iiint_V \nabla \cdot \vec{B} \cdot d\vec{V} = 0$$

Using divergence theorem,

$$\boxed{\iint_S \vec{B} \cdot d\vec{S} = 0}$$

(iii) $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$

⇒ Taking surface integral on both sides,

$$\iint_S (\nabla \times \vec{E})\, d\vec{s} = -\iint_S \frac{d\vec{B}}{dt} \cdot ds$$

Using stoke's theorem,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{s}$$

$$\Rightarrow \boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt}}$$ [Φ → Magnetic flux]

(iv) $\nabla \times \vec{H} = \left(\vec{J} + \frac{d\vec{D}}{dt}\right)$

⇒ Taking surface integral on both sides;

$$\iint_S (\nabla \times \vec{H}) \cdot d\vec{s} = \iint_S \left(\vec{J} + \frac{d\vec{D}}{dt}\right) d\vec{s}$$

Using stoke's theorem,

$$\boxed{\oint \vec{H} \cdot d\vec{l} = \iint_S \vec{J} \cdot d\vec{s} + \iint_S \frac{d\vec{D}}{dt} \cdot ds}$$

$$\Rightarrow \oint \vec{H} \cdot d\vec{l} = I + \frac{d}{dt} \iint_S \varepsilon_0 \vec{E} \cdot d\vec{s}$$

$$\Rightarrow \oint \vec{H} \cdot d\vec{l} = I + \varepsilon_0 \frac{d\Phi}{dt}$$ [Φ → Electric flux]

or, $\oint \frac{\vec{B}}{\mu_0} \cdot d\vec{l} = I + \varepsilon_0 \frac{d\Phi}{dt}$

$$\Rightarrow \boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi}{dt}}$$

(1) Wave Eqn in free space :-

⇒ In free space, $\rho = 0$, $\vec{J} = 0$, ε_0 & μ_0

Now, Maxwell eqn's are;

(i) $\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0$ --- (i)

(ii) $\nabla \cdot \vec{B} = 0$

(iii) $\nabla \times \vec{E} = -\frac{dB}{dt}$ --- (iii)

(iv) $\nabla \times \vec{H} = \frac{d\vec{D}}{dt}$ -

or, $\nabla \times \frac{\vec{B}}{\mu_0} = \varepsilon_0 \frac{d\vec{E}}{dt}$

or, $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$ ----- (iv)

Taking curl on both sides of eqn (iii)

$$\nabla \times (\nabla \times \vec{E}) = -\frac{d}{dt} (\nabla \times \vec{B})$$

We know that, the vector identity,

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B})$$

Now, $\nabla (\nabla \cdot \vec{E}) - \vec{E} (\nabla \cdot \nabla) = -\frac{d}{dt} (\nabla \times \vec{B})$

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Using eqⁿ (i) & (iv) :

$$\nabla \times 0 - \nabla^2 \vec{E} = -\frac{d}{dt}\left(\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}\right)$$

or $$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}}$$

Eqⁿ of electric field in free space.

Comparing this eqⁿ with, we get

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{d^2 E}{dt^2}$$

or $$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

or $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

So. The EM wave travel in free space with velocity of light.

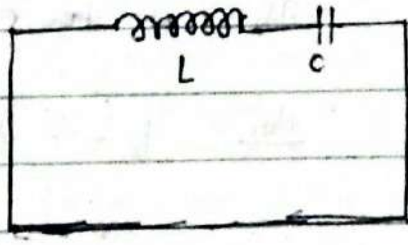
Similarly in terms of magnetic field

$$\boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{d^2 B}{dt^2}}$$

(ii) Wave eqⁿ in non-conducting medium ($\rho = 0, J = 0, \varepsilon_0 \& \mu_0$)

⊕ Electromagnetic Oscillation:-

(a) Free EM oscillation :-



⇒ The inductor and capacitor in series forms a free EM oscillation. Consider a inductor of inductance 'L' and capacitor of capacitance 'C' are connected in series.

The total energy is given by,

$$U = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} L I^2$$

since, the circuit has no resistance the total energy is always constant.

$$\frac{dU}{dt} = 0$$

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or $\frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2\right) = 0$

or, $\frac{1}{2} \times \frac{2Q}{C} \times \frac{dQ}{dt} + \frac{1}{2} L \times 2I \cdot \frac{dI}{dt} = 0$

or $\frac{Q \cdot I}{C} + LI \frac{dI}{dt} = 0$

or $\frac{Q}{LC} + \frac{dI}{dt} = 0$

or $\boxed{\frac{d^2Q}{dt^2} + \frac{1}{LC} Q = 0}$

This is the eqn of free LC oscillation.
its soln is,

$Q = Q_0 \sin(\omega t + \phi)$

This ~~is the~~ eqn is similar to, $\frac{d^2y}{dt^2} + \omega^2 y = 0$

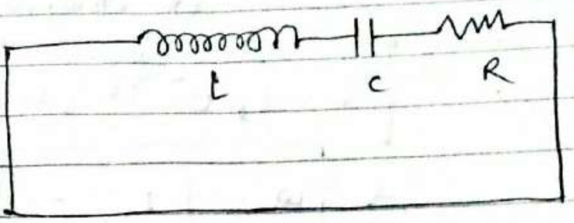
Now, $\omega^2 = 1/LC$

or $\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}}$

(4) Damped EM oscillation :-

⇒ The series combination of inductor 'L', capacitor 'C' and a resistor 'R' forms the damped EM oscillation.



Here,

$$V_L + V_C + V_R = 0$$

or. $$L\frac{dI}{dt} + \frac{Q}{C} + IR = 0$$

or $$\frac{dI}{dt} + \frac{Q}{LC} + \frac{R}{L}I = 0$$

⇒ $$\boxed{\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = 0}$$

This is the eq^n of damped EM oscillation.

This eq^n is similar to the eq^n,

$$\frac{d^2y}{dt^2} + 2\delta\frac{dy}{dt} + \omega_0^2 y = 0$$

then,

$$2\delta = R/L \quad , \quad \omega_0^2 = 1/LC$$

$$\Rightarrow \delta = \frac{R}{2L}$$

The sol^n is given by, $y = a e^{-\delta t}\sin(\omega t + \phi)$

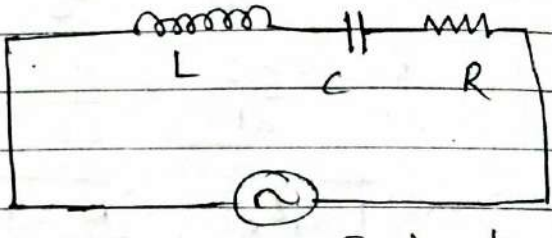
$$\& \quad \omega = \sqrt{\omega_0^2 - \delta^2}$$

The sol^n of damped EM oscillation,

$$\boxed{q = q_0 e^{-\frac{Rt}{2L}} \sin(\omega t + \phi)}$$

& $\boxed{\omega = \sqrt{1/LC - \frac{R^2}{4L^2}}}$

(4) Forced EM oscillations



⇒ The series combination of inductor 'L', capacitor 'C' and a resistor 'R' with an ac source forms the forced EM oscillation.

Here, $V_L + V_C + V_R = E_0 \sin\omega t$

or, $L\frac{dI}{dt} + \frac{Q}{C} + IR = E_0 \sin\omega t$

or, $\boxed{\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = f\sin\omega t}$ where $f = \frac{E_0}{L}$

This is the eq^n of forced EM oscillation.

This eq^n is similar to;

$$\frac{d^2y}{dt^2} + 2\delta\frac{dy}{dt} + \omega_0^2 y = f\sin\omega t$$

then,

$$2\delta = R/L \quad , \quad \omega_0^2 = 1/LC \quad , \quad f = \frac{E_0}{L}$$

$$\Rightarrow \delta = \frac{R}{2L}$$

The sol^n is given by,

$$y = y_m \sin(\omega t + \phi)$$

where

$$y_m = \frac{f}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\delta\omega)^2}}$$

The sol^n of forced EM oscillation.

$$q = Q_0 \sin(\omega t + \phi)$$

and

$$Q_0 = \frac{f}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\delta\omega)^2}}$$

or,

$$Q_0 = \frac{E_0/L}{\sqrt{(\omega^2 - 1/LC)^2 + \left(\frac{R\omega}{L}\right)^2}}$$

we know that,

$$I = \frac{dq}{dt} = Q_0 \omega \sin(\omega t + \phi)$$

or,

$$I = I_0 \sin\omega t$$

Where,

$$I_0 = Q_0 \omega$$

or $$I_0 = \frac{E_0 \omega}{L\sqrt{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{R\omega}{L}\right)^2}}$$

or $$I_0 = \frac{E_0}{\frac{L}{\omega} \times \sqrt{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{R\omega}{L}\right)^2}}$$

or $$I_0 = \frac{E_0}{\sqrt{\frac{L^2}{\omega^2}\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{R\omega}{L}\right)^2 \times \frac{L^2}{\omega^2}}}$$

or $$I_0 = \frac{E_0}{\sqrt{\left(\frac{L}{\omega} \times \omega^2 - \frac{L}{\omega} \times \frac{1}{LC}\right)^2 + \frac{R^2 \cancel{\omega^2}}{\cancel{L^2}} \times \frac{\cancel{L^2}}{\cancel{\omega^2}}}}$$

or $$I_0 = \frac{E_0}{\sqrt{\left(L\omega - \frac{1}{\omega C}\right)^2 + R^2}}$$

or, $$I_0 = \frac{E_0}{\sqrt{\left(X_L - X_C\right)^2 + R^2}}$$

or $$\boxed{I_0 = \frac{E_0}{Z}}$$

Where,

$Z = \sqrt{(X_L - X_C)^2 + R^2}$ is the impedence of LCR circuit.

(ii) Resonance :-

⇒ The resonance occurs when the maximum current flows through the circuit.

For maximum current in the circuit,

$X_L = X_C$

or $L\omega_r = \frac{1}{\omega_r C}$

or, $\omega_r^2 = \frac{1}{LC}$

or, $\omega_r^2 = \frac{1}{\sqrt{LC}}$

or, Resonance frequency is,

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}}$$

(ii) Quality Factor :-

⇒ The ratio of voltage drop across inductor or across capacitor to the voltage drop across resistor at resonance is called quality factor.

i.e. $Q = \frac{V_L \text{ or } V_R}{V_R}$ at resonance.

or, $Q = \frac{I \times X_L}{IR}$

or $Q = \frac{L\omega_r}{R}$

or, $Q = \frac{L}{R} \times \frac{1}{\sqrt{LC}}$

or. $\boxed{Q = \frac{1}{R} \sqrt{L/C}}$

THE END